Students' understanding of vectors in the context of forces

S. Flores-García, L.L. Alfaro-Avena, and O. Dena-Ornelas
Universidad Autónoma de Ciudad Juárez,
Avenida del charro 450 Nte., Col. Partido Romero, 32310 Ciudad Juárez Chih.
e-mail: sergiflo@hotmail.com, lalfaro@uacj.mx,
odena@uacj.mx

M.D. González-Quezada
Instituto Tecnológico de Ciudad Juárez, Avenida Tecnológico 1340,
Fracc. Crucero 32500, Ciudad Juárez Chih.
e-mail: doloresgo73@hotmail.com

Recibido el 20 de marzo de 2007; aceptado el 6 de junio de 2007

A functional understanding of Newton's second law as a vector equation requires that students be able to reason about forces as vectors. In this paper, we present data describing students' conceptual difficulties with forces as vectors. These data suggest that after traditional instruction in introductory physics, some students do not recognize the vector nature of this quantity. Other students do not have the necessary procedural knowledge to determine net force or acceleration, and are therefore unable to reason qualitatively about Newton's second law. We describe some specific procedural and reasoning difficulties we have observed in the students' use of vectors in the context of forces and Newton's second law. In addition, we encourage modifications in the instruction of mechanics that we designed on the basis of our research into student understanding. These modifications are intended to improve the students' understanding of vector addition and subtraction and to promote the students' use of vectors insolving mechanics problems.

Keywords: Newton's second law; conceptual difficulties; forces as vectors.

Un entendimiento funcional de la segunda ley de Newton como ecuación vectorial requiere que los estudiantes sean capaces de razonar acerca de las fuerzas como vectores. En este artículo, presentamos datos que describen las dificultades conceptuales de los estudiantes con las fuerzas como vectores. Estos datos sugieren que, después de una instrucción tradicional en los cursos introductorios de física, algunos estudiantes no reconocen la naturaleza vectorial de esta cantidad. Otros estudiantes no tienen el conocimiento procedimental requerido para determinar la fuerza neta o la aceleración, y por lo tanto, son incapaces de reconocer la segunda ley de Newton cualitativamente. Describimos algunas dificultades de procedimiento y razonamiento observadas acerca del uso de vectores en el contexto de las fuerzas y la segunda ley de Newton. Además, se promueven modificaciones en la instrucción de la mecánica observadas en base a esta investigación dentro del entendimiento del estudiante. Estas modificaciones tienden a mejorar el entendimiento de la suma y resta de vectores por parte de los estudiantes y a promover el uso de vectores en los cursos de mecánica.

Descriptores: Segunda ley de Newton; dificultades conceptuales; fuerzas como vectores.

PACS: 01.40.d; 01.40.Fk; 01.49.Ha

1. Introduction

An understanding of Newtonian mechanics as a coherent subject thus requires an understanding of vector addition (to find a net force), vector subtraction (to find an acceleration), and a recognition that Newton's second law connects these two independently determinable quantities.

A conceptual understanding of Newton's second law and the fundamental vector operations are essential in developing a meaningful understanding of forces as vectors. Many students in the introductory calculus-based and algebra-based physics courses do not develop a conceptual understanding of the vectorial nature of Newton's second law as a vector equation.

Most instructors of introductory physics courses recognize that thinking about physical quantities as vectors is difficult for students. Even when instructors consistently model solutions to problems in Newton's second law by starting with free-body diagrams, many students avoid these diagramming tools. There is a tendency, even among fairly capable students, to jump immediately to force components and to resort to memorizing what these components are in specific cases rather than deriving them from the geometry of the situation. Therefore, students have difficulties understanding those problems that require several steps in the solution process. These problems are called "multiple-step" problems.

In the process of our investigation into the student use of vectors in the context of forces, we observed additional difficulties with tension. This observation motivated an additional investigation into student understanding of tension that we shall describe in future articles.

2. Research techniques

There are two primary sources of data that we use to assess student understanding and to learn about students' ideas about physics topics and about the prevalence of these ideas in a given student population. These are individual student responses to questions in one-on-one interviews and student responses to written questions. We shall describe each of these in turn.

