

# Simulation of Michelson and Young experiments using moiré fringes

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The “visualization” of classical physics experiments for a classroom demonstration is highly stimulating and encouraging for students in a General Physics course. Optics is unique in this matter because it offers the opportunity to awe the class with fun and beauty. The use of moiré fringes is a motivating, easy way to fulfill these purposes. This paper shows how moiré fringes can be used to simulate wave interference by means of the Michelson and Young experiments. A computer code in Postscript is added to help in tracing a set of concentric circles as an example of spherical wavefronts. The validity of the “experimental measurements” are checked with the Young and Michelson theories.

*Keywords:* Temporal coherence; spatial coherence; optical interference.

La “visualización” de experimentos de física clásica como demostraciones de salón de clases estimula enormemente a los estudiantes del curso de Física General. La Óptica es única en este sentido, porque ofrece la oportunidad de llevar a clase diversión y belleza. El uso de las franjas de moiré es una manera sencilla de cumplir con este propósito. Este trabajo muestra como las franjas de moiré pueden usarse para simular la interferencia de ondas en los experimentos de Michelson y Young. Se incluye un código en Postscript que ayuda a trazar un juego concéntrico de círculos que ejemplifican los frentes de onda esféricos. La validez de las “mediciones experimentales” se comparan con las teorías de Young y Michelson.

*Descriptores:* Coherencia temporal; coherencia espacial; interferencia óptica.

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## 1. Introduction

Almost every physics lab at the undergraduate level includes the Michelson and Young experiments in the Optics program. This is important not only from the historical point of view but also for the didactic development of a General Physics course [1]. In this paper, these popular optical experiments are simulated using moiré fringes. The moiré effect is the optical effect of superimposing a repetitive or periodic design, such as a grid or fringe, on the same or a different design, oriented at an acute angle or displaced, to produce a pattern distinct from its components [2]. The French word refers to a watery and wavy appearance that is observed when layers of silk are pressed together at an angle [3]. The optical moiré effect has become a well-refined scientific tool mainly in metrology [3]. Moiré deflectometry is also a technique for testing optical components such as lenses and mirrors [4]. The moiré effect has the advantage of simplicity and complete control of the “experiment”. The idea is to have a static set of measurable fringes with a ruler for class demonstration of some wave interference phenomena. This set-up might help to visualize, and consequently acquire a better understanding, of coherence, optical path difference, phase stability, and other optical concepts.

Two identical concentric circular patterns printed on two transparency films for overhead projectors can be superimposed to produce different moiré fringes depending on the relative distance between their centers [4]. The idea of concentric circular patterns is to simulate spherical wavefronts emitted by secondary point sources in a real experiment. Therefore, the transparencies shown in Fig. 1 correspond

to a cross-sectional view of the spherical wavefronts. It is shown that if a “screen” (detection plane) is located parallel to the line connecting the centers of the concentric circles, Young fringes are produced on the “screen”. However, if the “screen” is perpendicular to the line connecting the centers of the concentric circles, Michelson fringes are obtained on the “screen”. These two effects are observed if the distance between these two planes is larger than the distance between the centers of the circles. The equivalence of these two experiments to the corresponding moiré fringes will be discussed in this paper.

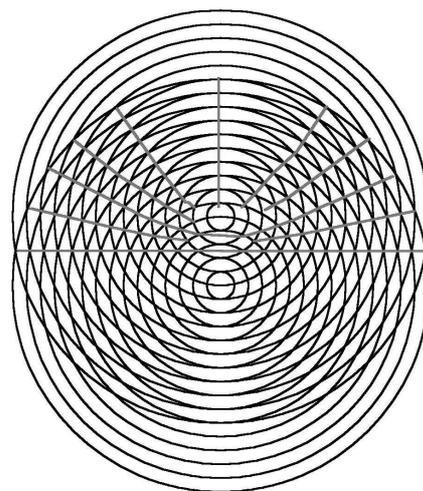


FIGURE 1. Moiré fringes after superposition of two identical circular patterns.

