

An experimental setup to verify stokes' law using an electronic balance

M. Dolz*, A. Casanovas, J. Delegido, and M.J. Hernández

*Departament de Termodinàmica, Facultat de Farmàcia i Física, Universitat de València,
46100 Burjassot, València, Spain,*

*e-mail: mdolz@uv.es, Alejandro.Casanovas@uv.es,
Jesús.Delegido@uv.es, mj@uv.es*

Recibido el 4 de marzo de 2003; aceptado el 9 de octubre de 2003

An experimental setup has been designed to allow direct measurement of the force acting upon a sphere immersed in a moving fluid. This experiment confirms the direct proportionality between the force and each of the variables involved (sphere–fluid relative velocity, sphere radius and fluid viscosity). A simple experimental demonstration has been made of Stokes' law for spheres immersed in fluids moving, for very small Reynolds numbers, along cylindrical tubes.

Keywords: Stokes law; students laboratory; particle drag.

Se ha diseñado un montaje experimental que permite la medida directa de la fuerza que actúa sobre una esfera sumergida en un líquido en movimiento. Mediante este experimento se confirma la relación de proporcionalidad directa entre la fuerza y cada una de las variables implicadas (velocidad relativa esfera-fluido, radio de la esfera y viscosidad del fluido). Se ha realizado una sencilla demostración experimental de la ley de Stokes para esferas introducidas en el seno de fluidos en movimiento en un tubo cilíndrico, para números de Reynolds muy pequeños.

Descriptores: Ley de Stokes; laboratorio docente; fuerza sobre partículas.

PACS: 47.15, 47.80

1. Introduction

Particle movement within fluids is involved in the study of transport properties [1,2], the determination of diffusion coefficients and molecular weights [3-5], the electric mobility [6], the analysis of particle sedimentation of poly-disperse systems [7], the gel electrophoresis, centrifugal methods [7,8] and erythrocyte sedimentation rate determinations [9]. These phenomena are of great interest both in basic sciences, like chemistry, physics or geology, and in biomedical sciences [10,11] or different areas of engineering. Therefore, it would be desirable for undergraduate students to have some understanding of the physical variables involved.

Particles moving through a fluid experience a resistive drag force, which is velocity and viscosity dependent. The most common and familiar expression for this kind of force is [2,4,12] the Stokes' law of the force acting on a sphere,

$$F = 6\pi R\eta v, \quad (1)$$

where R is the sphere radius, η is the fluid viscosity and v is the relative speed sphere-fluid. Expression (1) is derived for infinite media and a Reynolds number much lesser than unity (Stokes' flow) [7]. A way to obtain such reduced Reynolds number is considering very small solid spheres, as in the case of suspensions.

A number of expressions have been developed for correcting Stokes' law (1) for slightly greater Reynolds numbers or finite media (*e.g.*, end effects) [2,13,14]. In any case, the presence of a limiting wall increases the force factor 6π due to increased velocity gradients. Dimensional analysis [10,15] shows that for a sphere of radius R much less than the shortest distance to a vertical wall [13], the force is proportional

to R , η and v ,

$$F = KR\eta v, \quad (2)$$

where K is a constant to be determined by experiment [10]. In infinite media [case of (1)] K is exactly 6π .

Different methods for measuring Stokes' force have been developed, involving indirect approaches such as the use of high-cost falling ball viscometers [13] or direct, but imprecise, techniques such as falling spheres within appreciably long and wide tubes. A variation of the latter approach is to use an Atwood machine to control the sphere limit velocity, but with added pulley-friction problems [16].

For introductory courses students, it is well known the Stokes' law for the drag force on spheres. However, usually, this law is introduced in its final form without any demonstration or experimental verification, due to the difficulty of both things [12,17]. Students simply accept the law and its consequences and applications.

The importance of the phenomena related to this law substantiates the existence of a specific experiment to study it. It is, however, not easy to measure the force on a moving body [17]. We propose here a simple experiment for directly measuring the force acting upon a sphere immersed in a moving fluid using an electronic balance.

The proposed experiment shows the friction force for different fluid velocities, different fluid viscosities and different sphere radii. The experiment stands out the linear dependence of Stokes' force.

2. Material and methods

The present study describes a setup designed to allow, both, variations in solid-fluid velocity and a direct, and highly sensitive, measurement of the force acting upon the sphere. The measurement device (Fig. 1) consists of an electronic balance fitted with a hook for hydrostatic measurements at its lower end [18], from where a steel ball is suspended from a very thin nylon thread. The control of the fluid consists of a cylindrical glass flask (CF) joined by means of a capillary tube (T) to another glass flask (RF).

The electronic balance minimum resolution should be 1 mg, allowing for about 1×10^{-2} mN measurement error of the force acting upon the sphere. A Mettler Toledo PB 303 electronic balance with 0.001 g resolution is used.

