Propagation of the information in a one-way quantum computer

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Both linear momentum and Poynting vector associated with the propagation of information in a one-way quantum computer are studied. It is found that, within the so-called Mean Field Theory (MFT) approximation the total energy, the linear momentum and the Poynting vector associated with the propagation of information are invariant under arbitrary rotations of logical qubits. This means that propagation of the quantum information stored in the entangled state does not depend on the choice of the quantum gates. Due that the involved cluster of neighboring particles is large enough, last property satisfies the scalability test. As a consequence, quantum information in the one-way computer is read, written and processed independently of this choice, which suggests a simple hardware for it. When an external magnetic field is switched on, the invariance under arbitrary rotations of the logic qubits of these quantities is lost, that is, the field induces a preferential direction of propagation of the information which at the same time is optimized while more intense be the field.

Keywords: Information; propagation; Poynting vector; invariance.

Se estudia tanto el momento lineal como el vector de Poynting asociados a la propagación de la información en una computadora cuántica de un solo camino. Se encuentra que dentro de la aproximación de la Teoría del campo medio la energía total, el momento lineal y el vector de Poynting asociados a la propagación de la información son invariantes bajo rotaciones arbitrarias de qubits lógicos. Esto significa que la propagación de la información almacenada en el estado enredado no depende de la elección para las compuertas cuánticas. Debido a que el cúmulo involucrado de partículas vecinas is suficientemente grande, la anterior propiedad satisface la prueba de escalabilidad. Como consecuencia de lo anterior, la información cuántica en la computadora de un solo camino es leída, escrita y procesada independientemente de tal elección lo cual sugiere un hardware simple para ella. Cuando se enciende un campo electromágnetico externo, la invariancia del momento lineal y del vector de Poynting bajo rotaciones arbitrarias de los qubits lógicos se pierde, esto es, el campo induce una dirección preferente de propagación de la información la cual a la vez esta última es optimizada mientras más intenso sea el campo.

Descriptores: Información; propagación; vector de Poynting; invariancia.

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1. Introduction

Altough basic principles of a quantum computer have been demonstrated in the laboratory [1], scalability of these systems to a large number of qubits [2] in order to reach an operative quantum computer is still a challenge. The point is that the experimental devices tested so far employ sequences of highly controlled interactions between selected qubits according to the rules of quantum mechanics. In order to process information through the controlled manipulation of qubits, these experiments follow models of quantum computers as a network of quantum logic gates [3, 4].

The one-way quantum computer was proposed by Briegel and Raussendorf (BR) in Ref. 5 as an alternative model of universal, scalable quantum computation[i]. The BR model starts from the assumption that physical registers are contained in the spin σ_x^a while it acts on the multi-qubit entangled state (cluster C) $\otimes_{a\in C}|+\rangle_a$ (where $\sigma_x^{(a)}|\pm\rangle_a = \pm|\pm\rangle_a$). This observable property means that, in the process of computation (*i.e.* during the rounds of qubit measurement), all entanglement in the cluster states is destroyed, so that the cluster can be used only once. This is the origin of the name "one-way" quantum computer.

The BR model proposed in Ref. 5 is a serious attempt to account for the process of information in a quantum computer, seen it as a sequential network of quantum logic gates. However, it is worth pointing out that in this work it was not considered the concerning to the properties of the physical propagation of the information in the one-way computer[ii]. Furthermore, in Ref. 5 they did not consider any suggestion concerning the question of how complicated the respective hardware might be.

To shed light on the last two points, in this work we study, within the BR one-way computer model, both the linear momentum and the Poynting vector associated with the propagation of information at very low temperatures. By employing the MFT approximation [7] for the nearest-neighbors Ising hamiltonian, it is shown that both of the above quantities remain invariant under arbitrary rotation of the logic qubits. This last can be interpreted as an independence of the propagation of the information on the choice of the quantum gates. Consequently information in the one-way computer is read, written and processed in a simple one-way hardware. In addition it is shown that, when an external magnetic field is turned on, the invariance of both the linear momentum and the Poynting vector under arbitrary rotations of logic qubits