2.1. Written questions

Our primary source of data for our research was student responses to written questions. These questions were asked in homework assignments (both laboratory and lecture), as laboratory pretests, and on classroom quizzes and examinations. Since we were primarily interested in students' conceptual understanding of physics, the questions we asked primarily qualitative rather than quantitative. Student responses to these questions were typically analyzed and categorized on the basis of response and of the reasoning given for that response.

In our analysis of these written questions, we were looking for *patterns* of student responses, either correct or incorrect. These patterns could be patterns of incorrect ideas, a common tendency to focus on irrelevant features, patterns of reasoning, or patterns of procedure. Some features of common student responses that seemed to lead to correct responses could then form the basis for curriculum exercises that reinforced productive lines of reasoning. Conversely, other patterns of responses could indicate that there was a need for curriculum that elicited a common misconception or error of procedure and reasoning and then addressed this difficulty.

2.2. Interviews

Interviews were conducted at New Mexico State University and by colleagues at Arizona State University. These interviews were audio or videotaped, and the tapes and student written responses were later analyzed. At NMSU, we interviewed students from the introductory calculus-based mechanics courses intended for engineering majors. All of these students were volunteers. The interviews lasted about 30 minutes. We designed the interviews to probe students' conceptual reasoning. During the interview, students were asked questions about selected topics and were encouraged to explain the reasoning behind their responses.

3. Context for research

While the data presented here were collected primarily at New Mexico State University (NMSU), we collected additional data from the University of Washington (UW), Syracuse University (SU), and the Independent University of Juarez in Mexico (UACJ). In this study, student responses from UACJ have been translated from Spanish into English.

The courses used as information sources for this investigation were:

- NMSU: Physics 215 (Introductory calculus-based mechanics)
- NMSU: Physics 211 (Introductory algebra-based mechanics)
- NMSU: Physics 215 laboratory
- NMSU: Physics 211 laboratory
- Syracuse University: General Physics I (Calculus-based mechanics)
- University of Washington: Physics 121 (Calculus-based mechanics)
- Independent University of Juarez: General Physics I (Calculus-based mechanics).

Physics 215 is primarily intended for engineering majors. Instruction in introductory calculus-based physics courses at New Mexico State University consists of three 50-minute lectures. The sequence of topics in the lectures follows the sequence used in most textbooks. There is no recitation section.

Physics 211, the algebra-based physics course, covers more topics than the calculus-based course, but at a less rigorous mathematical level. The majors of the students enrolled in Physics 211 were approximately the following: 30% Engineering Technology, 30% Biology, 10% Agriculture, 5% Education, and 20% Other/Undeclared.

There is an associated 1-credit laboratory, Physics 211L and Physics 215L, that is required for some majors. About one-half of the students enrolled in the lecture portion of the course also take the laboratory. The 3-hour laboratory is graded separately from the lecture. All of the laboratory sessions are taught by graduate students. In the laboratory, students work in small groups on materials intended to strengthen connections between observed phenomena and mathematical formalism, to promote scientific reasoning skills, and to foster conceptual understanding. Instead of a laboratory report, students are assigned laboratory homework intended to reinforce and extend concepts underlying the laboratory. Students are encouraged to *predict*, *compare* or *rank* variables in physical situations. Most of the laboratory sessions for both the calculus-based and the algebra-based course were based on *Tutorials in Introductory Physics*[1]. We have modified these tutorials for use in the laboratory, and shall describe these modifications in Chapter 4.

Instruction at the University of Washington (Physics 121) consists of 150 minutes of lecture, a 3-hour compulsory laboratory, and a 50-minute recitation tutorial per week. Students enrolled in this course are primarily engineering majors. In the tutorial, students are encouraged to work in groups on conceptual exercises taken from *Tutorial in Introductory Physics*[1]. Instruction at Syracuse University is similar to instruction at the University of Washington.

