Teaching thermal wave physics with soils

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In this paper, we discuss the features of a possible student experiment related to the conduction of heat in soils excited by a natural periodically time dependent source, namely the daily periodical oscillations in the earth's temperature, which can be denoted as thermal waves. A measuring device was designed and constructed for automatic measurements of the daily time air temperature variations as well as of the daily time temperature variations, at different depths beneath the soil's surface. Measurements were performed using LM-335 solid state temperature sensors incorporated into a computer-controlled probe. The data acquisition software was developed using a programming environment LabVIEW from National Instruments. In order to obtain characteristic parameters governing the physical phenomena involved, the results of our measurements were fitted to a thermal wave like solution of the heat diffusion equation in the presence of periodical heat sources. The phase shift as well as the attenuation of the temperature waves with depth was demonstrated, as well as their dependence on soil thermal properties, in particular its thermal diffusivity.

Keywords: Thermal waves; soils; thermal properties; thermal diffusivity.

En este trabajo se discuten las peculiaridades de un posible experimento docente relacionado con la conducción del calor en suelos excitados por una fuente térmica variable periódicamente en el tiempo, como son las oscilaciones cíclicas diarias de la temperatura de la corteza terrestre, que pueden considerarse como ondas térmicas. Se diseñó y construyó un dispositivo para la medición automática de las variaciones periódicas en la temperatura del aire así como a diferentes profundidades de la superficie del suelo. Las mediciones fueron realizadas utilizando sensores de estado sólido LM 335 incorporados en una sonda controlada por ordenador. El programa de adquisición de datos fue desarrollado en el ambiente LabVIEW de National Instruments. Para obtener los parámetros característicos que gobiernan los fenómenos físicos involucrados, los resultados de las mediciones fueron ajustados a la solución tipo onda térmica de la ecuación de difusión del calor en presencia de fuentes periódicas. Fueron demostrados el corrimiento de fase y la atenuación de las ondas de temperatura con la profundidad, así como su dependencia de las propiedades térmicas del suelo, en particular su difusividad térmica.

Descriptores: Ondas térmicas; suelos; propiedades térmicas; difusividad térmica.

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1. Introduction

The conduction of heat is a diffusion-like process, where the propagation of thermal energy in a periodically heated solid is well described in terms of thermal waves [1]. These waves have become of great interest for the explanation of the photothermal (PT) phenomena, on which several non-destructive measurement techniques are based [2].

In these methods, thermal waves are generated in a given sample by means of a periodically varying heat source. The changes in the temperature of the sample or in temperature dependent parameters can be monitored. These changes depend, among other things, on the thermal properties of the material of the sample. Therefore, among their several applications, one active area of research nowadays is devoted to their use in the thermal characterisation of materials. Although Ångström in 1861 [3] proposed a temperature-wave method for measuring the thermal diffusivity of a solid in the form of a rod, it was not until the 1970's that practical applications of the photothermal techniques concerning the thermal characterization of solids appeared [4,5].

Consequently, with the development of these techniques, the use of a wave treatment of heat dates from the 1980's [6], although the concept of thermal wave first appeared about hundred years before. Fourier, in his *Analytical Theory of Heat* [7], published in 1822, showed that heat conduction problems could be solved by expanding temperature distributions as series of waves. Fourier, as well as Poisson, have used equations identical to those used today in describing thermal waves in order to estimate the thermal properties at the earth's crust, making use of the daily periodical temperature oscillations [7,8]. In the above-mentioned work [7] Fourier stated that "The problem of temperature at the earth's crust presents one of the most beautiful applications of the theory of heat".

As thermal properties constitute key parameters governing the behavior of many processes in nature, it is of great importance to deal with this theme with students at a college or university level of physics, materials sciences and engineering, as well as the development of student experiments for their measurement. As mentioned in a recent paper [9], the knowledge of thermal properties is of particular importance in the case of soils, because of the role that they play in our food, shelter, and well-being. Seeds, for example, require a certain temperature threshold in order to germinate and develop. Soil scientists are concerned with the effects on the soil properties, including thermal ones, of several natural or artificial processes, such as those of an environmental nature [10]. While in earlier times the effects of pollution were restricted to industrial areas, where increased concentrations of toxic compounds occurred near their sources, more recently, damages to nature over a large area, such as the acidification of soils due to acid rain and forest degradation, were related to the increase of pollution over large distances. On the other hand, soil provides an easily accessible physical system, where temperature studies can lead to a number of interesting observations, which can be carried out throughout a range of levels of sophistication.