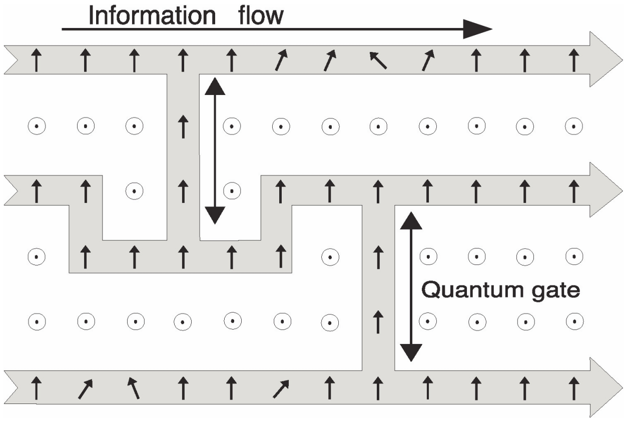

FIGURE 1. Diagram of processing of the information. Before measurements, the qubits are in a cluster state. Circles $\odot$ symbolize measurements of σ_x. Tilted arrows refer to measurements in the $x-y$ plane.

is lost. This means that the external magnetic field induces a preferent type of register for the transmission of the information. This behaviour can be used to arrange the device in order to optimize the propagation of the information.

The paper is organized as follows. In Sec. 2 are described the main features of the BR quantum computer model. Section 3 contains the central part of the present work since there, an effective expression is derived for both the linear momentum and the magnitude of the Poynting vector associated with the propagation of the information. In this same section, a proof is given of both the invariance of these quantities under arbitrary rotation of the logic qubits as well as the lost of their invariance in the presence of an external magnetic field. The work concludes with a discussion of the results.

2. The BR model

According to this scheme, the entire resource for quantum computation is provided by an entangled state called cluster state $|\phi>_C$ of a large number of qubits. Information is then written onto the cluster, processed and read out from the cluster by one-particle measurements only. The way the cluster states are prepared is through a lattice configuration with an Ising interaction between two-state particles at very low temperatures. Thus, by switching on the Ising interaction for an appropriate chosen finite time interval T, and switched off afterwards, the entangled state is activated.

Figure 1 illustrates the processing of the quantum information in the cluster C in a certain order and in a certain basis. The cluster qubits are displayed as dots $\odot$ or as arrows $\nearrow$, $\uparrow$, depending on the respective measured observables. These measurements will induce a quantum processing of logical qubits. The horizontal spatial axis on the cluster can be associated with the time axis with the direction of the 'information flow'. Measurement of the observable

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

will effectively remove the respective lattice qubit from the cluster htus breaking the entanglement of the quantum state. This property allows one to structure the cluster state on the lattice and imprint a network-like structure on it (the gray underlay in Fig. 1). For further details on the model we refer the reader to Ref. 5.

In the present work the MFT approximation, is employed and we consider considered two and three-dimensional arrays of qubits that interact through an Ising-type next-neighbors interaction described by the hamiltonian [7]

$$H = -\alpha(t) J \sum_{a,a'} \sigma_x^{(a)} \sigma_x^{(a')}, \tag{1}$$

whose dimensionless strength $\alpha(t)$ can be controlled externally. $[J]$ is in ergs. Energy H is switched on for an appropriate chosen finite time interval T, where $\int_0^T dt\alpha(t) = \pi T$.

To create a cluster state $|\phi>_C$ on cluster C from a product state $\otimes_{a\in C}|+>_a$, (where $\sigma_x^{(a)}|\pm>_a = \pm|\pm>_a$), the Ising-interaction is switched on for an appropriately chosen finite time interval T, and is switched off afterwards. Since the Ising Hamiltonian acts uniformly on the lattice, an entire cluster of neighboring particles becomes entangled in a single step.

The horizontal spatial axis on the cluster can be associated with the time axis of the implemented quantum circuit, *i.e.* with the direction of the "information flow". As will be explained, measurements of observables σ_z effectively remove the respective lattice qubit from the cluster. This property allows one to structure the cluster state on the lattice and imprint a network-like structure on it (displayed in Fig. 1 in gray underlay).