At the University of Juarez, the General Physics I course is a common introductory physics course. This course contains topics related to classical mechanics. Students attend two 90-minute lecture sessions a week. There is a 1-hour mandatory lab. There is no recitation section, and the students are primarily engineering majors. Students are taught in several groups. Each group has a different traditional-based instructor.

4. Previous research about student understanding of tension and forces as vectors

For the purposes of our study, it is important to note that, even though the *Force Concept Inventory*[2] has become the most common measure of conceptual understanding of mechanics, very little understanding of vectors is required to successfully answer the questions in it. While there is an extensive body of research into student understanding of forces and acceleration, most of these investigations do not explicitly look at student understanding of vectors.

As part of an a study in student understanding of vector use, Shaffer and McDermott[3] gave two matched, multiple-choice questions in several calculus-based lecture sections at the University of Washington (Fig. 1). These courses correspond to the General Physics I course at Juarez. In the first version, a car strikes and rebounds from a wall; students were asked to find the direction of the average acceleration. In the other version, students were asked to find the difference between the same two vectors with no physical context. Results were better for the version without a physical context: while only 45% of the 350 students gave the correct answer for the first version, 60% of the 115 students gave the correct answer for the second version.

To identify common student conceptual errors in recognizing the existence of passive forces such as the tension in a string, Sjoberg and Lie[4] of the University of Oslo administered a written questionnaire to over 1000 secondary school students, future teachers, university students and physics graduate students.

Figure 2 shows two pendulums, one stationary and one swinging through its equilibrium position. Sjoberg and Lie[4] asked students to indicate the forces acting on both pendulums. Results indicated that about 50% of the secondary-school students with one year of physics omitted the tension in the string. About 40% of the future teachers and about 10% of the graduate students omitted this force as well. A great number of students included a force in the direction of the motion of the swinging pendulum.

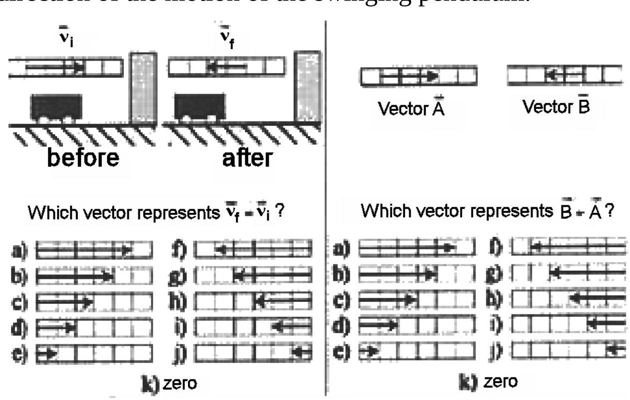


FIGURE 1. Vector questions asked by Shaffer and McDermott with and without physical context.

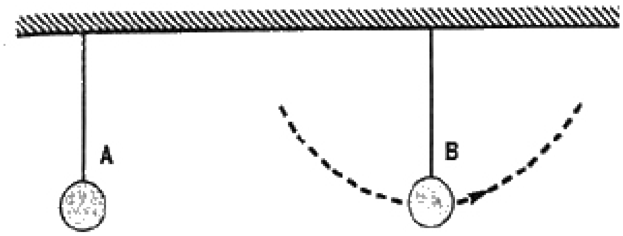


FIGURE 2. Experiment set used by Sjoberg and Lie to probe student difficulties with forces.

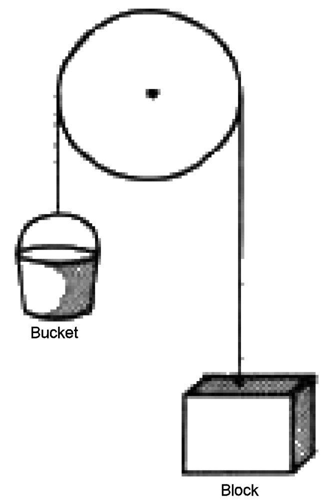


FIGURE 3. Experiment set used by Gunstone and White. The bucket and the block are suspended from a bicycle wheel.