From the several methods reported in the literature that should be used for this purpose, the photothermal techniques offer some advantages that made them suitable for student laboratories, which have certain features that should be taken into account in designing experiments. Among other things, the equipment must be as simple and inexpensive as possible and the physical-mathematical formalism involved must be understandable for a given student level.

The use of the daily periodical temperature oscillations at the earth's surface can be considered to be one of the technically simplest photothermal methods that satisfy the above-mentioned requirements. The experimental arrangement that we shall describe in this paper requires only minimal expense and is very instructive from the pedagogical point of view. It may help us to introduce students to the fascinating fields of thermal wave and soil physics.

The paper is organized as follows. In the next section we shall discuss some aspects of the thermal transport in the case of periodical heat sources. Section 3 is devoted to describing the experimental details of the proposed experiment. In Sec. 4, we shall present and discuss the results. In Sec. 5, we shall draw our conclusions.

2. Theory

Consider an isotropic homogeneous semi-infinite solid, whose surface is heated uniformly (in such a way that the one-dimensional approach used in what follows is valid) by a source (light, for example) of periodically modulated intensity $I_o(1 + \cos(\omega t))/2$, where I_o is the intensity of the light source, ω is the angular modulation frequency and t is the time.

The temperature distribution T(x,t) within the solid with thermal diffusivity α can be obtained by solving the heat diffusion equation [1,2],

$$\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} = 0 \quad , \; x \rangle 0 \, , \qquad t \rangle 0, \quad (1)$$

with the boundary condition

$$-k \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} = Re\left[\frac{I_o}{2} \exp(i\omega \, t)\right], \qquad (2)$$

where i=(-1)$^{1/2}$ and k is the thermal conductivity, related to the thermal diffusivity α through the relation

$$k = \alpha \, \rho \, c. \qquad (3)$$

Here ρ is the density and c is the specific heat at constant pressure.

Condition (2) expresses the fact that the thermal energy generated at the surface of the solid (for example by the absorption of light) is dissipated into its bulk by diffusion.

The solution of physical interest to the problem for applications in PT techniques is that related to the time dependent component. If we separate this component from the spatial distribution, the temperature can be expressed as:

$$T(x,t) = Re\left[\Theta(x)\exp(i\omega t)\right]. \qquad (4)$$

Substituting in Eq. (1) we obtain

$$\frac{d^2\Theta(x)}{dx^2} - q^2\Theta(x) = 0, \qquad (5)$$

where

$$q = \sqrt{\frac{i\omega}{\alpha}} = (1+i)\sqrt{\frac{\omega}{2\alpha}} = \frac{(1+i)}{\mu} \qquad (6)$$

and

$$\mu = \sqrt{\frac{2\alpha}{\omega}} \quad . \qquad (7)$$

The general solution of Eq. (5) using condition (2) has the form [1,2]

$$\Theta(x) = \frac{I_o}{2\varepsilon\sqrt{\omega}}\exp\left(-\frac{x}{\mu}\right)\exp\left[-i\left(\frac{x}{\mu}+\frac{\pi}{4}\right)\right]. \qquad (8)$$

Parameter ε is known as the thermal effusivity of the sample, given by

$$\varepsilon = {}^{k}\!/\!_{\sqrt{\alpha}}. \qquad (9)$$

An extended explanation of the physical relevance of the parameters governing the generation and propagation of heat in solids may be found in many books, monographs and articles to which the reader is referred [1,2,10,11]

Expression (8) has the meaning of a plane wave. Like other waves, it has an oscillatory spatial dependence of the form exp(iqx), with a wave vector q given by Eq. (6). Because it has several wave-like features, Eq. (8) represents a thermal or temperature wave propagating with phase velocity v_f given by:

$$v_f = \omega\mu = \sqrt{2\alpha\omega} \quad . \qquad (10)$$

Parameter μ represents the thermal diffusion length of the thermal wave, *i.e.* the distance at which the propagated wave amplitude decays e times its value at x=0. It is similar to that obtained for the amplitude of an electromagnetic wave in the surface skin depth of a metal [1]. As can be seen from Eq. (7), it depends on the thermal diffusivity α, the relevant parameter for time-dependent diffusion processes within homogeneous, isotropic materials, and on the light modulation frequency, ω. Eq. (8) represents, therefore, an attenuated wave. For a given sample, the attenuation of a propagating thermal wave varies following the changes in the periodicity of the heat source. The product $2\pi\mu$ is the thermal wavelength. Between the light excitation and the thermal response of the sample, there is a phase-lag given by the term (x/$\mu + \pi$/4) in the complex exponent.

