The wavelength of a laser diode and the birefringence of mica. The IPhO40 experimental exam

A. Cornejo-Rodríguez[a], R. Diaz-Uribe[b], R. Espejel-Morales[c], J. Jiménez-Mier[d],
N. Korneev[a], M.E. Ortiz[e], and V. Romero-Rochin[e]
[a]*Instituto Nacional de Astrofísica, Óptica y Electrónica.*
[b]*Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma México, 04510 México D.F.*
[c]*Facultad de Ciencias, Universidad Nacional Autónoma México, 04510 México D.F.*
[d]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma México, 04510 México D.F.*
[e]*Instituto de Física, Universidad Nacional Autónoma México,*
04510 México D.F.

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The experiments proposed and the equipment built for the 2009 International Physics Olympiad are described in this article. A list of the items used with details of the constructed elements, as well as an exhaustive discussion of both experiments, including the questions and the step by step solutions, are given.

Keywords: Physics education; diffraction; polarization; birefringence.

En este artículo se describen los experimentos propuestos y el equipo construido para la Olimpiada International de física 2009. Se da una lista de los aparatos usados y detalles de su construcción, así como una discusión exhaustiva de ambos experimentos, incluyendo las preguntas y las soluciones paso a paso.

Descriptores: Enseñanza de la física; difracción; polarización; birefrigencia.

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1. Introduction

The 40 International Physics Olympiad (IPhO40) was held in Mérida, México, in July 11-19, 2009. This competition consisted of two examinations, one theoretical and one experimental. This article is a description of the experimental part. The Physics Olympiads allow participant countries to develop strategies not only to prepare international competitive contestants but also to become aware of the level of knowledge of their high school students in this scientific discipline, in order to motivate their improvement. With this in mind,the Organizing Committee was especially concerned to prepare an experimental exam that was capable not only to test the technical abilities of the contestants, but also to evaluate their creativity, qualities not satisfied by a simple guided laboratory practice.

The experimental exam consisted of two problems, the aim of the first one being the measurement of the wavelength of a diode laser and the goal of the second one the determination of the birefringence of a mica crystal. All the contestants were given a kit with all the necessary elements that allowed them to carry out the measurements asked in the test. The competitors had to design and set up their experiment, align and calibrate the instruments by themselves.The actual examinations with solutions may be found at the official website of the International Physics Olympiads, http://www.jyu.fi/ipho/

The philosophy of the experiment consisted in giving the students the opportunity to develop their own experimental setup with given elements, rather than offering them the apparatus ready for doing the measurements. The financial limitations dictated minimal use of special laboratory equipment. Since both experiments were in the field of optics, they were performed on a very simple optical table made out of wood with a series of holes to fix elements on posts with screws and nuts. Common to both experiments were the wood optical table; a diode laser setup, including a diode laser with its power supply, a support post, and an ”S” clamp (Fig. 1, LABEL A in Fig. 3); and, a first surface mirror mounted on a movable mount with two adjusting knobs for fine alignment and support posts, (Label B), see Fig. 2. This setup allows the laser beam to be somewhat expanded by propagating freely through a distance of approximately 70 cm. The purpose of the movable mirror is to allow for alignment of the laser beam.

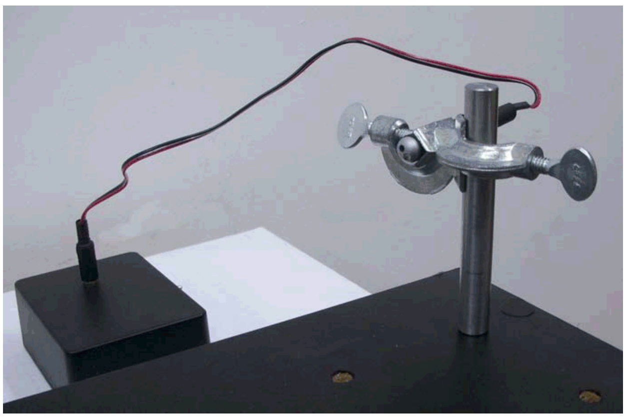

FIGURE 1. Diode laser, support post, “S” clamp and power supply box (LABEL A).

