On the teaching and learning of physics problem solving

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This article presents a six-step problem-solving strategy, aimed at addressing three major problems in the learning and teaching of physics: 1) the demand by physics instructors for effective teaching strategies that could help in the teaching of intuitive conceptual and quantitative reasoning in physics, and how to teach both aspects holistically; 2) the students' need for suitable methodology that could help students to fill the gap in textbooks on enhancing their mathematical reasoning abilities, which are essential for reinforcing students' knowledge of conceptual physics; and 3) a deficiency in the teaching of physics leading to students not being taught a coherent physics problem-solving strategy that would enable them to engage in both mathematical and conceptual reasoning.

After a review of publications made by the *Physics Education Research group* (PER), the importance of a structured, systemic methodology to solve physics problems is considered. Then a structured, systemic methodology for solving physics problems is described by extending the well-known problem-solving steps presented by Polya. The proposed strategy includes the following steps: 1. Understand the problem, 2. Provide a qualitative description of the problem, 3. Plan a solution, 4. Carry out the plan, 5. Verify the internal consistency and coherence of the equations used, and 6. Check and evaluate the obtained solution.

Finally, an illustrative example is provided: the calculation of the moment of inertia of a thin hollow right circular cone.

Keywords: Physics problem-solving; physics learning; teaching of physics; quantitative reasoning.

Este artículo presenta una metodología de seis pasos para resolver problemas, con la intención de abordar tres grandes problemas en el aprendizaje y la enseñanza de la física: 1) la demanda de los profesores de física de estrategias de enseñanza efectivas, que permitan ayudar en la enseñanza del razonamiento conceptual y cuantitativo en física, y cómo enseñar ambos aspectos de manera integral 2) la necesidad de los estudiantes de contar con una metodología adecuada que los ayude a cubrir las deficiencias de los textos en cuanto al fortalecimiento de sus habilidades de razonamiento matemático, las cuales son esenciales para consolidar el conocimiento conceptual de física y 3) una deficiencia en la enseñanza de la física en presentarle a los estudiantes una estrategia coherente de solución de problemas, que los involucre en razonamiento tanto cuantitativivo como cualitativo.

Después de presentar una revisión bibliográfica en cuanto a publicaciones sobre el tema aparecidas en el área de *Investigaciones sobre la Enseñanza de la Física* (PER, por sus siglas en inglés *Physics Education Research*), la importancia de una metodología estructurada y sistemática para resolver problemas de física es considerada. Luego, por extensión del conocido esquema de resolución de problemas de Polya, se describe una metodología sistémica y estructurada para resolver problemas de física. La estrategia propuesta incluye los siguientes pasos: 1. Comprender el problema, 2. Proporcionar una descripción cualitativa del problema, 3. Planificar la solución, 4. Ejecutar el plan, 5. Verificar la consistencia interna y la coherencia de las ecuaciones utilizadas, y 6. Inspeccionar y evaluar la solución obtenida.

Por último, se presenta un ejemplo ilustrativo: el cálculo del momento de inercia de un cono circular recto hueco muy delgado.

Descriptores: Resolución de problemas en física; aprendizaje de física; enseñanza de la física; razonamiento cuantitativo.

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1. Introduction

It is not difficult to find instructors of general physics courses anxiously perusing articles published by the Physics Education Research (PER) community, searching essentially for advice on how to approach the teaching of physics effectively. Despite the demand for an actually useful pedagogy of physics, PER has not produced so far any ultimate theory in this regard, and the great amount of published research on the subject, in addition to being controversial [1-9], might be overwhelming and even confusing to physics instructors in the sense that since physics is intrinsically a quantitative based subject, much of the recent research favors an overemphasis on qualitative (conceptual) physical aspects [5,10-12], while standard mathematical abilities, which are crucial for understanding physical processes, are not stressed, or even taught, because, rephrasing a passage from a recent editorial [13], they interfere with the students' emerging sense of physical insight. Consequently, physics instructors face the problem of finding suitable advice on how to approach the teaching of physics in the most efficient way and an answer to the question of how much time should be spent on intuitive conceptual reasoning and how much time in developing quantitative reasoning. Let us mention, in passing, that the aforementioned editorial [13] has produced an interesting debate [14-16] regarding the benefits and shortcomings of science education reform in the United States in relation to its influence on the development of reasoning skills on the students, expressed in the way students use or apply the materials learned in their courses.