Using Eqs. (4) and (8), the thermal wave equation can be expressed as:

$$T(x,t) = \frac{I_o}{2\varepsilon\sqrt{\omega}} \exp\left(-\frac{x}{\mu}\right) \cos\left(\omega t - \frac{x}{\mu} + \frac{\pi}{4}\right)$$
$$= T_a \cos\left(\omega t - \frac{x}{\mu} + \frac{\pi}{4}\right) \quad (11)$$

As in other wave phenomena, the phase velocity [Eq. (10)] is defined as the velocity of points of constant amplitude in a wave of the form given by the above expression. Since Eq. (5) is a linear ordinary differential equation describing the motion of a thermal wave, then the superposition of solutions will be also a solution of it (we have approximated the temperature distribution by just the first harmonic of that superposition because the higher harmonics damp out more quickly due to the damping coefficient increase with frequency). This superposition represents a group of waves with angular frequencies in the interval ω, ω+dω travelling in space as "packets" with a group velocity:

$$v_g = \frac{1}{dq_R/d\omega} = 2\sqrt{2\alpha\omega} = 2v_f, \quad (12)$$

where q_R =Re(q)=1/μ. This velocity is the phase velocity of the envelope, *i.e.* the velocity at which thermal energy propagates. In other words, it is the velocity of points of constant amplitude in a group of waves and is calculated from the dispersion relation [Eq. (6)] as usual. It is worth noticing that both phase and group velocities depend on the modulation frequency [see Eqs. (10) and (12)] in such a way that if ω tends to infinity, they would approach infinity as well, which is physically inadmissible. This apparent paradox is well resolved by taking into account the fact that the heat diffusion equation itself, in the form written above, implies an infinite speed of propagation and therefore is not valid in the whole frequency range [1]. The modification of the heat diffusion equation taking into account the time necessary for the onset of the heat flux after a temperature gradient is imposed on a sample, and its physical implications, are questions discussed recently in detail by Marin *et al.* [1].

The surface of the earth is exposed to more or less regular daily temperature fluctuations, owing to different causes, that can differ appreciably from day to day, from season to season, and from region to region. As a consequence, air and soil temperatures generally exhibit a diurnal cycle with a period T=2π/ω of near by 24 hours. The temperature variations T(x,t) over time, t, and with depth below the surface, x, can be described by equations like those derived above for thermal waves, *i.e.* Eq. (11). Measuring T(x,t) as a function of time t at different distances x from the earth's surface, and adjusting the results to Eq. (11), makes it possible to determine parameters such as the period of the temperature oscillations, the soil's thermal diffusion length and its thermal diffusivity.

3. Experimental details

The experimental setup used for the temperature vs. time measurements is shown schematically in Fig. 1, and consists of five LM335 [13] solid state temperature sensors located along a 40 cm long support in such a manner that, when introduced into the soil, the first sensor remains fixed at a distance of 5 cm above the soil's surface to measure the air temperature, whereas the others permit temperature readings at different depths of 5, 10, 15 and 20 cm, respectively.

The sensors are interfaced with a computer using a data acquisition board installed in a Pentium III based PC. The data acquisition control and processing software was developed using LabVIEW [14], a programming environment from National Instruments which facilitates the development of graphical programs called *Virtual Instruments (*VI). The user interface of such programs resembles the front panel of real laboratory instruments with dials, push buttons, toggles, and various digital and analogue readouts. Fig. 2 shows the LabVIEW VI front panel designed for our experiment and the block diagram programming code. The front panel includes controls to select the total measurement time and the time interval between successive readings, indicators to show the actual temperature as measured by a previously selected sensor and indicators to display, the temperature vs. time curves.