FIGURE 2. Mirror on a movable movable mount with two adjusting knobs and support post (LABEL B).

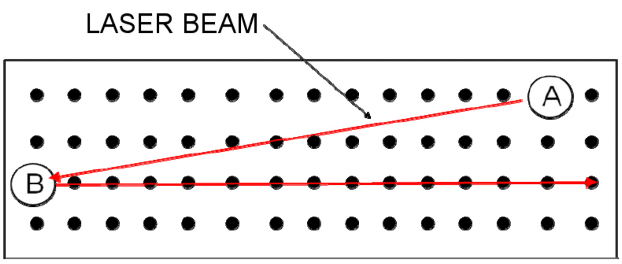

FIGURE 3. Mounting the laser and the mirror on the wooden optical table.

The source of the coherent light, namely, the diode laser,was a common inexpensive presentation laser pointer, red, with nominal wavelength 650nm, and with less than 1mW of output power. This information was not given to the contestants, of course. For the mirror holder we used a good quality mount (Thorlabs KMS); this was one of the most expensive parts of the kit. The mirrors were made at INAOE by evaporation of aluminum onto regular microscope slides. The mirrors were glued to the mounts. The power supply for the laser was designed and made at UNAM. It worked with 3 AA batteries, including a switch and a connector, and it was sufficient for the stable work of the laser during the 5 hours of the exam.

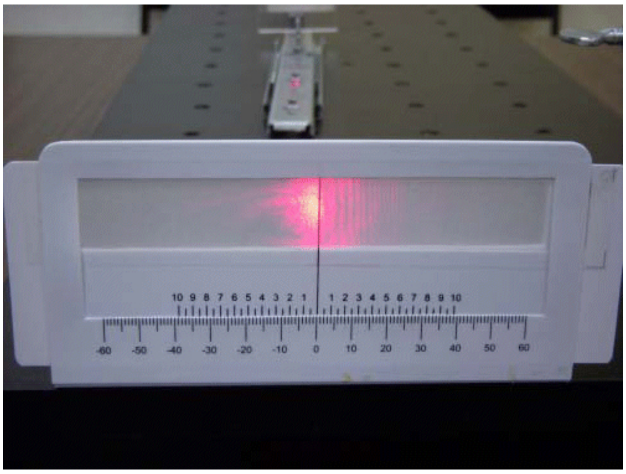

FIGURE 4. Typical fringes observed on the screen (LABEL E), produced by the interference of the beam passing through the focus of a lens and the diffraction caused by the sharp edge of a razor blade, see below.

2. First question: Determination of the wavelength of a laser diode

The students were asked to determine the wavelength of a diode laser without high precision micrometer range instruments, such as calibrated diffractive gratings. All the necessary measurements of the experiment had to be made with rulers and calipers, at most, at the scale of fractions of a millimeter. The wavelength had to be determined using light diffraction on a sharp edge of a razor blade.

Although the contestants had to design their own experimental setup, it was nevertheless explained that the basic phenomenon is obtained by placing the laser on an extreme of the optical table and directing the beam to a mirror, see Fig. 6. Once the laser beam (A) is reflected on the mirror (B), it must be made to pass through the center of a lens (C), which has a focal length of *a few centimeters*. It can now be assumed that the focus is a light point source from which a spherical wave is emitted. After the lens, and along its path, the laser beam hits a sharp razor blade edge as an obstacle. This can be considered to be a light source from which a cylindrical wave is emitted. These two waves interfere in the forward direction, creating a diffraction pattern that can be observed on a screen. See Fig. 4 with a photograph of a typical pattern.

One possible setup is given in Fig. 5. After the lens, a sliding rail supports a razor blade that has to move along a parallel line to the optical axis. For the rail we used a commercial kitchen drawer rail. The razor blade was mounted in a slide holder, which in turn was an acrylic support and fastened to the rail by means of a drilled plate to permit adjustments of the distance, see Fig. 7. To measure the diffraction pattern fringes comfortably, a special device was designed at UNAM, consisting of a translucent sliding screen with a caliper scale, in such a way, that it was easy to adjust the origin and to measure relative fringe positions, see Fig. 4.