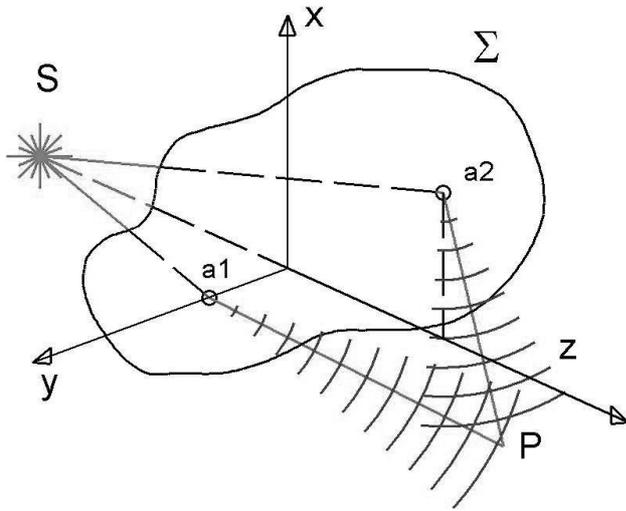


FIGURE 2. Huygens-Fresnel principle to show volume coherence.

## 2. The fringe pattern

The mathematical treatment of light interference is based on the Huygens-Fresnel principle. Temporal and spatial coherence are useful to express some relevant aspects of a light source. They both describe shape, emittance, and spectral details of a light source. The best way to assess light source coherence is overlapping two wavefronts of spherical waves selected from two arbitrary regions, as shown on Fig. 2.

The general procedure is to mask the region around the light source  $S$  with a screen  $\Sigma$  having two small apertures on it,  $a1$  and  $a2$ . According to the Huygens-Fresnel principle, a family of spherical waves radiates from each pinhole [5] and they eventually superimpose to each other at a given point  $P$  in space. If the pinholes are transverse to the wavefront propagation, at the same optical path length from the source  $S$ , the arrangement is known as the Young experiment. This experiment provides the spatial coherence of the source. On the other hand, if the pinholes are placed along the wavefront propagation, the setup is called the Michelson experiment. This experiment gives temporal coherence. In general, the pinholes might be placed in different arrangements by linearly combining the Young and Michelson schemes to result in volume coherence. The feasibility of this fact is based on the mixture of these two experiments.

Let us use a set of concentric circles printed on a transparency film as a representation of spherical waves. The spherical wavefronts can be generated by means of a simple postscript code using any text editor [4]. A computer code has been reproduced in the Appendix section to draw circular gratings to be used in the “experiment” described here. Consider each circle in the pattern as an optical or mechanical monochromatic wave propagating from the center, location of the perturbation point source. The crest of the wave (maximum) corresponds to the solid circle and the trough of the wave (minimum) to the imaginary blank circle in between. The wavelength  $\lambda$  will be the distance between two consec-

utive circles. To minimize the error in the measurement of wavelength, measure with a ruler from 10 to 20 consecutive fringes and take the average. The wavelength can be modified with a slight change in the code (line `/period1.92def`); with this code the waves cover as much area as possible of the transparency.

Take a set of four transparency films, two having the set of concentric circles and two blank ones. To observe the moiré effect, overlap both films with the circular patterns and slide one with respect to the other. Interesting and nice effects will be observed depending on the distance  $a$  between centers of the circles, as shown in Fig. 1. A family of hyperbolas will be produced, similar to the lines of force due to an electric or magnetic dipole, if the distance  $a$  is relatively large [6]. It is shown that if the distance  $a$  is small, a pattern resembling the one obtained by wave interference by two slits is observed: equally spaced fringes. In this case, each secondary source is monochromatic and their relative phase is constant when the circles are kept fixed, regarded as two coherent sources. The superposition of solid circles (crests) causes a “bright” fringe (maximum in an interference pattern), and the superposition of a solid circle (crest) with a blank circle (trough) cancel out to produce a “dark” fringe (minimum in an interference pattern). The superposition of blank circles (troughs) also produces a maximum in an interference pattern because the light intensity is the square of the field. The two blank films will be used later to draw the location of maxima and minima on the “screen” for every interference “experiment” described below.