On the other hand, physics instructors need also be mindful of the importance of selecting the most appropriately functional textbook, basically because innovative active-learning teaching methods require students to acquire basic and fundamental knowledge through reading textbooks. Correspondingly, innovative teaching strategies should be designed to help students in processing their ever thicker and heavier textbooks, which are laden with physical and mathematical insights [5,17-20]. Thus, the panorama regarding the learning of physics is even more dramatic on the side of the students. For one reason, in their struggle to fully participate in the process of learning, at the moment of trying to find suitable learning materials that could help them to go beyond classroom instruction (*i.e.* aiming to develop self-confidence on their own through exercising their role as active learners), students face the dilemma of deciding which textbook could be helpful: perhaps a conceptual physics textbook (*i.e.* [21]), the student might wonder; or maybe a calculus-based physics textbook (*i.e.* [22]); or why not an algebra-based physics textbook (*i.e.* [23]); or what about a combination of all of them? These questions could have the student to verify his/her pocket/handbag to see if the money in there could be enough to take some extra weight home (for a good account of the drama of choosing a textbook, see for instance [24] and references therein). For another, in a typical course work for students majoring in science and/or engineering, they usually need to take more than one physics class. It could happen that in one term his/her physics instructor may emphasize quantitative reasoning over conceptual analysis, and in another term the respective instructor could rather accentuate conceptual learning over quantitative analysis, likely causing confusion for students, leading them to wonder which emphasis is correct.

Finally, it is not difficult to find results published by the PER community in which the inability of students to express, interpret, and manipulate physical results in mathematical terms is shown directly or indirectly. That is, students shows a clear deficiency in their training to exploit the mathematical solution of a problem (which sometimes could be obtained mechanically or by rote procedures) to enhance their knowledge regarding conceptual physics [2,3,11,25-28]. More importantly, the analysis of published excerpts of student's responses to interviews conducted by some researchers to further understand students' way of reasoning while solving physics problems, shows that students lack a structured methodology for solving physics problems [2,11,12,26]. These findings can not be surprising at all. In fact, none of the most commonly recommended physics textbooks (*i.e.* [22,29,30]) make use of a consistent, clear problem-solving methodology when presenting the solution of the textbook worked out for illustrative examples [31]. Moreover, the lack of a coherent problem-solving strategy can also be found in the solutions given in both the student and instructor manuals that usually accompany textbooks. Generally, problem-solving strategies in standard textbooks encourage the use of a formula-based scheme as compiled by the formula summary found at the end of each chapter of the text, and this strategy seems to be common even in classroom teaching [32]. Consequently, students merely imitate the way in which problems are handled in the textbooks. Furthermore, from the aforementioned student interview excerpts, one can also appreciate the lack of reasoning skills trying to associate or connect a way to solving a problem with the solution of other similar problems from another previous context (*i.e.* by using analogies). Again, the absence of this skill can not be surprising at all because students are just mirroring the unrelated way in which commonly used physics textbooks present the themes (*i.e.* the use of analogies is not fostered) [33-35].

2. On the importance of a structured, systemic methodology to solve physics problems

To further motivate the subsequent discussion, let us summarize our introductory commentaries. We are essentially pointing out three major problems in the learning and teaching of physics:

1) the demand by physics instructors for effective teaching strategies that would explain how much time should be spent on teaching intuitive conceptual reasoning and how much time on developing students' quantitative reasoning, and how to teach both aspects holistically;

2) the students' need for suitable textbooks that will help them develop mathematical abilities reasoning, which are essential for enhancing their knowledge of conceptual physics; and

3) a deficiency in the teaching of physics leading to students not being taught a coherent physics problem-solving strategy that would enable them to engage in both mathematical and conceptual reasoning.

A moment of thought about the above summarized difficulties leads us to postulate the need for a systemic [36,37] approach which, from an operational point of view, could help instructors and students to achieve a better performance in the process of teaching and learning physics.