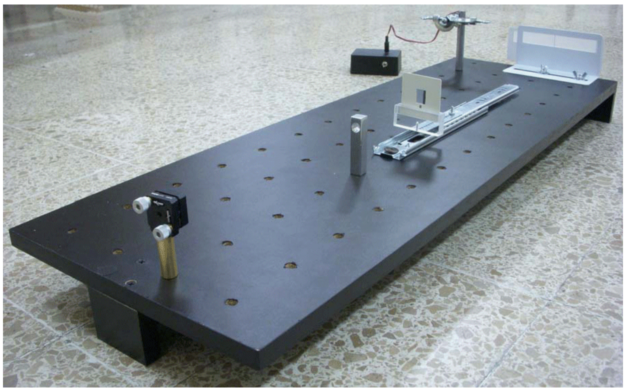

FIGURE 5. Photograph of a typical setup for determining the wavelength of a laser diode.

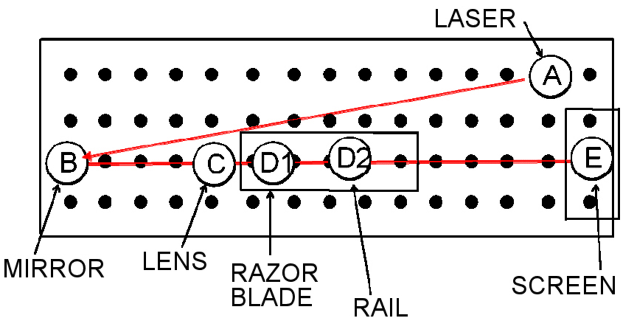

FIGURE 6. Scheme of the setup for determining the wave wavelength of a laser diode.

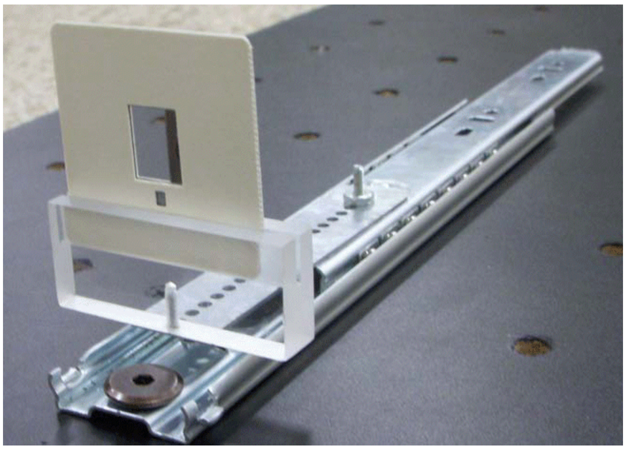

FIGURE 7. Razor blade in a slide holder, placed in an acrylic support (LABEL D1) and mounted on a sliding rail (LABEL D2).

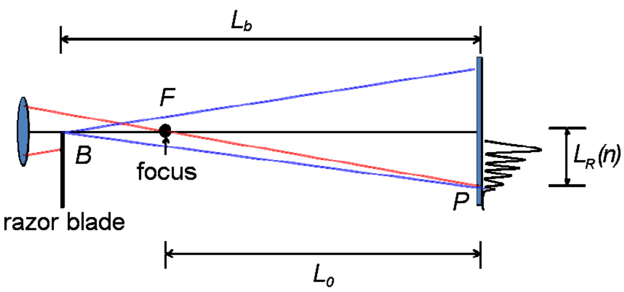

FIGURE 8. Case (I). The razor blade is *before* the focus of the lens. Figure is not to scale.