As part of an study in students' understanding of gravity, Gunstone and White[5] asked 463 first year physics students at Monash University to compare the weight of a bucket with the weight of a block when they are hanging from a string stretched around a pulley as shown in Fig. 3. About one-half of the students correctly concluded that the weights are equal. About one-fourth stated that the block is heavier. The most common reason for this response was that "the block is nearer the floor". There was a version of other reasons given. For example, "In the string used to link both the bucket and the block together over the pulley, tension exists at both its ends. At the end towards the bucket, the tension is less than at the end towards the block. This then causes the block, to pull itself down, thereby raising the bucket."

Clement[6] described many difficulties that students have with the concepts of force and acceleration. Students from introductory mechanics courses were asked about the direction of each force acting on the pendulum bob at a point where the bob was moving along a circular trajectory. He noticed that students often include a force in the direction of motion. Most of the incorrect responses contained a force parallel to the trajectory of the bob. An example of an incorrect student explanation is: "If this force were not there, the pendulum could never move up to the top of its swing". It seems that this force is seen as the force that makes the pendulum travel along the path.

Arons[7] made the observation that "massless strings are a source of significant conceptual trouble for many students". He also states that "students have no intelligible operational definition of *massless;* they fail to see why the forces of tension should have equal magnitude at either end; they proceed to memorize problem-solving procedures without understanding what they are doing".

This observation led to an investigation by McDermott, Shaffer and Somers[8] into some specific student difficulties with tension in the context of the Atwood's machine. Figure 4 shows a physical situation used in this investigation. The string and the pulley are massless. In interviews, most students predicted that the heavier mass would fall and the lighter mass would rise. Although all recognized that the tension in the string acting on block A is greater than the weight of this block, on the free-body diagrams many showed different magnitudes for the tension exerted by the string on the two blocks.

As a part of the same investigation, a written question based on Fig. 5 was administered to students in three calculus-based courses. Students were asked to compare the magnitude of the tension at the middle of the strings in cases (a) and (b). Only about half of the students predicted that the two strings would have the same tension. Many students responded that the tension in the string attached to the two blocks would be twice that in the other string. Two common difficulties found were: 1) the belief that tension is the sum of the forces exerted at the two ends; and 2) the belief that an inanimate object, such as a wall, does not exert a force on a string.

McDermott, Shaffer and Somers[8] concluded that student performance on simple qualitative questions that were asked after lecture instruction suggested that traditional instruction in the Atwood's machine did not improve their understanding of dynamics. Practice in only one context such as the Atwood's machine is not enough to develop a functional understanding of the concept of tension.

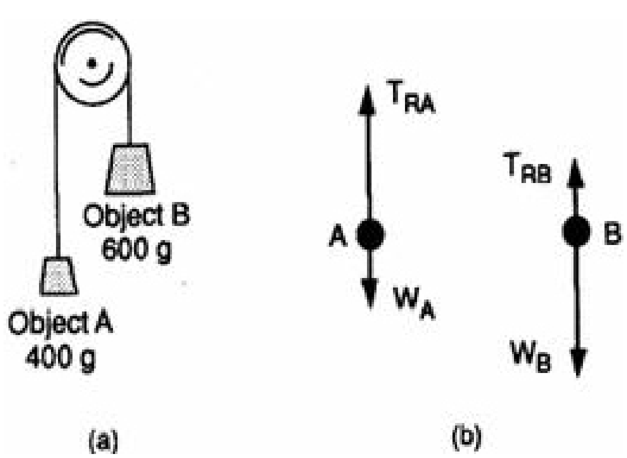


FIGURE 4. Physical system used by McDermott, Shaffer and Somers. a) Original Atwood's machine. b) Typical incorrect free-body diagram drawn by students to represent the forces exerted on blocks A and B.