On the instructor's side the need for a systemic approach in the teaching of physics could be justified by the advantage of using a methodology which would help them to incorporate both conceptual and mathematical reasoning systematically in their teaching. In this way, students will obtain the necessary training in their computational skills while learning how to use mathematical formulae to obtain the physics in the equations, even when they can obtain the mathematical solutions to a problem by rote procedures. In other words, students could apply "higher-order thinking skills." [38] via the mathematical understanding of a physics problem, which in turns involves meaningful learning which goes beyond the mere application of rote procedures. Moreover, using properly designed quantitative problems that require students to illustrate their conceptual learning and understanding will reveal much to instructors about their students' learning and will provide invaluable feedback [17,39-41]. Such problems can also be a powerful way to help students to understand the concepts of physics [38,41], a point emphasized by the great Nobel prize-winning, physicist Lev Davidovich Landau on the importance of first mastering the techniques of working in the field of interest because "fine points will come by themselves." In Landau's words, "You must start with mathematics which, you know, is the foundation of our science. [...] Bear in mind that by 'knowledge of mathematics' we mean not just all kinds of theorems, but a practical ability to integrate and to solve in quadratures ordinary differential equations, etc." [42] To further enhance their reasoning skills, the students would have the opportunity to increase their intuitive conceptual skills in the physics laboratory, where conceptual learning is reinforced by experience [43,44].

On the student's side, the need for a systemic approach in the learning of physics could be justified by the usefulness of applying a working methodology which could help them to approach the learning of physics from a interrelated point of view. That means that his/her knowledge of mathematics is useful for mastering ideas from physics, and that the use of analogies are important in approaching the solution of physical, mathematical and engineering problems. In short, this kind of practical, unified problem-solving strategy will help students to formulate and address any kind of problem. In other words, with such an approach, students would internalize the fact that it is in physics classes where they can start to apply what they have learned in their math classes and to find new non-formal approaches to performing computations [45]. To paraphrase Heron and Meltzer, learning to approach problems in a systematic way starts from teaching and learning the interrelationships among conceptual knowledge, mathematical skills and logical reasoning [46]. The problem that arises after the opening of the Millennium Bridge can further illustrate the needs for teaching and learning based on a systemic approach which recognizes the interrelatedness of every aspect of a physical process (physics, mathematics, and engineering design) [47].

3. A systemic, structured methodology for solving physics problems

Earlier work on the importance and necessity of a problem-solving strategy can be found in the work of the great mathematician George Polya [48,49], who placed emphasis on the relevance of the systematicity of a problem-solving strategy for productive thinking, discovery and invention. Some of his views, either provocative or encouraging, about teaching and learning can be found in some PER publications, like for instance his statements that *teaching is not a science* (*i.e.* [2]) and that *teaching is an art* (*i.e.* [50]), and his views on *the aim of teaching* (*i.e.* [6,38]) and on the importance of *problem-solving skills* (*i.e.* [3,39,40,51]), etc. For a further detailed account of Polya's work please refer to [48,49,52-55].

In *How to Solve It*, Polya set four general steps to be followed as a problem-solving strategy:

P1 *Understanding the problem*,

P2 *Devising a plan*,

P3 *Carrying out the plan*, and

P4 *Looking back*.

Surprisingly, these steps encompass "the mental processes and unconscious questions experts explore as they themselves approach problem solving" [55]. These four steps also form the basis for some computational models devised to "model and explore scientific discovery processes" [55]. Nevertheless, even though the aforementioned four steps seems very simple, their generality makes it hard for novices to follow them.

Thus, in order to have a more approachable problem-solving strategy for students, we extended the four-step problem-solving strategy into a six-step strategy. We made our choice based on empirical observations after experimenting with a five-step strategy reported in Ref. 51. Justification for having a more detailed problem-solving strategy can be found in the words of Schoenfeld: "First, the strategies are more complex than their simple descriptions would seem to indicate. If we want students to use them, we must describe them in detail and teach them with the same seriousness that we would teach any other mathematics" [56]. In addition, we shall further rationalize below the need for explicitly including the new step in our proposed methodology (see item 5 below). Accordingly, our proposed six step problem-solving strategy is as follows:

1. **Understand the problem**: some considerations to develop at this step involve drawing a figure and asking questions like What is the unknown? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? That is, at this stage students need to actually be sure what the problem is. In addition to making drawings to get a grasp of the problem, students might need to reformulate the problem in their own words, making sure that they are obtaining all the given information needed for solving the problem. This is a crucial step in the sense that *if we do not know where are we going, any route will take us there*.

2. **Provide a qualitative description of the problem**: at this stage students need to think and write down the laws, principles, or possible formulations that could help them to solve the problem. For instance students need to consider any possible framework of analysis that could help them to represent or describe the problem in terms of the principles of physics (*i.e.* Newtons law, energy conservation, momentum conservation, theorem of parallel axis for computing inertia moment, non-inertial reference system, etc.) If necessary,

the drawings of the previous step could be complemented by the corresponding free-body and/or vector diagram.