Thus, the experiment consists of an interference pattern produced by a beam emerging from the focus of the lens, considered as a point light source, and by the diffracted part of the laser beam caused by the razor blade in its path. Formally, the fringes are commonly explained in terms of diffraction theory in the Fresnel regime [1]. However, a more or less rigorous treatment in this case goes far beyond the mathematical level required for Olympiad problems. Nevertheless, it is possible to develop a simplified approach which allows for a good approximation and remains within the elementary mathematics and basic concepts of physics. The key idea here is the fact that the interference of two waves, with the same wavelength, depends on the optical paths difference. The complicated aspect is that there exist additional non-trivial phase differences, but these ones were given as explained below. As we discuss now, the experiment requires measuring the diffraction patterns produced in two different positions: first placing the razor blade between the lens and its focus and second placing it after the focus.

Referring to Figs. 8 and 9 above, there are five basic lengths:

L_0: distance from the focus to the screen.

L_b: distance from the razor blade to the screen, Case I.

L_a: distance from the razor blade to the screen, Case II.

$L_R(n)$: position of the n-th **dark** fringe for Case I.

$L_L(n)$: position of the n-th **dark** fringe for Case II.

The first dark fringe, for both Cases I and II, is the broadest one and corresponds to $n = 0$.

The experimental setup must be such that $L_R(n) \ll L_0$, L_b for Case I and $L_L(n) \ll L_0,\ L_a$ for Case II.

The phenomenon of wave interference is due to the difference in optical paths of a wave starting at the same point. Depending on their phase difference, the waves may cancel each other (destructive interference) giving rise to dark fringes; or the waves may add (constructive interference) yielding bright fringes.

A detailed analysis of the interference of these waves gives rise to the following condition to obtain a **dark** fringe, for Case I:

$$\Delta_I(n) = \left(n + \frac{5}{8}\right)\lambda \quad \text{with } n = 0, 1, 2, \ldots \qquad (1)$$

and for Case II:

$$\Delta_{II}(n) = \left(n + \frac{7}{8}\right)\lambda \quad \text{with } n = 0, 1, 2, \ldots \qquad (2)$$

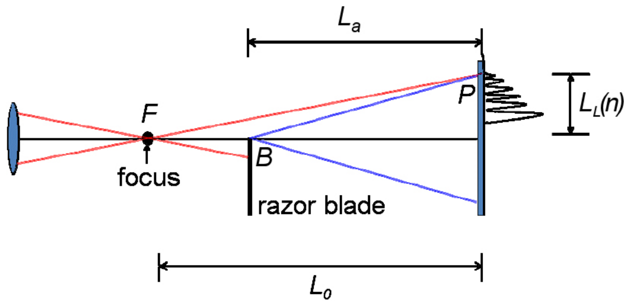

FIGURE 9. Case (II).The razor blade is *after* the focus of the lens. Figure is not to scale.

where λ is the wavelength of the laser beam. The condition for dark fringes, we recall, is that the optical path difference is a semi-integer multiple of the wavelength. However, a careful analysis [1] shows that additional phases are acquired. That is, by passing by the focus of the lens, a phase of π is added, while diffracting by a metallic edge, a phase of $5\pi/4$ is acquired. Fig. 10 indicates how these give rise to the phase differences of equations (1) and (2).

The difference in optical paths for Case I above is,

$$\Delta_I(n) = (BF + FP) - BP \quad \text{with } n = 0, 1, 2, \ldots.$$

while for Case II is,

$$\Delta_{II}(n) = (FB + BP) - FP \quad \text{with } n = 0, 1, 2, \ldots.$$

With this information, the students had to derive approximated expressions for the above optical paths differences. One may proceed as follows,

Case I:

$$\Delta_I(n) = (BF + FP) - BP$$
$$= (L_b - L_0) + \sqrt{L_0^2 + L_R^2(n)} - \sqrt{L_b^2 + L_R^2(n)}$$
$$= (L_b - L_0) + L_0\sqrt{1 + \frac{L_R^2(n)}{L_0^2}} - L_b\sqrt{1 + \frac{L_R^2(n)}{L_b^2}}$$

using $\sqrt{1+x} \approx 1 + x/2$ yields

$$\Delta_I(n) \approx \frac{1}{2}L_R^2(n)\left(\frac{1}{L_0} - \frac{1}{L_b}\right)$$