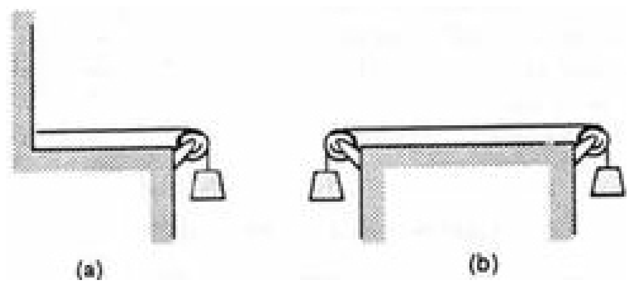


FIGURE 5. Physical situation used by McDermott, Shaffer and Somers. Students were asked to compare the tension in the two strings in cases a and b.

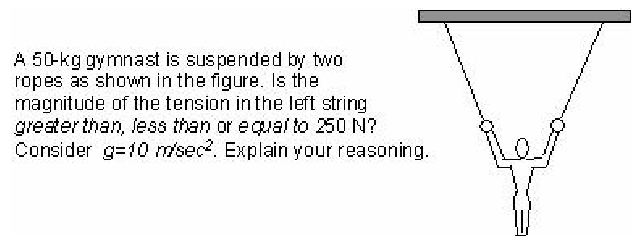


FIGURE 6. The static gymnast question.

We have found (as the results above suggest) that ideas about vector addition and subtraction are often more difficult for students in a physical context. In this next section we shall describe student use of vectors when asked qualitative questions about forces.

5. Research questions

5.1. The static gymnast question

The question shown in Fig. 6 was included on examinations and was also been asked in interviews. Students were given a drawing of a gymnast suspended at rest by two non-vertical ropes, and asked whether the magnitude of the tension in the left rope was *greater than*, *less than*, or *equal to* one-half of the gymnast's weight. All the questions used in this investigation were administered in the 2003 spring semester.

This question can be answered by considering the vertical components of the forces, or by graphically adding the forces acting on the gymnast. The triangle of forces obtained can be split into two right triangles. The longest sides of the triangles represent the magnitude of the tensions into the ropes. As shown in Fig. 7, one-half of the weight of the gymnast (250 N) is less than the magnitude of the tension in the left string.

This question was also asked to 191 students after traditional instruction in a calculus-based physics course at the University of Juarez in Mexico. There was no emphasis placed in the lectures on graphical vector methods. Only 4 students gave a correct answer with correct reasoning, with no students including a graphical argument as part of their answer.

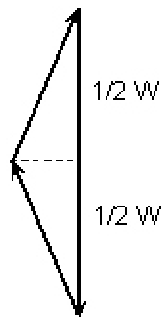


FIGURE 7. Graphical procedure leading to correct solution.

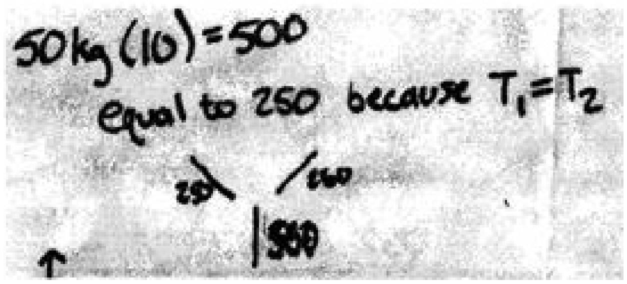


FIGURE 8. Example of an answer in adding the magnitudes of tension.

FIGURE 9. Free-body diagram of the gymnast for the case when ropes are vertical.

After modifications to instruction, students still had difficulties in answering this question. At Syracuse University, about 20% correctly answered that tension in the left rope was greater than 250 N. About 70% stated that the tension in the left rope was equal to 250 N, with most of these students reasoning that, because the angles of the ropes were equal, the tensions would be equal to each other and therefore equal to one-half of the weight.

Instruction was modified on the basis of a conceptual approach in all topics covered during the courses. In all sessions, instructors used the technique called *Elicit, Confront and Resolve*. This technique confronts the students with their mistakes, to allow them to resolve these errors during instruction. In addition, the problems used in class are not common textbook problems. Most of these exercises are designed on an conceptual basis. Students must understand the core of the concept to develop a *functional understanding* of forces as vectors.