The glass system formed by CF (60 mm internal diameter and 250 mm in height) and RF (110 mm diameter and 160 mm height), yielding a compact system, has been manufactured at the University of Valencia glass-workshop. A and B are Teflon valves.

A vacuum pump with vacuum regulator (Dinko D95) is used in RF to keep the fluid descent-rate in CF practically constant. By using a pump, the pressure difference due to fluid height changes in CF, ρgh , (about 0.01 atm) is negligible compared with the pressure difference (about 1 atm) found between the two ends of the tube T.

The steel balls used have radius of 5, 7.5 and 10 mm.

As the experiment requires using liquids with viscosities that are independent of fluid velocity (Newtonian liquids), we have chosen solutions of glycerol and water. Moreover, these solutions are convenient because they allow an easy cleaning of the system. In the test of the setup we used chemical

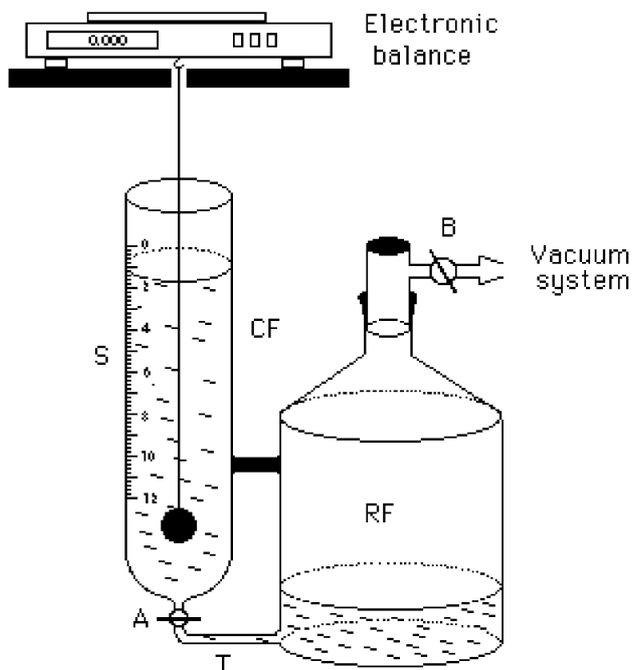


FIGURE 1. Experimental setup for measuring Stokes' fluid friction force upon a sphere.

TABLE I. Viscosity of the different mixtures of glycerol and water used, at room temperature.

Solution	$\eta / \text{Pa s} \pm 0.010$
1	0.774
2	0.478
3	0.254
4	0.200
5	0.141

analytical grade glycerol R.P. NORMAPUR AR from PRO-LABO, batch number J326. Anyway, any other Newtonian liquid, not necessarily of analytical grade, can be used. Oils give good results, but are more difficult to clean.

One advantage of glycerol is its high viscosity (about 1000 times the viscosity of water), which decreases easily when mixing it with very small amount of water. Five mixtures of glycerol and water have been used, with viscosity ranging from about 0.8 to 0.1 Pa s. The most dilute mixture corresponds to a proportion of about 1/10 (w/w) water in glycerol. The viscosities of these solutions, numbered from 1 to 5 (Table I) has been determined at room temperature with a rotary viscometer, Brookfield DV-II.

The measurement method is as follows:

First, the balance must be zeroed after suspending the sphere from the lower hook of the balance and immersing it, centered, in the CF tube filled with the test liquid.

Afterwards, the vacuum is turned on while opening valve B and keeping valve A closed. Then, when valve A is opened, the fluid flows from CF to RF at constant speed (regulated by Poiseuille law), and the force (in mg units) acting upon the sphere can be read on the balance display and recorded.

Simultaneously, the time t required for the fluid surface to travel a distance d marked on a scale S on the CF tube must be measured. The time is measured between two marks, to avoid any initial transitory. The ratio d/t provides an estimate of the relative fluid-sphere velocity, based on the average fluid velocity. A broad range of velocities can be obtained with different openings of the valve A, or if you have a vacuum regulator, with the vacuum level.

To repeat the measurement the fluid is returned from RF to CF. To this end, the vacuum system is inverted to increase the pressure in RF, and the fluid will come back through T.

Changing of the measured fluid requires, thorough cleaning of the entire system: CF, RF, tube T and sphere.

3. Results and discussion

3.1. Dependency of viscous force on fluid velocity and sphere radius

The former described method has been used to measure the force acting upon the sphere immersed in glycerol (solution 1) for five different fluid velocities and the three different

spheres. Figure 2 shows the measured force as a function of fluid velocity. For each sphere radius, the experimental points are on straight lines going through the origin:

$$F = K_1 v. \tag{3}$$

The best values for K_1 were obtained by the least squares method. These ones, together with the correlation coefficients for each of the spheres are shown in Table II. Therefore, we can show by the experiment that for a specific liquid and sphere radius, the viscous force acting upon the sphere is directly proportional to the relative sphere-fluid velocity.