3. **Plan a solution**: some considerations to have in mind in order to develop this step involve looking at the unknown and trying to think of a familiar problem having the same or a similar unknown. Some questions to be asked are: Have you seen this before? Or have you seen the same problem in a slightly different form? Once the student as all the many possibilities of approaching approach the problem, he/she only needs to pick one strategy of solution and write down the corresponding mathematical formulation of the problem, avoiding as much as possible plugging numbers into the respective equations. Also, they need to think whether the information at hand would be enough to find a solution (*i.e.* if a set of algebraic equations is under-or over-determined, or if the number of boundary conditions provided is enough to solve a differential equation).

4. **Carrying out the plan**: at this stage the student will try to find a solution to the mathematical formulation of the problem sketched according to the previous steps, and perhaps will need to go back in order to find an easier mathematical formulation of the problem. This can be facilitated if the students have written down alternative solutions as they were supposed to do on item 2.

5. **Verify the internal consistency and coherence of the equations used**: at the moment of finding a solution to the mathematical equations involved, students need to verify whether the equations are consistent with what they represent (*i.e.* are the equations dimensionally correct? Do they represent a volume or a surface?). Though this seems to be an unnecessary step, experience shows that students too often do not verify the internal consistency and coherence of the equations they solve. And this mistake is also found to be performed by textbook writers, as discussed in a recent editorial [57]. After verifying no inconsistencies are found in the mathematical solution to the problem, students could then plug numbers into the obtained results to find, whether required or not, a numerical solution which in turn could be used in the next step to further evaluate the obtained result. In the next section, by means of an illustrative example, we shall show how the right answer to the problem posed could be obtained, even though the internal consistency of an equation used is not right [58].

6. **Check and evaluate the obtained solution**: once a solution has been obtained, its plausibility needs to be evaluated. Some questions could be asked in this regard: can the results be derived differently? Can the result or the method be applied to solve or fully understand other problems? Can the solution be used to write down the solution of a less general problem? Can the solution be used to further understand the qualitative behavior of the problem? Is it possible to have a division by zero by changing a given parameter? Does it makes sense?, and so forth.

A first comment on our six-step problem-solving strategy is that it provides a unified, systemic way of approaching the solution to a physical problem encompassing both qualitative (steps 1-3) and quantitative (steps 4-6) reasoning. In this sense, instructors could place as much emphasis as they choose on any of the set of steps, providing the students with a structured recipe on how to approach in detail the other side of the problem's solution. That is, if the instructor decides to emphasize steps 1-3, students could still follow steps 4-6 at their own pace, and viceversa. Second, comparing our six-step problem-solving strategy with Polya's four-step scheme, it could be appreciated that we have explicitly divided Polya's step one (P1) into two steps (1-2), and Polya's step three (P3) into two steps (4-5). A further comment on our problem-solving strategy is that we prefer to call the second step (2) *Provide a qualitative description of the problem* rather than *Physics description* as in Ref. 51, because one shares the idea that students tend to think that, by providing a qualitative analysis of a problem, they are also providing the solution required by a physicist, and that the mathematical solution to the problem is just uninteresting mathematics. Instead, we place emphasis on the fact that a physical solution to a problem is a combination of both qualitative and quantitative reasoning. As stressed by the great physicist Lord Kelvin: "I often say that when you can measure something and express it in numbers, you know something about it. When you can not measure it, when you can not express it in numbers, your knowledge is of a meager and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of science, whatever it may be." [21] Freeman Dyson was more eloquent: "...mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created" [59].

4. Illustrative example

In the following example, we shall present an approach on how to introduce students to the use of our proposed six-step problem-solving strategy. Though each one of the steps has its importance, we shall provide further evidence of why step five needs to be taught explicitly. It is pertinent to point out that, paraphrasing Polya's words, by proper training students could absorb the steps of our problem-solving strategy in such a way that they could perform the corresponding operations mentally, naturally, and vigorously. The dynamics of teaching is left to the instructor. In this article we are not claiming to show how the teaching should be carried out. Innovative teaching strategies can be found elsewere [5,17-20,51,60].