Students often seemed to fail to take the vector nature of forces into account. About 11% believed that the tensions had magnitudes equal to 250 N because the gymnast was in equilibrium (suggesting scalar reasoning) as shown in these examples:
"*Equal, because the weight is uniform like the position of the ropes*."
"*Is equal because the gymnast is in the center and therefore the weight is equal in both ropes*."

At New Mexico State, after modified instruction, about 45% of 65 students were able to give a correct answer with correct reasoning. Only 20% answered that the tensions were equal to 250 N. More than half of these students included a free-body diagram as part of their answer, and about a third attempted to add the tensions and weight graphically to reach a net force equal to zero. A few of students gave reasoning based on the lengths of the ropes to justify their answer. One of these students wrote that "*If the lengths of ropes are equal, then there is an equal amount of tension force in each rope*". It seems that, even after the extensive conceptual emphasis on this topic during instruction, some students who showed a correct vector sum were unable to reason geometrically about the magnitudes of the vectors.

In addition to written results, five interviews that included this question after traditional instruction (calculus-based course) were conducted at New Mexico State. Three students who were interviewed said that the magnitude of the tension in the left rope was equal to 250 N. They argued that the ropes "shared" the weight of the gymnast. The answer of one student is shown in Fig. 8. It seems that this student attempted to add the magnitudes of forces as scalars.

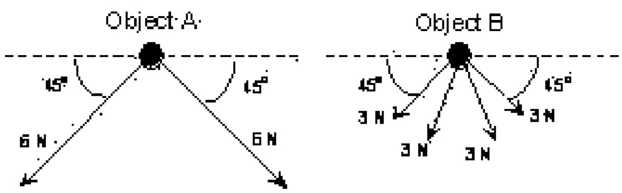
For students who answered that the tension was one-half the weight, we asked a follow-up question about the tension if the situation were changed so that the ropes were vertical. One student interviewed said that the tension in the ropes in this case must be greater than one-half of the weight of the gymnast. In the following excerpt from this interview, the interviewer is indicated by an "I", the student by an "S".

I: "*Are the magnitudes of tensions greater than, less than or equal to 250N?*"
S: "*They are equal to*."
I: "*... Why do you think that T_1 and T_2 are equal to 250N?*"
S: "*Because if T_1 is equal to T_2, then T_1 is equal to one-half of the weight and T_2 is equal to one half of the weight*."
I: "*In the previous case you told me that T_i, is equal to T_2 equal to 250N. Now in this case what are the values of both tensions?*
Be careful because now both ropes are vertical."
S: "*If the free body diagram is* (Fig. 9 shows the diagram the student drew), *then T_1 and T_2 are going to be greater*."
I: "*Why?*"
S: "*Because T_1 and T_2 are both in the same direction. That's why tensions are greater*."

During the interview, this student's responses changed with changes to the directions of the ropes. Initially, he responded that the weight is shared by the two ropes. However, the following responses suggest that he associated the magnitudes of the tensions in the ropes with their direction. As shown in the excerpt, he seems to believe that the more vertical the ropes, the greater the tensions.

5.2. The superposition question

In the *superposition question* (Fig. 10), students were asked to compare the magnitudes of the net force acting on two objects. Two forces of magnitude 6 N are exerted on object A, while four forces of magnitude 3 N are exerted on object B in the directions shown. In each case, the horizontal components of the forces acting on the objects cancel each other, so the net force acting on each object is vertically down. Figure 11 shows a graphical addition of the force vectors. We expected that students would reason about the vertical components of these vectors to determine that the magnitude of the resultant vector in case B is greater than in case A.



Shown above are free-body diagrams for two different objects, A and B. Is the magnitude of the net force on objetct. A *greater then* , *less than*, or *equal to* the magnitude of the net force on object B? Explain and show how you determined your answer.

FIGURE 10. Context of forces: *Superposition question*.