Case II:

$$\Delta_{II}(n) = (FB + BP) - FP$$
$$= (L_0 - L_a) + \sqrt{L_a^2 + L_L^2(n)} - \sqrt{L_0^2 + L_L^2(n)}$$
$$\approx (L_0 - L_a) + L_a\sqrt{1 + \frac{L_L^2(n)}{L_a^2}} - L_0\sqrt{1 + \frac{L_L^2(n)}{L_0^2}}$$

gives

$$\Delta_{II}(n) \approx \frac{1}{2}L_L^2(n)\left(\frac{1}{L_a} - \frac{1}{L_0}\right)$$

with $n = 0, 1, 2 \ldots$

In both cases, the dark fringe condition is approximate, and one can expect that the first dark fringe, with $n = 0$ has the biggest deviation in position. The measurement consists in taking the coordinates of consecutive dark fringes. However, it is not straightforward to measure L_0, $L_R(n)$ and $L_L(n)$ accurately. The first one because it is not easy to find the position of the focus of the lens, and the other two because the origin from which those lengths are defined may be very hard to find due to misalignments of the optical devices. The following is a suggestion that works very well.

To solve the difficulties with $L_R(n)$ and $L_L(n)$ first choose the zero (0) of the scale of the screen (LABEL E) as the origin for all the measurements of the fringes. Denote as and the (unknown) positions from which $L_R(n)$ and $L_L(m)$ are actually defined. And now, call $l_R(n)$ and $l_L(n)$ the positions of the fringes as measured from the chosen origin (0). We then have,

$$L_R(n) = l_R(n) - l_{0R} \quad \text{and} \quad L_L(n) = l_L(n) - l_{0L} \quad (3)$$

The point is that with these considerations one can devise a strategy to arrange the experimental data for a linear regression analysis - the students had to arrive to this conclusion. A possible solution is as follows.

From the condition of dark fringes, we have

$$\frac{1}{2}L_R^2(n)\left(\frac{1}{L_0} - \frac{1}{L_b}\right) = \left(n + \frac{5}{8}\right)\lambda$$

and

$$\frac{1}{2}L_L^2(n)\left(\frac{1}{L_a} - \frac{1}{L_0}\right) = \left(n + \frac{7}{8}\right)\lambda.$$

Using (3), $L_R(n)=l_R(n)-l_{0R}$ and $L_L(n)=l_L(n)-l_{0L}$ we can rewrite

$$\frac{1}{2}\left(l_R(n) - l_{0R}\right)^2\left(\frac{1}{L_0} - \frac{1}{L_b}\right) = \left(n + \frac{5}{8}\right)\lambda$$
$$\Rightarrow l_R(n) = \sqrt{\frac{2L_bL_0}{L_b - L_0}\lambda}\sqrt{n + \frac{5}{8}} + l_{0R}$$

and

$$\frac{1}{2}\left(l_L(n) - l_{0L}\right)^2\left(\frac{1}{L_a} - \frac{1}{L_0}\right) = \left(n + \frac{7}{8}\right)\lambda$$
$$\Rightarrow l_L(n) = \sqrt{\frac{2L_aL_0}{L_0 - L_a}\lambda}\sqrt{n + \frac{7}{8}} + l_{0L}$$

These can be cast as equations of a straight line, $y{=}mx{+}b$.

Case I:

$$y_R = l_R x_R = \sqrt{n + \frac{5}{8}} m_R = \sqrt{\frac{2L_bL_0}{L_b - L_0}\lambda} b_R = l_{0R}$$

Case II:

$$y_L = l_L x_L = \sqrt{n + \frac{7}{8}} m_L = \sqrt{\frac{2L_a L_0}{L_0 - L_a} \lambda} b_L = l_{0L}$$

Thus, by combining the equations for the slopes of the straight lines one obtains the length L_0 and the wavelength,

$$L_0 = L_a L_b \frac{m_R^2 + m_L^2}{L_a m_R^2 + L_b m_L^2}$$

$$\lambda = \frac{L_b - L_a}{2L_a L_b} \frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$

Therefore, in order to determine the wavelength, one must measure the distances from the blade to the screen L_b and L_a, and the dark fringes positions to obtain the slopes m_R and m_L. As we shall see below, one obtains a significant improvement on the uncertainty of the wavelength by also measuring directly the difference $L_b - L_a$ with a caliper.