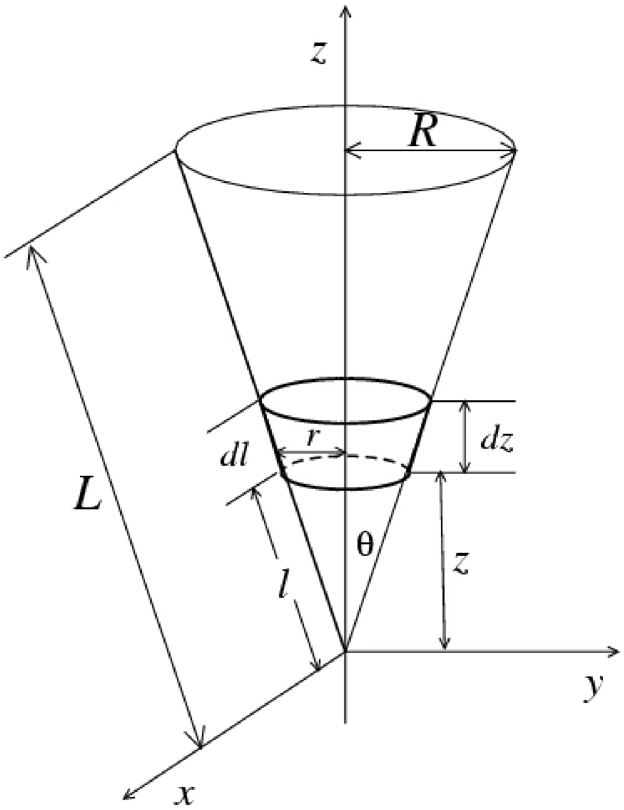

FIGURE 1. A hollow right circular cone with radius R, lateral length L, and uniform mass M. The cone's high is $H{=}\sqrt{L^2-R^2}$. The figure also shows at the lateral distance l, measured from the cone's apex at the origin of the coordinate system, an infinitesimal ring of lateral length dl and radius r. Two useful geometrical relations among some of the dimensions shown in the figure are $r=(R/L)\,l$ and $r=(R/H)\,z$.

4.1. The problem statement

Problem: About its central axis, find the moment of inertia of a thin hollow right circular cone with radius R, lateral length L, and mass M uniformly distributed on its surface with density σ.

1. **Understand the problem**: "It is foolish to answer a question that you do not understand. $\cdots$ But he should not only understand it, he should also desire its solution" [48]. Following Polya's commentary, before attempting to solve this problem, students need to have been exposed to a basic theory on computing moment of inertia (I). Particularly, students need to be familiar with the computation of I for a thin circular ring about its main symmetric axis. To further understand the geometry of the present problem, students could, for example, have a discussion about the shape of an empty ice cream cone. After some talk, a drawing better than the one shown in Fig. 1 could be presented on the board. Let's mention that additional ways of presenting each step in meaningful ways can be found in Refs. 48 and 51.

2. **Provide a qualitative description of the problem**: In this step one could further motivate the discussion by associating the computation of I with rotational motion quantities (*i.e.* kinetic energy, angular momentum, torque, etc.). One can even motivate the qualitative discussion by considering the hollow cone as a first crude approximation of a symmetric top or of a cone concrete mixer. The drawing of Fig. 1 could even be made more explicative.

3. **Plan a solution**: "We have a plan when we know, or know at least in outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown. ... We know, of course, that it is hard to have a good idea if we have little knowledge of the subject, and impossible to have it if we have no knowledge. ... Mere remembering is not enough for a good idea, but we cannot have any good idea without recollecting some pertinent facts" [48]. Accordingly, at this stage instructors could point out the superposition principle to solve the problem by slicing the hollow cone into a set of small, infinitesimal, rings distributed along the symmetrical axis of the cone. Thus, each infinitesimal ring will have in common the same rotational axis about which the moment of inertia of them is already known: $dI = r^2dm = r^2\sigma dS$, where r is the radius of each ring, while dS represents the respective infinitesimal surface of each ring.