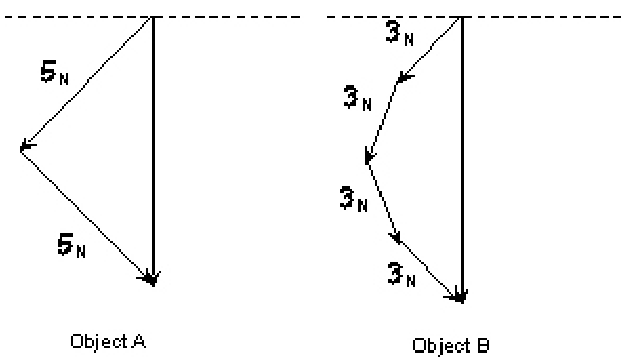


FIGURE 11. Correct graphical addition of force vectors for objects A and B.

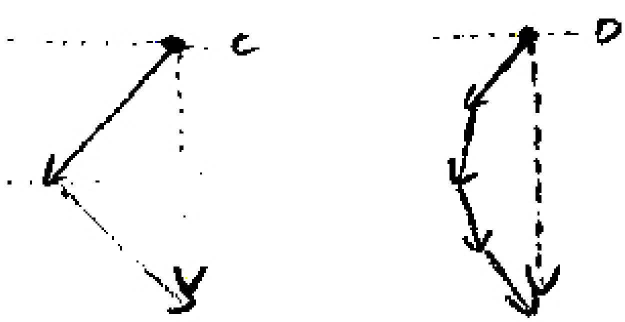


FIGURE 12. Correct student response to the superposition question.

This question was asked after modified instruction at New Mexico State and at the University of Washington. About 55% of 142 students at New Mexico State and about 50% of 616 students at the University of Washington answered this question correctly, stating that the magnitude of the net force acting on object A is less than the magnitude of the net force on object B. Figure 12 shows an example of a correct student response. About 30% at New Mexico State and about 10% at the University of Washington answered that the magnitude of the net force in case A is greater, and about 10% at New Mexico State and about 35% at the University of Washington stated that the magnitudes of the net forces were the same.

For this question, many students simply added all the forces as scalars. These students concluded that the magnitude of the net force was equal to 12 N. For example, one student answered, "*equal because both the net force of A and B are 12 N*". Other students seemed to realize that the horizontal components of the forces cancelled each other out, but still answered that the magnitudes of the net forces were equal to 12 N in both cases:

"*Same. As in object A, the horizontal forces cancelled through symmetry so it is just the sum of the vertical forces that is taken into account for the net force: 6+6=3+3+3+3=12*."

Other students used graphical methods to add forces. However, some of these students concluded that the magnitude is greater in case A or that the magnitudes are equal despite including a correct answer in a vector sum diagram. An example is shown in Fig. 13, where the students correctly added the forces by using head-to-tail method, but incorrectly stated that the vector sum was equal to 12 N.

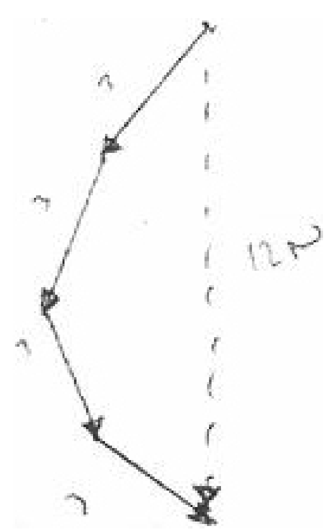


FIGURE 13. An example of an incorrect student response in the superposition question.

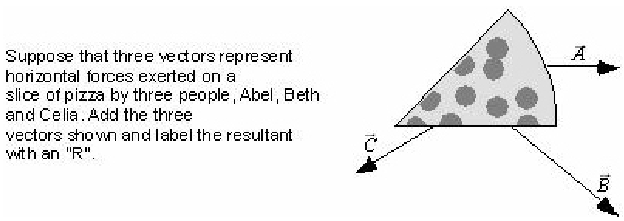


FIGURE 14. The pizza slice question.

