Non-endoreversible Carnot refrigerator at maximum cooling power

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Within the context of the so-called finite time thermodynamics a Carnot's refrigerator is studied. It is used the non-endoreversibility concept and it is found an expression for the coefficient of performance, w, that permits to obtain values near to the experimental values reported in the literature.

Keywords: Carnot's refrigerator; yield; finite time thermodynamics.

Se estudia el refrigerador de Carnot en el contexto de la termodinámica de tiempo finito, utilizando el concepto de no-endorreversibilidad, y se encuentra una expresión del rendimiento, w, que permite calcular valores de él más cercanos a los valores reales reportados en la literatura.

Descriptores: Refrigerador de Carnot; rendimiento; termodinámica de tiempos finitos.

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1. Introduction

All transformations of energy occurring in nature are irreversible processes. Some of these irreversibilities must be included in a realistic description of such processes. Endoreversible thermodynamics is a non-equilibrium approach to these irreversible processes by taking a network of internally reversible subsystems exchanging energy in an irreversible fashion. An endoreversible system consists of a number of subsystems which interact with each other and with the surroundings. The subsystems can be chosen to insure that each one undergoes only reversible processes. The whole irreversibility of the processes is confined to the interaction between the subsystems and the surroundings. An endoreversible system is thus defined by the properties of its subsystems and of its interactions. The processes of such systems are called endoreversible processes.

As unlike classical equilibrium thermodynamics, these processes are considered reversible. Classical equilibrium thermodynamics has been very important in the study of thermal engines, and its main role in thermal engine analysis has consisted in providing upper bounds for variables of processes, such as efficiency, work, heat and others. However, the classical equilibrium thermodynamics bounds are usually far from typical real values. Moreover, the problem of leaving a system from a given initial state to a given final state, while producing a minimum of entropy or a minimum loss of availability leads, to reversible processes. These processes are equivalent and have zero value both entropy and loss of availability, but need infinitely long process time.

From the Novikov [1] and Curzon and Ahlborn [2] results many other results have been published in "finite time" thermodynamics, or endoreversible thermodynamics. Finite time thermodynamics can be considered an extension of classical equilibrium thermodynamics for thermal engines that include time dependence of the interaction processes with the external sources while excluding irreversible effects within the working substance. The exclusion of intrinsic irreversible effects in the substance, knowing as the endoreversibility hypothesis, is considered for cases in which the internal relaxation times of the working substance are negligibly short compared to the total time of the cycle. The Curzon and Alhborn-Novikov engine is a Carnot-type cycle, in which there is no thermal equilibrium between the working fluid and the reservoirs at the isothermal branches of the cycle and in which the adiabatic branches are taken as instantaneous processes.

In the context of finite time thermodynamics it can be built, theoretical cycles with non null power output by using the Newton heat transfer law. Particularly by the named ecological optimization it is qualify the performance of the Curzon and Ahlborn-Novikov engine[3,4]; endoreversible thermodynamic potentials can be built to obtain the efficiency of the Curzon and Ahlborn-Novikov engine[5]; and Diesel cycle has been studied at finite time of exchanging heat[6]. Using a non linear heat transfer law it is possible to make an analysis of the Curzon and Ahlborn-Novikov engine by maximization of power output [7] like it is in references [3,4]. Moreover, Curzon and Ahlborn-Novikov engine can be analyzed by taking a van der Waals gas as working substance [8].

Finite time thermodynamics also permits to model a refrigeration system as an endoreversible refrigerator driven by an endoreversible heat engine, so that some authors have studied the performance of the irreversible Carnot refrigeration cycle. Particularly Agrawal and Menon [9] applied the Curzon and Ahlborn [2] method to calculate the yield of a refrigerator, and they shown that it is necesary to modify the time of the adiabatic branches to take a model of thermal engine with values of yield near to the real values of the experimental engines. This idea leads to the function $a = Q_H/Q_C$, where Q_H and Q_C are the heat to the hot reservoir and the heat from the cool reservoir, respectively, that permits to obtain values of the yield coeficient w near to the experimental values. The total entropy production is considered only associated to the irreversible part. It is necesary to find a form to include the effect of the internal entropy production. Ozkaynak *et al.* [10] and Chen [11] proposed an alternative method to approximate a refrigerator to a non-endoreversible engine operating at finite time; even more, they proposed [10,12] the parameter