3. Propagation of the information

Before studying in detail the characteristics of the propagation of the information a proof is given first of the invariance of the Ising hamiltonian of Eq. (1) under the following arbitrary rotations of the logic qubits of the cluster C:

$$\mathcal{R} = \otimes_{b\in C}(\cos\varphi_b \sigma_x^{(b)} \pm \sin\varphi_b \sigma_y^{(b)}), \tag{2}$$

where

$$\sigma_x^{(b)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and

$$\sigma_y^{(b)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

are Pauli matrices^{iii}.

It is obvious that $\mathcal{R}$ is a self-adjoint operator: $\mathcal{R}^H = \mathcal{R}$. In addition, it also has the property of unitarity, that is,

$$\mathcal{R}^H\mathcal{R} = \otimes_{b\in C}(\cos\varphi_b\sigma_x^{(b)} \pm \sin\varphi_b\sigma_y^{(b)}) \otimes_{b'\in C} (\cos\varphi_{b'}\sigma_x^{(b')} \pm \sin\varphi_{b'}\sigma_y^{(b')})$$
$$= \otimes_{b\in C}(\cos\varphi_b\sigma_x^{(b)} \pm \sin\varphi_b\sigma_y^{(b)})(\cos\varphi_b\sigma_x^{(b)} \pm \sin\varphi_b\sigma_y^{(b)}) = \otimes_{b\in C}(\cos^2\varphi_b + \sin^2\varphi_b)\mathbf{I}^{(b)} = \mathbf{I} \quad (3)$$

3.1. Invariance of the hamiltonian

From the properties of Pauli matrices, it is easy to see that the hamiltonian of Eq. (1) is invariant under the arbitrary rotations of logic qubits (2); indeed,

$$\mathcal{R}^H H\mathcal{R} = \otimes_{b\in C}(\cos\varphi_b\sigma_x^{(b)} \pm \sin\varphi_b\sigma_y^{(b)})\left[-\alpha(t)J\sum_{a,a'}\sigma_x^{(a)}\sigma_x^{(a')}\right]\cdot\otimes_{b'\in C}(\cos\varphi_{b'}\sigma_x^{(b')} \pm \sin\varphi_{b'}\sigma_y^{(b')})$$
$$= \otimes_{b\in C}\mathbf{I}^{(b)} \quad \left[-\alpha(t)J\sum_{a,a'}\sigma_x^{(a)}\sigma_x^{(a')}\right] = H. \quad (4)$$

The meaning of the above result is that the energy associated with the propagation of the information does not depend on the choice of the logic qubits (quantum gates).

From the relation $P = H/c$, where P is the linear momentum associated to the propagation, it is inferred from Eq. (4) that the information is processed independently of the choice above mentioned.

At this stage, it needs to be emphasized that the Ising hamiltonian (1) acts uniformly on the lattice; thus the entangled state composite by the neighboring particles is large enough, and consequently the invariance of the above quantities passes the test of scalability.

3.2. Invariance of the Poynting vector

Within the so-called Mean Field Theory (MFT) approximation [7], the resultant force acting on a given particle is teplaced by an effective external field. Here, it will be assumed that the role of the neighboring particles is to create an average molecular field that acts on the particle under study. This situation is illustrated in Fig. 2.

According with the MFT approximation, the force exerted on the qubit $\sigma_x^{(a)}$ due to the nearest neighbors is

$$-\frac{\partial H}{\partial\sigma_x^{(a)}} = \alpha(t)J\sum_{a'}\sigma_x^{(a')}; \quad (5)$$

consequently, the instantaneous magnetic field acting on $\sigma^{(a)}$ is

$$B^{(a)} = \alpha(t)J\sum_{a'}\sigma_x^{(a')}; \quad (6)$$

The Poynting vector associated with the propagation of the information is then

$$\mathbf{S} = \frac{c}{\mu_o}\left(B^{(a)}\right)^2\hat{\mathbf{e}}_1 = \frac{c}{\mu_o}\alpha(t)^2J^2\sum_{a',a''}\sigma_x^{(a')}\sigma_x^{(a'')}\hat{\mathbf{e}}_1. \quad (7)$$