The experiment corroborates linear dependence of Eqs. (1) and (2) with very good approximation for constant values of η and R , which is our case. Besides, Table II clearly shows that the K_1 values increase for spheres of increasing radius. Representing K_1 values as a function of sphere radius, we obtain again a straight line passing through the origin. This can be assured, e.g., by adding the point (0,0). In fact, (0,0) is an obvious point, because it is no force at $R=0$. This is more pedagogical than using an 'ad hoc' least square fit without an independent term. The linear fit

$$K_1 = K_2 R, \tag{4}$$

gives $K_2 = (70.4 \pm 0.3) \text{Pa s}$, with $r^2 = 0.998$. Therefore, combination of the relations (3) and (4) provides

$$F = K_2 R v, \tag{5}$$

for η constant, in agreement with Eq. (2). Therefore the student can conclude, by experiment, that the force is not only linearly related to velocity but also to the radius of the sphere.

TABLE II. Straight-line slopes (K_1) corresponding to each of the spheres (R = radius).

$R/10^{-3}m$	K_1/Nsm^{-1}	r^2 coefficient
5.0	0.289 ± 0.005	0.998
7.5	0.543 ± 0.007	0.998
10.0	0.725 ± 0.004	0.999

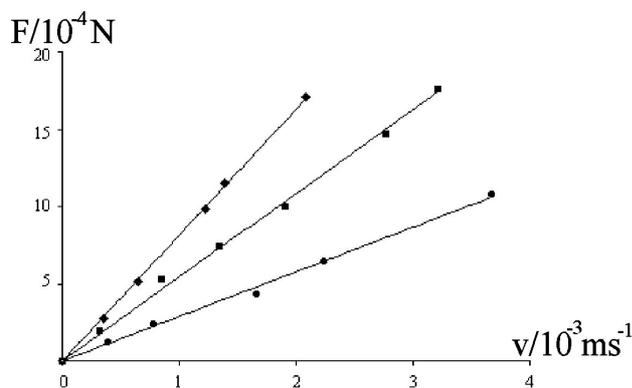


FIGURE 2. Friction force as a function of relative sphere-fluid velocity, for spheres of radius: (●) 5 mm; (■) 7.5mm; (◆)10 mm.

3.2. Dependency of viscous force upon fluid viscosity

Now, in order to complete the check of the linear dependence of Eq. (1), now on viscosity, the radius R will be kept unchanged and the experiment will be repeated using five different fluids, varying the viscosity of the medium.

Therefore, measurements have been made of the force acting upon the 1 cm radius sphere for five different fluid velocities and the five glycerol aqueous solutions indicated in Table I. The data of the viscous force, as a function of velocity v , for the different mixtures are represented in Fig. 3, and the fit of

$$F = K_3 v \tag{6}$$

yielded correlation coefficients over 0.997 in all cases.

As in the previous section, the K_3 slopes have been plotted in Fig. 4 as a function of viscosity. The linear least squares fitting provides the following equation:

$$K_3 = K_4 \eta \tag{7}$$

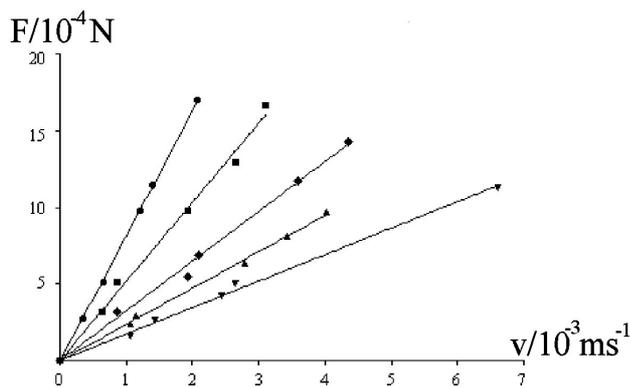


FIGURE 3. Friction force upon a sphere of $R = 10$ mm versus velocity, for the glycerol aqueous solutions indicated in Table I: (●) solution 1; (■) solution 2; (◆)solution 3 (▲) solution 4; (▼) solution 5.