$$r = \left| \frac{\Delta S_{1w}}{\Delta S_{2w}} \right|, \qquad (1)$$

whose values are in the interval $0 < r \leq 1$. ΔS_{1w} is the change of entropy in the hot isothermal branch and ΔS_{2w} is the change of entropy in the cool isothermal branch. The results obtained in Refs. 10 and 12 reduce to the Curzon and Ahlborn results when $r = 1$[2].

On the other hand, the optimal performance in a class of heat-driven pumps affected by some irreversibilities, such as finite-rate heat transfer between the working fluid and the external heat reservoirs [13], and the optimal performance related to energy of a Carnot refrigerator under the influence of thermal resistances [14] had also been investigated.

As one can see, because it exists a relationship between refrigeration systems and pollution, the analisys on the performance of refrigerator engines is important. In the present work, we propose a modified model of the endoreversibility hypothesis [15] which permits to consider a thermal irreversible engine, taking into account to sides of the question: an internal cycle with entropy production and an external irreversible one that includes the environments surrounding and the coupling with the working substance (heat flows). We also write the coefficient w of a finite time Carnot refrigerator in terms of a parameter, as Ozkaynak *et al.* [10] has propose, which includes internal irreversibilities of the engine. As a result obtain values of the yield coefficient, w, that improve the one obtained by Agrawal and Menon[9] for the same refrigerator.

2. Endoreversible Carnot refrigerator

Carnot's refrigerator had been analyzed in the context of finite time thermodynamics [16]. Particularly, Agrawal and Menon[9] use the cooling power P, definite by $P = Q_3/\tau$, where Q_3 is the heat from the cool reservoir to the refrigerator at working temperature $T_3 - y$, and τ is the total time of the cycle performance, as it is shown in Fig. 1.

Let us take the Clausius inequality,

$$\Delta S_{1w} + \Delta S_{3w} \leq 0, \qquad (2)$$

where ΔS_{1w} and ΔS_{3w} are the increase of entropy in the heat transfer processes between the reservoirs and the engine. Now, in a reversible engine we have,

$$\frac{Q_1}{T_{1w}} = \frac{Q_3}{T_{3w}}, \qquad (3)$$

and, from the Newton cooling law, we have

$$\frac{dQ_1}{dt} = \alpha(T_{1w} - T_1), \quad \frac{dQ_3}{dt} = \beta(T_3 - T_{3w}), \qquad (4)$$

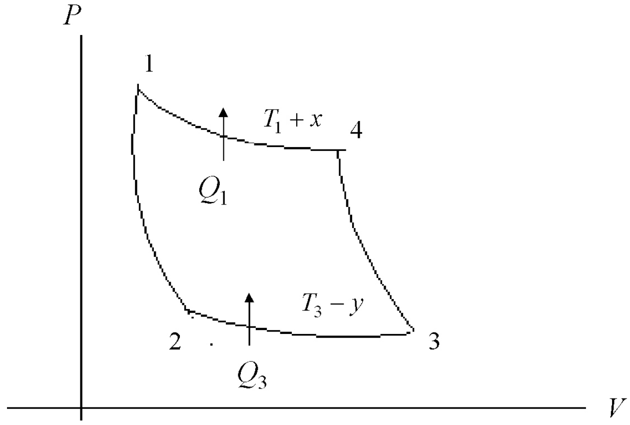

FIGURE 1. Inverse Carnot cycle like a refrigerator engine working at cool temperature $T_3 - y$ and hot temperature $T_1 + x$.

where α and β are the thermal conductances. So that, taking $T_{1w} - T_1 = x$ and $T_3 - T_{3w} = y$, Eq. (3) can be writen in such a way that permits to define the parameter a, as

$$\frac{Q_1}{Q_3} = \frac{T_{1w}}{T_{3w}} = \frac{T_1 + x}{T_3 - y} = a\,. \qquad (5)$$