4. **Carrying out the plan**: To carry out the plan, it won't be a surprise to choose the wrong dS. In fact, it is not difficult, at first sight, to choose wrongly (see Fig. 1): $dS = 2\pi r dz = 2\pi(R/H)zdz$, which leads to $S = \pi RH$, as the hollow cone surface (this result is of course wrong). Using this surface element, the momemnt of inertia for the small ring takes the form $dI{=}2\pi\sigma(H/R)r^3dr$, which leads to $I{=}2\pi\sigma(H/R)(R^4/4){=}(1/2)(\sigma S)R^2{=}MR^2/2$, as the required moment of inertia of the hollow cone (which is the right answer). It is not difficult to get students performing this sort of computations and they become uneasy when trying to convince them that in spite of having found a correct result, it is specious because it was obtained via a wrong choice for dS. Eventually students might agree on the incorrectness of their procedure if asked to compute explicitly the cone's mass.

5. **Verify the internal consistency and coherence of the used equations**: "Check each step. Can you see clearly that the step is correct? Can you prove that it is correct? ... Many mistakes can be avoided if, carrying out his plan, the student checks each step" [48]. Judging from our teaching experience, it is only too easy for students to perform without hesitation the incorrect computations, as presented in the previous step. And it is not easy to get students to realize their mistake. For God's Sake, they have computed the right answer!!!:

for a hollow thin cone, rotating about its symmetric axis, $I = MR^2/2$!!!. In this situation, to make students aware of their mistake, the easy way is the experiment. Instructors could unfold several hollow cones to actually show the students that the respective surface is $S = \pi RL$, instead of the wrongly obtained $S = \pi RH$. Accordingly, we hope to have provided enough evidence for the need to, explicitly and repeatedly, remind to students of the need to check each computational step, including checking for dimensionality correctness. In this case, the right approach is to consider $dS = 2\pi r dl = 2\pi l(R/L)dl$, which yields $S = \pi RL$, the right answer for S. This choice for dS leads to $dI{=}2\pi\sigma(L/R)r^3 dr$, which yields $I{=}2\pi\sigma(L/R)(R^4/4){=}(1/2)(\sigma S)R^2{=}MR^2/2$, the right answer.

Considering that it is not hard to find stories on reported wrong results due to wrong or incomplete computations [47,61], this problem could also be used as an example of how computations of a physical quantity (the surface of a cone shell) can be used to judge a mathematical result (the wrong value for S) that is used in additional computations yielding a right answer..

6. **Check and evaluate the obtained solution**: "Some of the best effects may be lost if the student fails to reexamine and to reconsider the completed solution" [48]. After gaining confidence with the obtained solution of the problem, it is necessary to spend some time in evaluating its plausibility. Examining the solution to our problem one could ask: it is not striking that the rotational inertia for a hollow cone about its symmetric axis is the same as for a solid disk having the same uniformly distributed mass M and radius equal to the cone's base? Is it not a counter example to the statement that rotational inertia only depends on how the mass is distributed around the axis of rotation? Furthermore, if for some reason the wrong choice for the dS was not caught in the previous step, it could be detected by analyzing the case of having a non-constant σ. A further interpretation of the result can be found at [62].

5. Concluding remarks

Previous work by researchers in *Physics Education Research* [3,39,40,45,46,51] shows clear evidence that when applied via active teaching and learning strategies [51,63-65], a problem-solving methodology increases students' performance in solving physics problems quantitatively and help them to enhance their conceptual understanding of physics concepts.

Nevertheless, the number of published "Comments on . . . " and "Reply to . . . " articles, in which much of the discussion is about the incorrectness of the physical interpretation of a concept or an idea, is indicative of the fact that the qualitatively understanding of the concepts of physics is a very elusive task, which even experienced researchers can fail to grasp [66].

Let's finish by recalling a particular point of view which the great mathematician Polya stressed very much in his writings and that, in some sense, can be considered as an "axiomatic thought" about the art of teaching and learning. He was emphatic regarding the fact that "*for efficient learning, the learner should be interested in the material to be learnt and find pleasure in the activity of learning*." In other words, inspiration to learn is without doubt a necessary condition in order to have an efficient and effective teaching and learning environment. This, is of course, by no means a new discovery, and, paraphrasing Schoenfeld [64], some ideas for circumventing a few of the barriers between the dedicated instructor and his/her students' attitudes in "learning" the subject that is being taught have been set forward in Refs. 64, 63 and 65. Nevertheless, one should keep in mind that "we know from painful experience that a perfectly unambiguous and correct exposition can be far from satisfactory and may appear uninspiring, tiresome or disappointing, even if the subject-matter presented is interesting in itself. The most conspicuous blemish of an otherwise acceptable presentation is the 'deus ex machina' "[67].

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