4. Results and Discussion

Figure 3 shows a temperature vs. time curve measured in air over a period of 24 hours on March 6, 2002, a typical clear winter day in Havana, Cuba. The solid curve is the best fit of the experimental data to the sinusoidal function:

$$T(t) = \Delta T \cos(\omega t) + T_0, \quad (13)$$

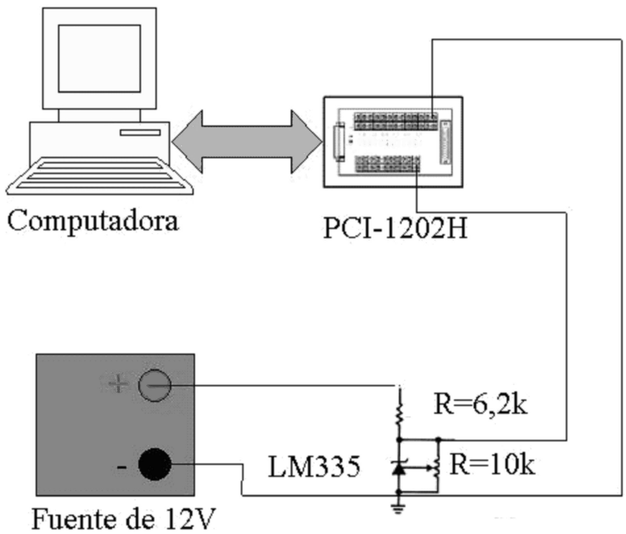

FIGURE 1. The experimental setup used for the measurements of the temperature as a function of time.