## 3. Young experiment

The most popular interference experiment by wavefront division is the Young experiment. Monochromatic, coherent light passing through two small slits with a distance  $a$  in between become the secondary sources to produce an interference pattern. The waves coming from the secondary sources interfere constructively or destructively, depending on the path difference, on the detection plane (screen) if  $a$  is smaller than the distance from the slits to the detection plane,  $s$ . The position  $y_m$  of dark fringes (constructive interference) is given by [7]

$$y_m = sm\lambda/a, \quad (1)$$

where  $m$  is the order of the dark fringe on the screen,  $m = 0, 1, 2, \dots$ . From (1), two consecutive bright fringes are separated by

$$\Delta y_m = y_{m+1} - y_m = s\lambda/a. \quad (2)$$

These two expressions are valid only when  $a/s \ll 1$ . Next, we are going to demonstrate that the relationship given by (2) holds true for the “Young experiment” performed by means of moiré fringes.

After choosing the proper distance  $a$  between slits (centers), fix the transparencies on a flat and smooth surface. Now, superpose one blank transparency on the top of the two

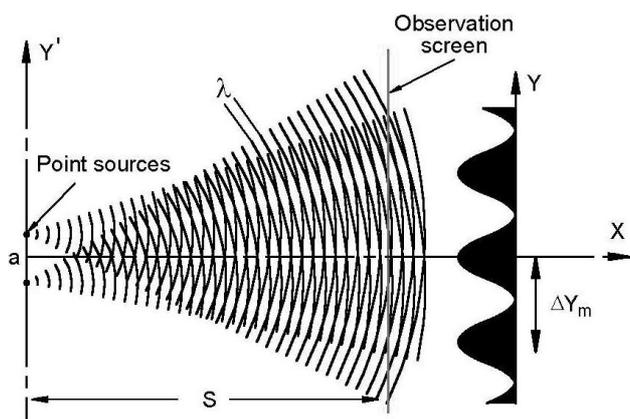


FIGURE 3. Young interference simulated with moiré fringes.

printed transparencies and fix it. Draw a solid line along the centers, where the sources are located, and measure  $a$  with a ruler. Lengthen out this solid line as a long solid-dash line to be used as the  $Y$  axis of a reference system. Exactly at the midpoint of the solid line, draw a second long solid-dash line perpendicular to the solid line to be used as the  $X$  axis. Next, draw a long solid line parallel to the  $Y$  axis as far as possible from the “slits”. This line represents the “screen” (detection plane), located at a distance  $s$  from the midpoint of the slits, as shown in Fig. 3. Measure and record the distance  $s$ . The bright and dark fringes are determined on the screen following the procedure described above. The positions are measured using a ruler, and the  $\Delta y$  separations are recorded. Notice that if the observer moves away from the  $Y$  axis, the fringe separation is not constant as it occurs near the axial region. Substitute the data from the “experiment” into (2), and the validity of this equation is verified.

**4. Michelson experiment**

Probably the best known interferometer is the Michelson interferometer due to the famous Michelson-Morley experiment to prove the existence of the luminiferous aether [8]. Wave interference in this instrument is based on amplitude division rather than wavefront division. It considers superposition of waves coming from the same source launched with a finite time delay. The Michelson interferometer produces this condition by means of a beam splitter; the light beam is split through two different trajectories with different optical path lengths. Having one optical path length larger than the other is equivalent to have two secondary sources with a time delay in between. As in the previous case, interference fringes are produced due to constructive and destructive interference of the waves. The angle  $\theta_m$  defined by secondary source 1 (the one with shorter optical path) and the  $m$ -th fringe is related to the experimental parameters by [9]

$$\theta_m = (m\lambda/a)^{1/2}. \tag{3}$$

The resulting fringes for the same angle  $\theta_m$  are called Haidinger fringes [10].