Transfer of heat from the cool reservoir to the hot reservoir by the engine, Q_1 and Q_3, become

$$Q_1 = \alpha x t_1, \quad Q_3 = \beta y t_3. \qquad (6)$$

It could be assumed that there is a piston with a certain velocity, u, (change of volume per time) into a cylinder as the engine. Without into account aceleration at last of motion, we have, by definition, the time for each process as

$$t_1 = \frac{Q_1}{\alpha x} = \frac{V_2 - V_1}{u}, \quad t_4 = \frac{V_4 - V_1}{u},$$
$$t_3 = \frac{Q_3}{\beta y} = \frac{V_3 - V_4}{u}, \quad t_2 = \frac{V_3 - V_2}{u}, \qquad (7)$$

where V_j, $j = 1, 2, 3, 4$, are the volumes in Fig. 1. Equations (5) and (7) permit to take

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} = \left[\frac{T_1 + x}{T_3 - y}\right]^C, \qquad (8)$$

where $C = C_v/R_g$, with the molar heater capacity C_v at v constant and the universal constant of gases R_g, and the total time of cycle, definite as

$$\tau = t_1 + t_2 + t_3 + t_4, \qquad (9)$$

can be writen as

$$\tau = \frac{2Q_3\left(1 - \frac{V_1}{V_2} a^{-C}\right)}{\beta y \left(1 - \frac{V_1}{V_2}\right)}, \qquad (10)$$

where

$$y = \frac{T_3 - \frac{T_1}{a}}{1 + \frac{\beta a^C}{\alpha}}. \quad (11)$$

Then, the cooling power expresion is

$$P = \frac{\alpha\left(1 - \frac{V_1}{V_2}\right)\left(T_3 - \frac{T_1}{a}\right)}{2\left[a^C - \frac{V_1}{V_2} + \frac{\alpha}{\beta} - \frac{\alpha V_1}{\beta V_2}a^{-C}\right]}. \quad (12)$$

The maximization of Eq. (12) gives a function for a,

$$f(a) = \frac{T_1}{a^2}\left[a^C - \frac{V_1}{V_2} + \frac{\alpha}{\beta} - \frac{\alpha V_1}{\beta V_2}a^{-C}\right] - C(T_3 - \frac{T_1}{a})\left[a^{C-1} + \frac{\alpha V_1}{\beta V_2}a^{-C-1}\right], \quad (13)$$

where $f(a) = 0$ makes it posible to obtain certain values for a. Now, the yield of engine[17], $w = Q_1/(Q_3 - Q_1)$, by Eq. (5), leads to

$$w = \frac{1}{a - 1}. \quad (14)$$

Equation (14) gives w through the obtained values of a, from (13). Figure 2 shows the behavior of P as a convex curve with a unique maximum value, to plotting P/α, for the variable a.

3. Non-Endoreversible Carnot refrigerator

Now, the Agrawal and Menon results[9] can be used to modify the endoreversibility hypothesis. Let us Clausius inequality given by Eq. (2). Parameter r definite in Eq. (1) permits change the Clausius inequality by

$$\frac{Q_1}{T_{1w}} = r\frac{Q_3}{T_{3w}}, \quad (15)$$

and with the Ozkaynak et al criterium [10], we obtain equivalent equations like founded equations in Sec. 2,

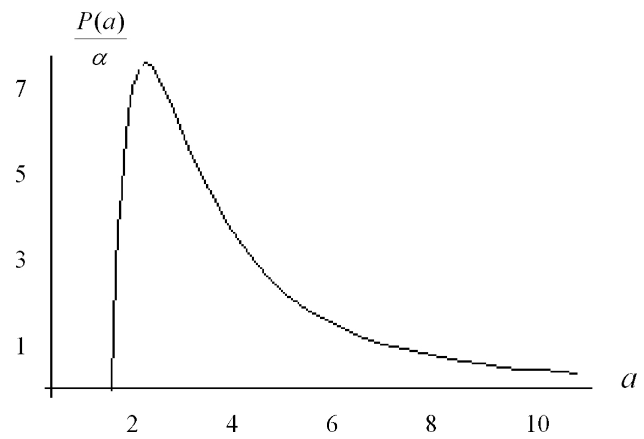

FIGURE 2. Performance of cooling power $P(a)/\alpha$ when $a \to \infty$.