The students were given a set of tasks to guide them and to obtain good quality results. These included the following steps.

1. First, the students needed to install correctly all elements and align the beam. For this, it was necessary to place the laser spot on the mirror at the height of the lens center, about 5 cm from the optical table surface, and along the direction defined by a line of holes in the projection on the optical table plane.

2. After this, the approximate direction was given to the beam by rotating the mirror post. The fine adjustment of the beam direction was performed with the screws on the mirror mount.

3. The correct alignment of the blade was a nontrivial procedure, and proved difficult even for some of the team leaders who tried it at the demonstration the day before the exam. It is necessary that the blade remains within the narrow beam for two positions of the holder, thus, the rail direction has to be parallel to the beam path within small error limits. The rail alignment can be done either by blind trial and error procedure, which can easily take 20 minutes or more, or by a coordinated adjustment of the rail and blade holder, which is much faster.

The measurement of the positions of the fringes is quite straightforward once the setup is well aligned. The best results are obtained by observing the fringes from the back side of the screen with a magnifying glass.

The following results, Table I and graphical analysis, Figs. 11 and 12, are the result of a typical run of the experiment.

Positions of the blade and their difference with higher precision:

$L_b \pm \Delta L_b (653 \pm 1) \times 10^{-3}$ m with the measuring tape.

$L_a \pm \Delta L_a (628 \pm 1) \times 10^{-3}$ m with the measuring tape.

$d = L_b - L_a = (24.6 \pm 0.1) \times 10^{-3}$ m with the caliper.

TABLE I.

$$x_R = \sqrt{n + \frac{5}{8}} \; x_L = \sqrt{n + \frac{7}{8}}$$

n	$(l_R(n) \pm 0.1) \times 10^{-3}$ m	$(l_L(n) \pm 0.1) \times 10^{-3}$ m	x_R	X_L
0	-7.5	1.1	0.791	0.935
1	-10.1	3.7	1.275	1.369
2	-12.4	6.4	1.620	1.696
3	-14.0	8.2	1.903	1.968
4	-15.6	10.0	2.151	2.208
5	-17.2	11.4	2.372	2.424
6	-18.4	12.2	2.574	2.622
7	-19.7		2.761	
8	-20.7		2.937	
9	-22.0		3.102	
10	-23.0		3.260	
11	-24.1		3.410	

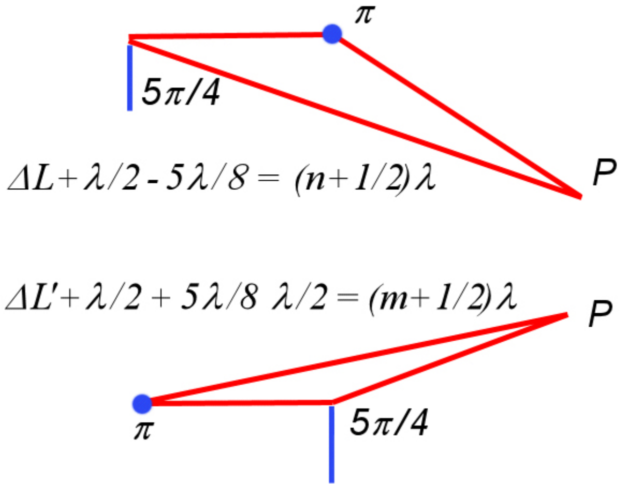

FIGURE 10. Scheme to indicate the additional phases acquired in its path by the laser beam as it crosses the focus of the length, phase π, and as it is diffracted by the razor blade, phase $5\pi/4$.