After choosing the proper distance  $a$  between sources (centers), fix the transparencies on a flat, smooth surface. Now, superpose the second blank transparency on the top of two printed transparencies, and secure it. Draw a solid line along the centers, where the sources are located, and measure  $a$  with a ruler. Extend this solid line as a long solid-dash line to be used as the  $X$  axis of a reference system. Exactly at the midpoint of the solid line, draw a second long solid-dash line perpendicular to the solid line to be used as the  $Y$  axis. Next, draw a long solid line parallel to the  $Y$  axis as far as possible from the “sources”. This line represents the “observation screen” (detection plane), located at a distance  $s_o$  from the midpoint of the sources, as shown in Fig. 4. These two light sources delayed in time produce different moiré fringes, corresponding to the interference pattern. The bright and dark fringes are determined on the screen following the procedure described above. Considering that the central fringe is bright, as shown in Fig. 4, then  $\theta_m = 0$  for  $m = 0$ . Taking this angle as a reference, the other  $\theta_m$  angles are measured by tracing a dotted line from the second bright fringe to source 1 for  $m = 1$ , as shown in Fig. 4, etc. Using a protractor, the  $\theta_m$  angles are measured and recorded. Substitute the data from the “experiment” into (3), and check the validity of this equation.

**5. Final remarks**

The arguments shown in this paper are valid only for perfectly coherent sources; the phase relationship between them

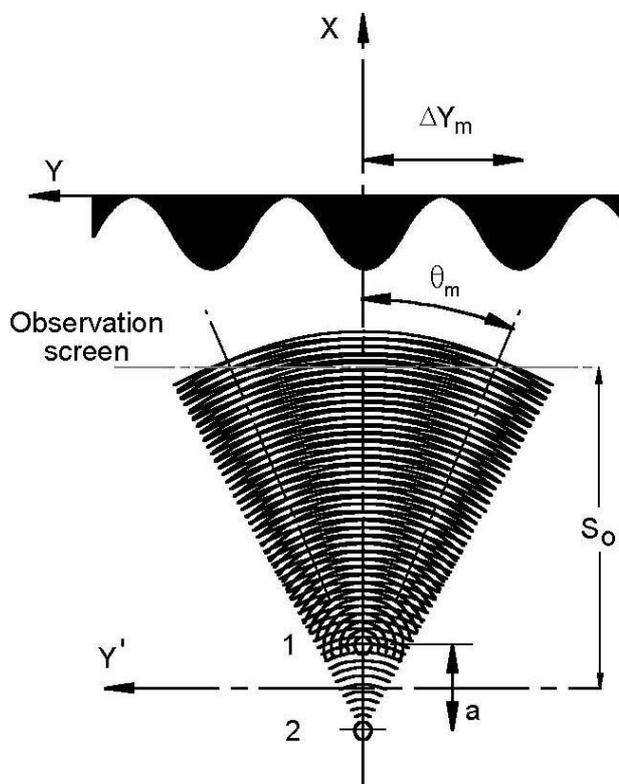


FIGURE 4. Michelson interference simulated with moiré fringes.

never changes. If this relationship is not constant, the fringes disappear. It happens when one transparency moves with respect to the other with a speed higher than the detector response.

## Appendix

The following is the computer code to trace a pattern of concentric circles according to the computer code found in reference [3]. Use your favorite editor to create a text-only file that contains the PostScript program shown below. Then, copy and paste this file directly to the printer using the appropriate command, and save it as “myfile.ps”,

```
%! Circular grating
/period 1.92 def
% period of grating
```

```
/maxradius 500 def %
% radius of largest circle to be ruled
/rulewidth 0.96 def
% width of lines (normal 1.92)
306 396 translate
% (x,y) location of center of circles
0 0 rulewidth 2 div
0 360 arc 0 setgray
fill rulewidth
setlinewidth 0
period maxradius
{0 0 3 -1 roll 0 360 arc stroke}
for showpage
```

The file is transferred to the printer using the command  
copy myfile.ps lpt1:  
assuming the printer understands PostScript code.

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