$$\frac{Q_1}{Q_3} = \frac{T_1 + x}{T_3 - y} = ra \quad (16)$$

and

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} = \left[\frac{T_1 + x}{T_3 - y}\right]^C = (ra)^C, \quad (17)$$

where C is the same as in Sec. 2. Also the period of cycle is modified as

$$\tau = \frac{2Q_3}{\beta y}\frac{1 - \left(\frac{V_1}{V_2}\right)(ra)^{-C}}{1 - \left(\frac{V_1}{V_2}\right)}, \quad (18)$$

with $y = [T_3 - T_1/(ra)]/[1 + \beta(ra)^C/\alpha]$.

Finally we obtain the cooling power for the parameter r as

$$P = \frac{Q_1}{\tau} = \frac{\alpha\left(1 - \frac{V_1}{V_2}\right)\left(T_3 - \frac{T_1}{ra}\right)}{2\left[(ra)^C - \frac{V_1}{V_2} + \frac{\alpha}{\beta} - \frac{\alpha V_1}{\beta V_2}(ra)^{-C}\right]}. \quad (19)$$

Now we derive P respect a for given α, β, T_3, T_1, r, C. Optimization condition, $\partial P/\partial a = 0$, gives a funcion for the argument ra,

$$f(ra) = \left[(ra)^C - \frac{V_1}{V_2} + \frac{\alpha}{\beta} - \frac{\alpha V_1}{\beta V_2 (ra)^C}\right] - C\left(\frac{T_3}{T_1}ra^2 - a\right)\left[(ra)^{C-1} + \frac{\alpha}{\beta} - \frac{\alpha V_1}{\beta V_2 (ra)^{C+1}}\right]. \quad (20)$$

Equation (20) reduces to the Agrawal and Menon result[9] when $r = 1$. The yield coeficient is written now as

$$w = \frac{1}{ra - 1}. \quad (21)$$

4. Numerical results

Now we can calculate the yield coeficiente by Eq. (21) in the Carnot cycle, taking a cooler gas which was not condensed during the processes. The best example is air [18], according to the literature. This cooler gas was used approximate by 100 years ago, with a yield of 0.75. At present, due to advanced technics approx in the Carnot cycle values of the yield near to 1.75 [19] had been obtained. Agrawal and Menon, using $T_1 = 316$ K, $T_3 = 275$ K, $C = 2.558$, $V_1/V_2 = 1/16$ and $\alpha/\beta = 2$, obtain $w = 1.27$, and $a = 1.79$. These values are acceptable compared with the experimental values of w, which are in the interval $(0.75, 1.75)$ [9]. Table I (a and b) shows the rank of variation of w and ra, according to the proposal values of r. Figures 3 and 4 show the behavior of cooling power $P(ra)$, to plotting $P(ra)/\alpha$, and the behavior of function $f(ra)$, with $r = 0.9$, when $a \to \infty$. Finaly, Fig. 5 shows a comparison between $p(a)$ and $P(ra)$, to plotting $P(a)/\alpha$ and $P(ra)/\alpha$ together, with $r = 0.9$.