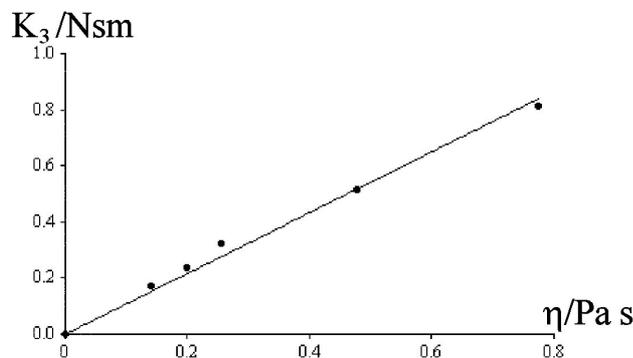


FIGURE 4. Slope of the straight lines in Fig.3 versus fluid viscosity.

with $K_4 = (1.08 \pm 0.03)m$, and $r^2 = 0.995$. That means the force is also directly linearly related to the fluid viscosity, *i. e.*,

$$F = K_4 \eta v \quad (8)$$

Expression again in agreement with Eqs. (1) and (22) for a constant value of sphere radius.

Therefore, at this point, after having obtained expressions (5) and (8), the student confirms by the experiment that the force acting upon a sphere in relative movement with respect to the surrounding fluid is directly proportional to sphere radius, viscosity and fluid velocity, as Stokes' law indicates.

4. Conclusions

Laboratory experiments in introductory courses are mainly thought to obtain a numerical result, a measurement of a physical quantity. However, introducing students in physics research is also one of the goals of laboratory classes. In that sense we think it is interesting to be aware of the relationships between the different variables involved in a physical law and to verify them experimentally, *i. e.*, to visualize them.

The conducted experiences show by experiment the linear dependence of the force acting upon a solid sphere moving in a fluid at very low Reynolds numbers. The experimental setup shows this force to be directly proportional to sphere radius, viscosity and fluid velocity, allowing for an elegant and relatively simple experimental verification of Stokes' fluid resistance force in tubes of finite diameter. Linearity has been confirmed for sphere radii of less than 1/3 the radius of the tube containing it.

The experimental setup has been designed to use inexpensive, like chronometers and glassware, or ubiquitous (like the balance) laboratory materials. The setup directly measures the force acting upon a sphere moving in a fluid, and the fluid velocity can be precisely controlled by simple means. The viscosity of the fluid medium and the radius of the sphere are parameters that can also be modified. Finally, the setup is practical for the amount of fluid involved and the size of the tubes of the experimental setup, much smaller than in previous arrangements.

Acknowledgments

We thank Marcelo Arias for the accurate glass model, Vicente Zorrilla for his helpful technical collaboration and Ignacio Gascó for his enthusiastic support.

*. To whom correspondence should be addressed.

1. K.E. Van Holde, *Physical Biochemistry* (Prentice-Hall Inc. New Jersey, 1971) p. 122.
2. R.B. Bird, W.E. Stewart, and E.N. Lightfoot, *Transport Phenomena* (John Wiley & Sons, New York, 1960) p. 6.11.
3. P.W. Atkins, *Physical Chemistry* (Oxford University Press. Oxford, 1998) p. 688.
4. J.I. Tinoco, K. Sauer, and J.C. Wang, *Physical Chemistry* (Prentice-Hall, New Jersey, 1995) p. 277.
5. J.T. Edward, *J. Chem. Educ.* **47** (1970) 261.
6. G.K. Vemulapalli, *Physical Chemistry* (Prentice Hall International, New Jersey, 1993) p. 944.
7. E.D. Schukin, A.V. Pertsov, and E.A. Améline, *Química coloidal* (Mir. Moscow, 1988) p. 166.
8. H.B. Hansall, and J.R. Wermeling, *J. Chem. Educ.* **59** (1982) 1076.
9. J.R. Shanebrock, *Am. J. Phys.* **46** (1978) 306.
10. J.W. Kane and M.M. Sternheim, *Física* (Reverté, Barcelona, 1989) p. 324.
11. J. Jou, J.E. Llebot, and C.P. García, *Física para las ciencias de la vida* (McGraw-Hill. Madrid, 1994) p. 167.
12. M.A. Lauffer, *J. Chem. Educ.* **58** (1981) 250.
13. J.R. van Wazer, J.W. Lyons, K.Y. Kim, and R.E. Colwell, *Viscosity and Flow Measurement. A Laboratory Handbook of Rheology* Interscience Publishers (John Wiley & Sons, New York, 1963) p. 272.
14. D. Auerbach, *Am. J. Phys.* **56** (1988) 850.
15. L. Landau and E. Lifchitz, *Física Teórica 6. Mecánica de Fluidos* (Reverté, Barcelona, 1986) p. 74.
16. M.S. Greenwood, F. Fazio, M. Russotto, and A. Wilkosz, *Am. J. Phys.* **54** (1986) 904.
17. G.T. Fox, *Am. J. Phys.* **39** (1971) 947.
18. M. Dolz, J. Delegido, M.J. Hernández, and J. Pellicer, *J. Chem. Educ.* **78** (2001) 1257.