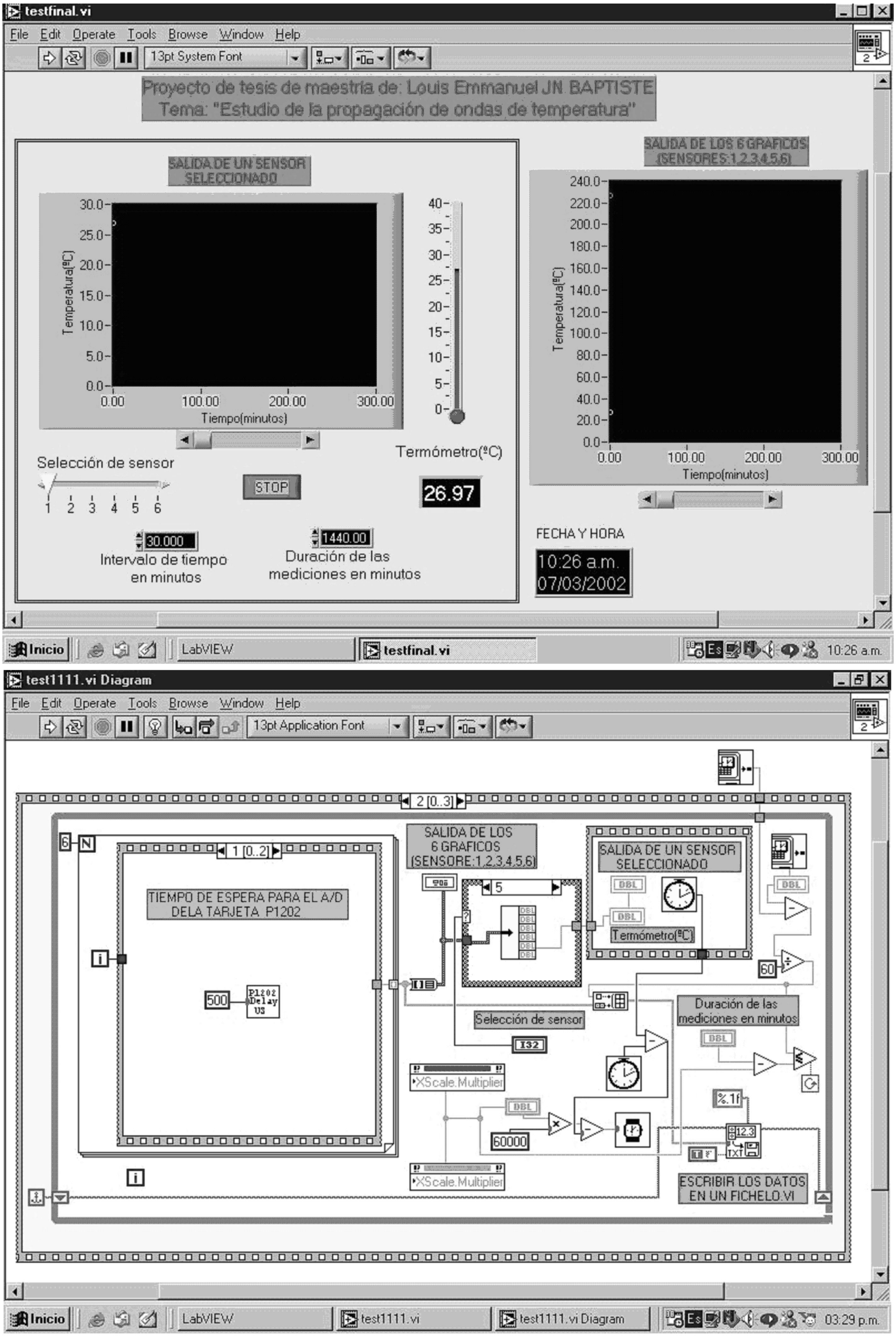

FIGURE 2. a) Front panel of the LABVIEW virtual-instrument designed for our experiment. b) The block diagram program code.

TABLE I. Values of the relevant fitting parameters.

Parameter	**d = 5 cm**	**d = 10 cm**	**D = 15 cm**
ω/**h**$^{-1}$	0.26±0.01	0.279±0.005	0.275±0.0003
μ/**cm**	8±2	7.7±0.5	8.2±0.2

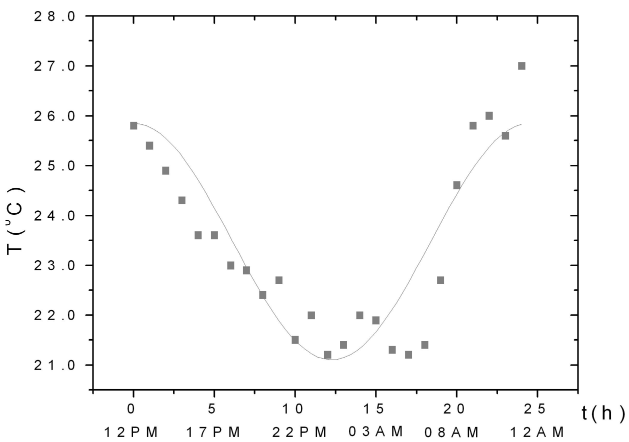

FIGURE 3. Hourly air temperature. The solid curve is the best fit of the data to Eq. (13).

taking ΔT, T_0 and ω as adjustable parameters. Note that this expression is an Eq. (11) like a solution of the heat diffusion equation, for a given depth x and assuming zero phase shift. Parameter ΔT takes into account the amplitude of the temperature variation around value T_0. The value obtained for the angular frequency was ω =(0.256±0.008)h^{-1}. One can observe from the figure that the variation in air temperature is approximately sinusoidal, but that obvious differences exist. Although the period of the temperature oscillation was T=2π/ω =(24.54 ± 0.03)h, the maximum and minimum of the air temperature do not occur at half an oscillation (about 12 hours) apart. The principal cause of this behavior is the deviation of the heat source from the assumed harmonic function of time. However, taking into account that the value obtained for the period is very close to the expected value for this magnitude, *i.e.* 1 day, our approximation can be considered a good assumption.

Figure 4 presents the hourly soil temperature measured at various depths beneath the surface. One can easily see the attenuation of the temperature wave (*i.e.* the reduction in the wave amplitude) as well as the phase shift between the maximum and minimum values of the curves with depth. The experimental data were fitted to Eq. (11), which was written in the form of the attenuated sinusoidal function

$$T(t) = \Delta T e^{-d/\mu} \cos(\omega t - d/\mu) + T_0, \qquad (14)$$

where d represents the depth beneath the surface at which the temperature measurements were taken. The results of the fits, taking ΔT, T_0, ω and the thermal diffusion length μ as adjustable parameters, are represented as solid curves in the figure. The obtained values of the relevant fit parameters are summarized in Table I.

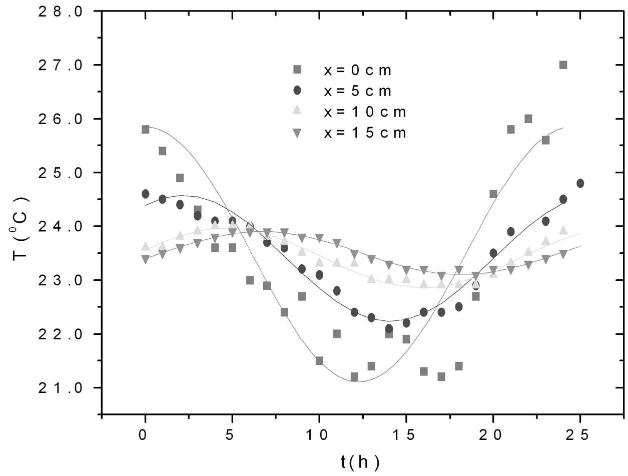

FIGURE 4. Soil Temperature as a function of time at different depths beneath the earth's surface and hourly air temperature. The solid curves represent best fits to Eq. (14) The air temperature (x=0) versus time curve is also represented for comparison.