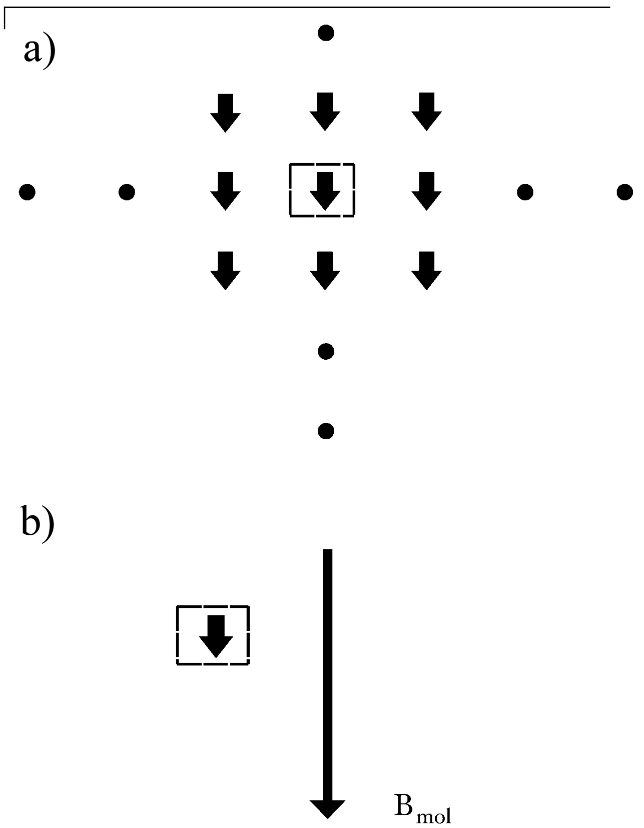

FIGURE 2. Sketch of Mean Field Theory Effective Interaction. (2a) Nearest neighbor interaction. (2b) Resultant Effective Field.

By using Eq. (4), it is straightforward to conclude that the Poynting vector associated with the propagation of the information is also invariant with respect to an arbitrary rotations of the logic qubits given by (1); indeed,

$$\mathcal{R}^H\mathbf{S}\mathcal{R} = \mathbf{S}. \quad (8)$$

Because the cluster C is large enough, the property of invariance of $\mathbf{S}$ is scalable.

From the above equation, it follows that, in the MFT version of the one-way quantum computer, the information is read, written, and processed independently of the logic qubits of both propagation and CNOT gate, which suggest a simple hardware for it.

3.3. An external magnetic field is switched on

When an external magnetic field is applied to the one-way computer along the direction of the propagation of the information, the respective hamiltonian will be

$$H_B = -\alpha(t) J \sum_{a,a'} \sigma_x^{(a)} \sigma_x^{(a')} - \beta(t) B \sum_a \sigma_x^{(a)}, \qquad (9)$$

where $\beta(t)$ is the dimensionless strength which is both controlled externally and activated within an interval of time T such that

$$\int_0^T dt \beta(t) = \pi T.$$

In the present case, it is straightforward to show that the invariance of the respective hamiltonian with respect to the arbitrary rotation of the logic qubits is lost; in fact applying (3) to (9), we obtain

$$\mathcal{R}^H H_B \mathcal{R} = - \otimes_{b\in C} (\cos\varphi_b \sigma_x^{(b)} \pm \sin\varphi_b \sigma_y^{(b)}) \Big[\alpha(t) J \sum_{a,a'} \sigma_x^{(a)} \sigma_x^{(a')} + \beta(t) B \sum_a \sigma_x^{(a)}\Big] \cdot \otimes_{b'\in C} (\cos\varphi_{b'} \sigma_x^{(b')} \pm \sin\varphi_{b'} \sigma_y^{(b')}) I^{(b')}$$

$$= -\alpha(t) J \sum_{a,a'} \sigma_x^{(a)} \sigma_x^{(a')} - \beta(t) B \otimes_{b\in C} (\cos^2\varphi_b - \sin^2\varphi_b) I^{(b)} \sum_a \sigma_x^{(a)}. \qquad (10)$$

This equation says that the invariance of the energy is satisfied only if $\varphi_b = 0, \ \pi$.

From Eq. (10), it is possible to conclude that an external magnetic field applied to the one-way computer induces a preferential direction of propagation of the information along the direction of the field.