TABLE I. Numerical values obtained for ra and w, according to given values of r. $V_1/V_2 = 1/16$ and $\alpha/\beta = 2$, at given T_1 and T_3 temperarures. a) $T_1 = 316K$ and $T_3 = 275K$; b) $T_1 = 298K$ and $T_3 = 233K$.

a)			b)		
r	ra	ω	r	ra	ω
1.00	1.79	1.27	1.00	1.95	1.05
0.98	1.77	1.28	0.98	1.93	1.07
0.96	1.76	1.31	0.96	1.91	1.09
0.94	1.74	1.34	0.94	1.89	1.11
0.92	1.72	1.37	0.92	1.87	1.14
0.90	1.71	1.40	0.90	1.85	1.16
0.88	1.69	1.44	0.88	1.78	1.28

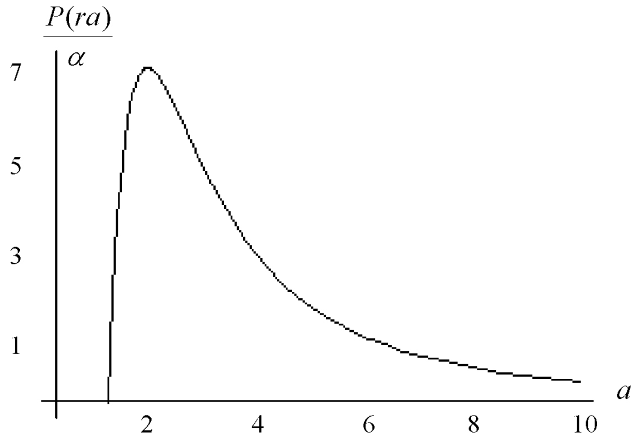

FIGURE 3. Performance of cooling power $P(ra)/\alpha$ when $a \to \infty$ and $r = 0.9$.

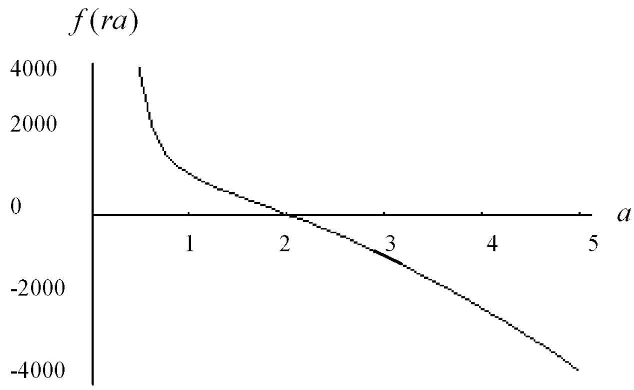

FIGURE 4. Performance of cooling power $f(ra)$ when $a \to \infty$ and $r = 0.9$. The value of a when $f(ra) = 0$ is the same as the correspondig value at maximum of cooling power.

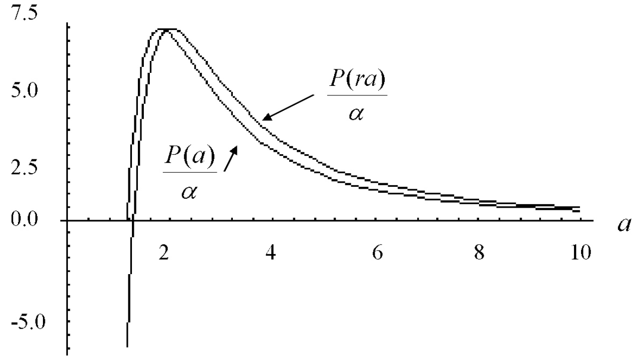

FIGURE 5. Comparison of $P(a)/\alpha$ and $P(ra)/\alpha$ at $r = 0.9$ when $a \to \infty$.

5. Conclusions

Finite time thermodynamics has shown to be an appropriate extension of classical equilibrium termodynamics for the treatment of processes in which changes in important quantities of the system are taken into account, respect to time. For a refrigerator, as it is shown, the obtained results lead to values of yield coefficient, w, which are near to experimental values. The model used here also gives convex curves with a unique maximum for the cooling power, too. With the endoreversible model of Agrawal and Menon, it values of w has been obtained with in the rank of the experimental values. It has been an improve over the reversible Carnot refrigerator, whose values of w were far away from real values. Agrawal and Menon model, gives only a unique value of w for known temperatures of the reservoirs. The proposed model here, taks into account internal irreversibilities trough the paramenter r, leading to a set of values of w (knowing the reservoirs temperatures), which are also near to the experimental values. So we conclude that our model is less restrictive than the Agrawal and Menon one.

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