As can be seen, the values of both parameters, for each depth, agree very well within the range of experimental error. From the mean value of the thermal diffusion lengths, $\langle\mu\rangle$ = (8±2)cm, the soil's thermal diffusivity can be calculated using Eq. (7). We obtained

$$\alpha{=}\omega\mu^2/2{=}(9.0 \pm 0.3)\text{cm}^2/\text{h}{=}(0.0022 \pm 0.0008)\text{cm}^2/\text{s},$$

in agreement with previous reported values [15]. It is worth noticing that this value is an effective value for the thermal diffusivity of soil, in reality a depth dependent parameter, which was considered constant in our model.

In an attempt to give a more quantitative description of the phase shift of the temperature oscillations with depth, we show in Fig. 5. the relationship between the time at which the minimum temperature appears and the depth at which the measurements were performed. The slope of the straight line can be interpreted as the phase velocity defined by Eq. (10), *i.e.*, the velocity of the temperature minimum (maximum) of the waves into the soils. From the linear fit of the data, we obtained the value v_f=(2.4±0.1) cm/h, similar to that reported by other authors in similar experiments [9] and to the value that one can obtain from the phase velocity definition using the values of ω and μ listed in Table I. Substituting these coefficients in Eq. (10), we obtained v_f=(2.2±0.5) cm/h .

To describe the attenuation of the thermal waves with depth we show, in Fig. 6, the logarithm of the difference between maximum and minimum temperature values, *i.e.* the wave amplitudes, as the function of depth. The wave amplitude is given by the parameter T_a accompanying the cosine function in Eq. (11). One can see that from this semi log plot, one obtains a straight line whose slope is the inverse of the thermal diffusion coefficient, μ. We have obtained then, from the linear fit, the value μ=(8.3±0.7) cm, in agreement (within the range of experimental uncertainty) with the values obtained from the individual fits of the temperature vs. time curves showed in Table I.

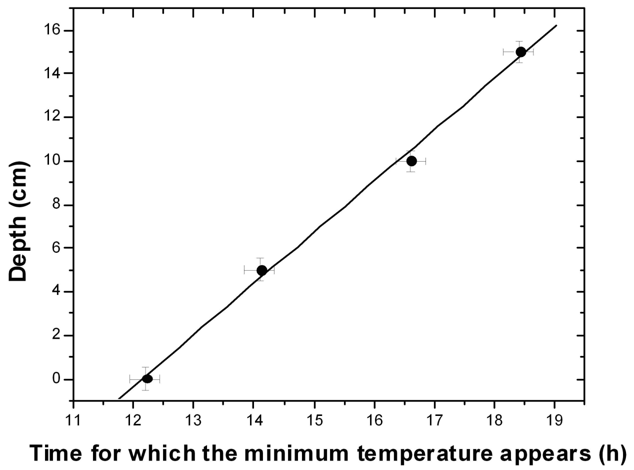

FIGURE 5. Depth at which the measurements were performed at a function of time at which the minimum temperature appears. The solid line is the best linear fit for the data.