The MFT magnetic field acting on the qubit $\sigma_x^{(a)}$ should, be in this case

$$-\frac{\partial H_B}{\partial \sigma_x^{(a)}} = \alpha(t) J \sum_{a'} \sigma_x^{(a')} + \beta(t) B, \qquad (11)$$

where use has been made of Eq. (9).

From the above equation the Poynting vector associated with the one-way computer in the presence of an external magnetic field will be

$$\mathbf{S}_B = \frac{c}{\mu_o} \Bigg[\alpha(t)^2 J^2 \sum_{a,a'} \sigma_x^{(a)} \sigma_x^{(a')} + 2\alpha(t)\beta(t) JB \sum_a \sigma_x^{(a)} + \beta^2(t) B^2\Bigg] \hat{\mathbf{e}}_1. \qquad (12)$$

By exactly the same arguments that led to Eq. (10), it follows that

$$\mathcal{R}^H \mathbf{S}_B \mathcal{R} = \frac{c}{\mu_o} \Bigg[\alpha(t)^2 J^2 \sum_{a,a'} \sigma^{(a)} \sigma^{(a')} + 2 \otimes_{b'\in C} (\cos\varphi_{b'} \sigma_x^{(b')} \pm \sin\varphi_{b'} \sigma_y^{(b')}) I^{(b')} \alpha(t)\beta(t) JB \sum_a \sigma^{(a)} + \beta^2(t) B^2\Bigg] \hat{\mathbf{e}}_1, \qquad (13)$$

which means that, in this case, the invariance of the Poynting vector associated with the propagation of the information under arbitrary rotations of logic qubits is also lost.

4. Conclusions

From the above statements, it is clear that the processing of the information in the one-way quantum computer is possible even though the result of every individual measurement in any direction of the Bloch sphere is completely random. This is made explicit to check here that the respective hamiltonian, the linear momentum and the Poynting vector associated with the one-way quantum computer are invariant under rotation of the logical qubits. This result means that quantum information is propagated through the cluster and processed without the choice of the CNOT-gate between two logical qubits. Due that the involved number of qubits is large enough, this property is scalable.

At this point it is stressed that, independently of the rotation of the observables σ_x and σ_y, the observables σ_z will effectively remove the respective lattice qubits from the cluster. As it was shown above, the invariance of the hamiltonian and Poynting vector is lost whenever an external magnetic field is turned on. This result is that the magnetic field induces a preferent direction of propagation of the information, along the direction of the field itself. In other words, the external field induces a preferent register of information. Within the present scheme, one must distinguish between physical qubits in the cluster and the logical qubits. The physical qubits or cluster qubits form an entangled resource where their one-qubit measurement state operates the computation. On the other hand, the logical qubits constitute the quantum information being processed. At this stage, one question arises and it is related to the behaviour of the respective hamiltonian and Poynting vector under rotations of the physical qubits. The answer to this question is that Ising-type quantities present are written and formulated in terms of logical qubits, and not in terms of logical qubits; consequently, they do not suffer changes under rotations of physical qubits. To conclude, it is pointed out that, if the external magnetic field B is considered as a parameter in the expression for the Poynting vector as it is given in Eq. (12) then $\mathbf{S}$ is always an increasing function of B reaching, its minimal value at

$$B = \frac{\alpha(t)}{\beta(t)} J \sum_a \sigma^{(a)}.$$

Thus, in order to optimize the propagation of the information in the one-way quantum computer it is recommendable subject to this to an intense external magnetic field.

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i The basic features of this model will be described in the next section.

ii In another paper published later [6], BR considered an arbitrary one-qubit rotation applied to a chain of 5 qubits with the intention of stating a universal set of gates. In such a work there was no discussion about the consequences on the propagation of the information due to the rotations of the logic qubits.

iii The Pauli matrices satisfy the ordinary algebra:

$$\sigma_x^{(b)\,2} = \sigma_y^{(b)\,2} = \sigma_z^{(b)\,2} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

$$[\sigma_n^{(b)}, \sigma_m^{(b)}] = 2i\epsilon_{nml}\sigma_l^{(b)}$$

and

$$\{\sigma_n^{(b)}, \sigma_m^{(b)}\} = 2\delta_{nm}\mathbf{I}.$$

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