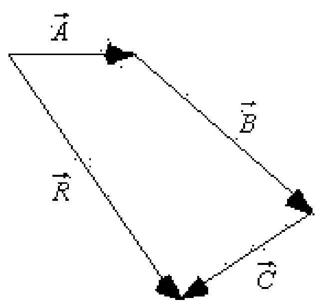


FIGURE 15. Correct answer for the pizza slice question.

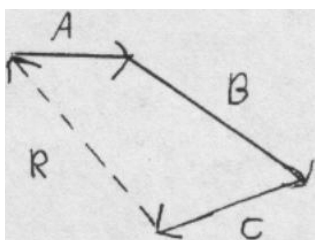


FIGURE 16. Example of the procedural difficulty in *closing the loop*.

5.3. The pizza slice question

This question (Fig. 14) was included on a midterm examination at New Mexico State University to probe the students' understanding of force addition for a situation where the net force is not zero. After modified instruction, 104 students from a calculus-based physics course were asked. Unlike the superposition question, for this question the magnitudes of the forces were not given explicitly. Figure 15 shows a correct answer.

About 60% gave an answer that correctly indicated the magnitude and the direction of the resultant force acting on the slice of pizza. About one-third gave a force opposite to the correct direction. All of the students who gave this answer drew a resultant that *closed the loop,* connecting the head of the resultant to the tail of the first force located. All of the students who *closed the loop* also gave the correct magnitude. Figure 16 shows an example of this reasoning difficulty. It may be that these students fail to differentiate between situations where the net force on an object is zero and situations in which there is a resultant force.

6. Conclusions

Many of the same difficulties that were observed when we asked students to add vectors without any context also appeared for questions about the addition of forces. However, additional difficulties were prompted by the context of force addition. After traditional instruction, many students still fail to recognize the vector nature of forces in situations where there is a net force or where the net force is zero.

A number of procedural and reasoning difficulties associated with the addition and subtraction of forces persist after instruction. The use of *Tutorials in Introductory Physics*[1] at New Mexico State provides an opportunity to practice the addition of vectors in the context of forces, and the subtraction of vectors in the context of kinematics. In modified lecture instruction, the focus on conceptual development includes an emphasis on the geometrical manipulation of vectors. Although student performance in questions about forces and Newton's second law has improved as a consequence, it is still disappointing.

It seems that many students hold incorrect beliefs about the nature of specific forces that interfere with their ability to reason about these forces correctly. After modified instruction that includes an emphasis on graphical methods, students still fail to recognize the relationship between the magnitudes and directions of forces, and some of them are not able to use vectors in solving problems about forces. We have described students' incorrect beliefs about the nature of specific forces that interfere with their ability to reason about forces correctly. For example, in the gymnast question, some students relate the tension in the ropes to their lengths. For this reason, we began a separate study into student understanding of tension in several contexts.

Finally, as Flores[9] said, "most of the undergraduate students have a problem understanding the fundamental physics concepts, primarily with vector operations. The development of the mathematical objects that represent physical concepts determine a cognitive evolution of the student's mathematical structures in the learning physical concepts".

1. L.C. McDermott, P. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice-Hall, New Jersey, 2002).
2. D. Hestenes, M. Wells, and G. Sawckhamer, *Phys. Teach.* **30** (1992) 14.
3. P.S. Shaffer and L.C. McDermott, *Am. J. Phys.* **73** (2005) 921.
4. S. Sjoberg and S. Lie, "Ideas about force and movement among Norwegian pupils and students," Report 81-11, The Centre for School Science, University of Oslo, Blindere, Oslo 3, Norway, 1981.

5. R.F. Gunstone and R.t. White, *Sci. Educ.* **65** (1981) 291.

6. J. Clement, *Am. J. Phys.* **50** (1982) 66.

7. A. Arons, "A guide to Introductory Physics Teaching," (Wiley, New York, 1990).

8. L.C. McDermott, P.S. Shaffer, and M.D. Somers, *Am. J. Phys.* **62** (1994) 46.

9. S. Flores, "Student use of vectors in mechanics", dissertation to get the Physics PhD degree, New Mexico State University, July 1007.