# Comment on "Continuous groups of transformations and time-dependent invariants"

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Some errors in a recent paper (*Rev. Mex. Fís. E* **53** (2007) 112) dealing with the Noether theorem are pointed out.

*Keywords:* Noether's theorem; Lagrangian; continuous symmetries.

Se señalan algunos errores en un artículo reciente (*Rev. Mex. Fís. E* **53** (2007) 112) relacionado con el teorema de Noether.

*Descriptores:* Teorema de Noether; lagrangiana; simetrías continuas.

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In a recent paper [1] the relation between continuous groups of transformations and constants of motion for a system described by a Lagrangian is considered. Apart from the carelessness in the presentation of the theory and the lack of appropriate explanations in Ref. 1, the example given in the paper, which involves the usual Lagrangian for a one-dimensional harmonic oscillator in Newtonian mechanics, seems suspicious at first sight because the group of transformations employed consists of Lorentz transformations. In fact, making use of the formulas given in Ref. 1, one can readily verify that the condition for the existence of a constant of motion associated with a one-parameter group of point transformations (Eq. (16) of Ref. 1) is not satisfied and a direct computation shows that the derivative with respect to the time of the "constant" of motion obtained there (Eq. (26) of Ref. 1) does not vanish.

The aim of this note is to point out in some detail the most important flaws in the presentation of Ref. 1 and, in some cases, add a comment to clarify the issue. A very clear and detailed treatment of the subject can be found in Ref. 2.

At the beginning of Sec. 2, a one-parameter group of transformations is considered, which means that the functions $f^j$ appearing in Eqs. (1) and (2) are given. However, immediately after Eq. (2), the functions $f^j$ are not assumed given, but defined as the solutions to a system of ordinary differential equations [Eqs. (3)], but the objects appearing on the right-hand side of Eqs. (3) are not defined at all.

Similarly, the functions $\xi^j_a$, appearing on the right-hand side of Eq. (6), are never defined. One can verify that the expression given by Eq. (6) is the solution to Eqs. (5) only if the $\xi^j_a$ are constants and coincide with the $\xi^j$. It seems that in Eq. (7) and throughout Ref. 1, there is summation over repeated indices, though there is no explicit mention of that convention.

There is a superscript $j$ missing on the $f$ appearing on the right-hand side of Eq. (8). Equation (11) and the following displayed equation involve new undefined symbols $x^{2'}$ and $\partial x^{2'}$; furthermore, the origin or meaning of the relations given by the displayed equation after Eq. (11) is not explained at all.

From Eqs. (15) one might guess that the two variables $x^1$ and $x^2$ are no longer independent, but that $x^2$ is somehow considered as a function of $x^1$. Also, the origin or meaning of Eq. (16) is not given and there is no indication as to whether, given a Lagrangian, there exist $\xi$ and $\eta$ such that Eq. (16) holds and how one could find them, if they exist. The right-hand side of Eq. (16) contains a function $f$ that might be confused with one of the $f$s employed before; however, such an identification would be wrong, the $f$ appearing in Eq. (16) is a new, arbitrary function. In fact, making use of Eqs. (14) and (16) one can readily verify that Eq. (18) holds for any differentiable function $f$.

As shown in Ref. 2 (Sec. 10.3), when $f$ is a function of $q$ and $t$ only, Eq. (16) amounts to the existence of a one-parameter group of point transformations that leave the Lagrangian $L(q, \dot{q}, t)$ invariant up to the total derivative of a function of $q$ and $t$ only.

It is not explained in which way the *physical meaning* of the invariant (19) may depend on "the selection of the parameters of the equation of motion", as claimed.

The meaning of the symbols appearing in Eqs. (21) is never explained (take, for instance, the $x_a$). The meaning of $\delta v^a/\delta t$ appearing in Eqs. (22) is not given and it is not even mentioned of what space the $e^a$ form a basis. The origin of the minus sign appearing on the right-hand side of Eq. (23) accompanying the undefined symbol $\bar{\nabla}$ is not given.

As already pointed out above, the "constant" of motion obtained in Ref. 1 [Eq. (26)] is not a constant of motion as one can readily verify making use of the equation of motion $\ddot{x} = -\omega^2 x$. Furthermore, the terms on the right-hand side of Eq. (26) do not correspond (owing to the factor $x$ in front of the bracket) to the description given in the paper. In connection with this fact, it is pertinent to add two comments. When the method is applied correctly, with each one-parameter group of point transformations that leave the Lagrangian invariant up to the total derivative of a function, one obtains a constant of motion, which may be the sum of several terms; however, each of these terms does not necessarily have to be a constant of motion separately. On the other hand, a mechanical system with $n$ degrees of freedom can have, at most, $2n - 1$ independent constants of motion; thus, in the case where $n = 1$ (as in the example considered in Ref. 1), there is, at most, only *one* independent constant of motion (in the case of the harmonic oscillator, this constant is the total energy of the oscillator or a function of it).

It is not clear if the parameter $m$ appearing in the relation between $k$ and $\omega$ given at the end of the paragraph containing Eq. (26) is different from the $m_0$ introduced, without explanation, in Eq. (26). Finally, by contrast with the claim in Sec. 4 of Ref. 1, Eq. (24), *defines*, essentially, the angular momentum but does not provide the conservation law of angular momentum.

1. A.L. Gelover-Santiago and M.G. Corona-Galindo, *Rev. Mex. Fís. E* **53** (2007) 112.
2. H. Stephani, *Differential equations: their solution using symmetries* (Cambridge University Press, Cambridge, 1989).