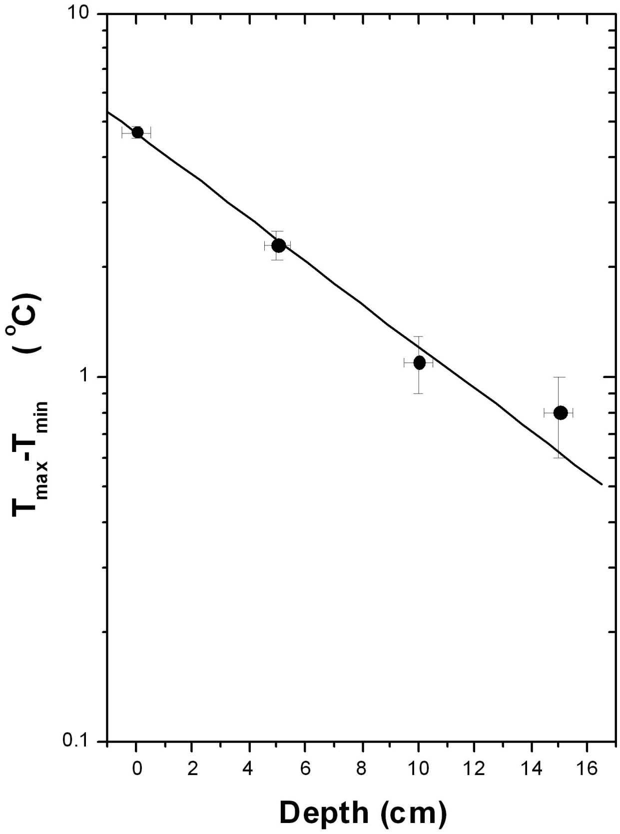

FIGURE 6. Logarithm of the thermal waves amplitudes as a function of depth. The solid line is the best linear fit for the data.

It is worth noticing that the annual variations in the soil temperature due to seasonal changes penetrate the soil more deeply than the daily changes. The cause of this phenomenon is the fact that the period of the temperature oscillations is greater in the former case, and is the thermal diffusion length, leading thus to less damping. However, the observation of this phenomenon is not possible in a one- or two-day laboratory session, although it should be presented to the students using previous recorded data or graphs of this behavior available in specialized literature.

Inspired by the above mentioned work of McIntosh and Sharratt [9], and taking into account the results described above, we have developed several exercises to introduce undergraduate students of physics and engineering to the very fascinating fields of thermal wave and soil-physics [16]. The experiments are designed to teach different technical aspects ranging from computer interfacing and programming to data acquisition, analysis and processing, graphical methods such as semi-log plots handle data, linear fit procedures, error propagation concepts and so on, as well as the physics related to the problem.

The automation of these experiments significantly reduces the amount of time spent by a student to collecting data, as well as providing digitalized formats that lend themselves to the easy display, analysis and comparison of data. Together, these enhancements of the experiments enable students to focus more directly on the physics of the research.

Our experimental device provides fast and accurate data collection, as well as real-time display of the Temperature vs. time curves with precise instrument control and automated data collection. The acquired data can be saved in a data file, permitting detailed data analysis and display using software packages such as ORIGIN [17] and MATHEMATICA [18]. Although LabVIEW routines can be written to analyze the acquired data and calculate the results, we left the data analysis portion out of our program to avoid making the experiment a kind of "black box."

There are several methods based on the same physical principles as those described above that can be also used for teaching the basics of thermal wave propagation in solids. For example, Bodas *et al.* [19] have reported on an experiment based on the Ångström method [3], where a thin bar of copper is heated periodically at one end using a soldering iron embedded in the sample and connected to a circuit made of timers and switches. The temperature changes along the bar were measured at different distances from the heating point using small thermal inertia thermistors. However, our approach is simpler because it uses a naturally existing thermal wave source and propagation medium. On the other hand soil constitutes a less explored physical system than copper. It is a very interesting system: as the knowledge of its thermal properties is of great importance in many fields of research, *e.g.* in the agricultural and environmental sciences, the development of techniques for their measurement is always a priority, as well as the introduction of students to their physical foundations.

The proposed experiments can be used, with proper modifications, to teach physics not only at the undergraduate and postgraduate university level, but also at the secondary, high-school, and technical level.

5. Conclusions

In this work we have described a laboratory experiment, based on the phenomenon of thermal wave propagation, for the measurement of the key parameters governing heat propagation in soils, such as thermal diffusivity. Our experimental setup also makes possible the study of the mean properties of thermal waves and their propagation in solids.

Because the experiment is computer controlled, time that is typically spent doing routine data collection can instead be used to explore the physics of the experiments and to perform a greater number of experiments. Students gain valuable experience using research-grade instrument control, data acquisition, and data analysis software, which can prove useful in future scientific or industrial research. The whole experimental apparatus can be easily handled by students, and it is possible within one laboratory session to program the measurement of several T vs. t curves. The results of the measurements can be collected the next day in an electronic format and they can be processed by the students in home sessions.

The experimental setup described in this paper is technically simple, low cost, and instructive from a pedagogical point of view. It can be useful to introduce students to the basic ideas and principles of the photothermal methods and of the soil physics.

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