As in any experiment, and those of the Olympiad are no exception, a statistical analysis is required. It is expected that the contestants are familiar with the technique of least squares data analysis and with common error propagation to determine the uncertainty of their results. For the above set of data the results obtained with least square analysis are

$$m_R \pm \Delta m_R = (-6.39 \pm 0.07) \times 10^{-3} \text{ m}$$
$$m_L \pm \Delta m_L = (6.83 \pm 0.19) \times 10^{-3} \text{ m}$$

and (values of l_{0R} and l_{0L})

$$l_{0R} \pm \Delta l_{0R} = b_R \pm \Delta b_R = (-2.06 \pm 0.17) \times 10^{-3} \text{ m}$$
$$l_{0L} \pm \Delta l_{0L} = b_L \pm \Delta b_L = (-5.32 \pm 0.36) \times 10^{-3} \text{ m}$$

Finally, to obtain the wavelength and its error, using the suggestion to replace $d = L_b - L_a$, one gets

$$\lambda = \frac{d}{2L_a L_b} \frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$

resulting

$$\lambda \pm \Delta\lambda = (663 \pm 25) \times 10^{-9} \text{ m}$$

The emission wavelength of several laser diodes used at the Olympiad were measured with a professional apparatus (ACTON SP2150i) and the value obtained was $\lambda \pm \Delta\lambda = (655 \pm 1) \times 10^{-9}$ m. The result is therefore excellent, and one can appreciate the smallness of the error, 4 to 5 %, given the simple measuring instruments.

The calculation of the uncertainty may be obtained from the exact formula,

$$\Delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial d}\right)^2 \Delta d^2 + \left(\frac{\partial\lambda}{\partial L_a}\right)^2 \Delta L_a^2 + \left(\frac{\partial\lambda}{\partial L_b}\right)^2 \Delta m_R^2 + \left(\frac{\partial\lambda}{\partial m_L}\right)^2 \Delta m_L^2}$$

with

$$\frac{\partial\lambda}{\partial d} = \frac{\lambda}{d}, \quad \frac{\partial\lambda}{\partial L_b} = \frac{\lambda}{L_b}, \quad \frac{\partial\lambda}{\partial L_b} = \frac{\lambda}{L_a},$$
$$\frac{\partial\lambda}{\partial m_R} = \frac{2m_L^2}{m_R} \frac{\lambda}{m_L^2 + m_R^2}$$

and analogously for the other slope.

However, one may note that the errors due to L_a, L_b and d are negligible, the latter due to the measurement with the caliper. Moreover, $m_R^2 \approx m_L^2$ and$L_a \approx L_b$. This implies,

$$\frac{\partial\lambda}{\partial m_R} \approx \frac{\lambda}{m_R} \approx \frac{\partial\lambda}{\partial m_L}.$$

Thus,

$$\Delta\lambda \approx \sqrt{2}\frac{\lambda}{m_L}\Delta m_L \approx \left(25 \times 10^{-9}\right) \text{ m}$$

3. Second question: Birefringence of mica

For the second question the students were required to measure the birefringence of optical grade Muscovite mica. The idea was to use the same basic setup of the laser beam and the movable mirror, as a source of coherent light to be made normally incident on a thin plate of mica, after polarizing such a beam. By measuring appropriate transmitted intensities of the light by a simple detector, an analysis can be carried out to infer the difference of indexes of refraction of the material, namely, the birefringence of the crystal. The measurement of this quantity is the goal of this problem. Together with polarizers, birefringent materials are useful for the control of light polarization states.

Birefringence is, by no means, a simple phenomenon. A material that shows such a property is typically an anisotropic crystal in which the velocity of propagation of electromagnetic waves depends on the direction that the wave electric field vector makes with respect to a preferred direction in the crystal [3], called the optical axis. Thus, in general, polarized light incident on a slab made out of this material, will be transmitted elliptically polarized. This is in contrast with common transparent materials, such as window glass, which transmit polarized light in the same direction as the incident one, because its index of refraction does not

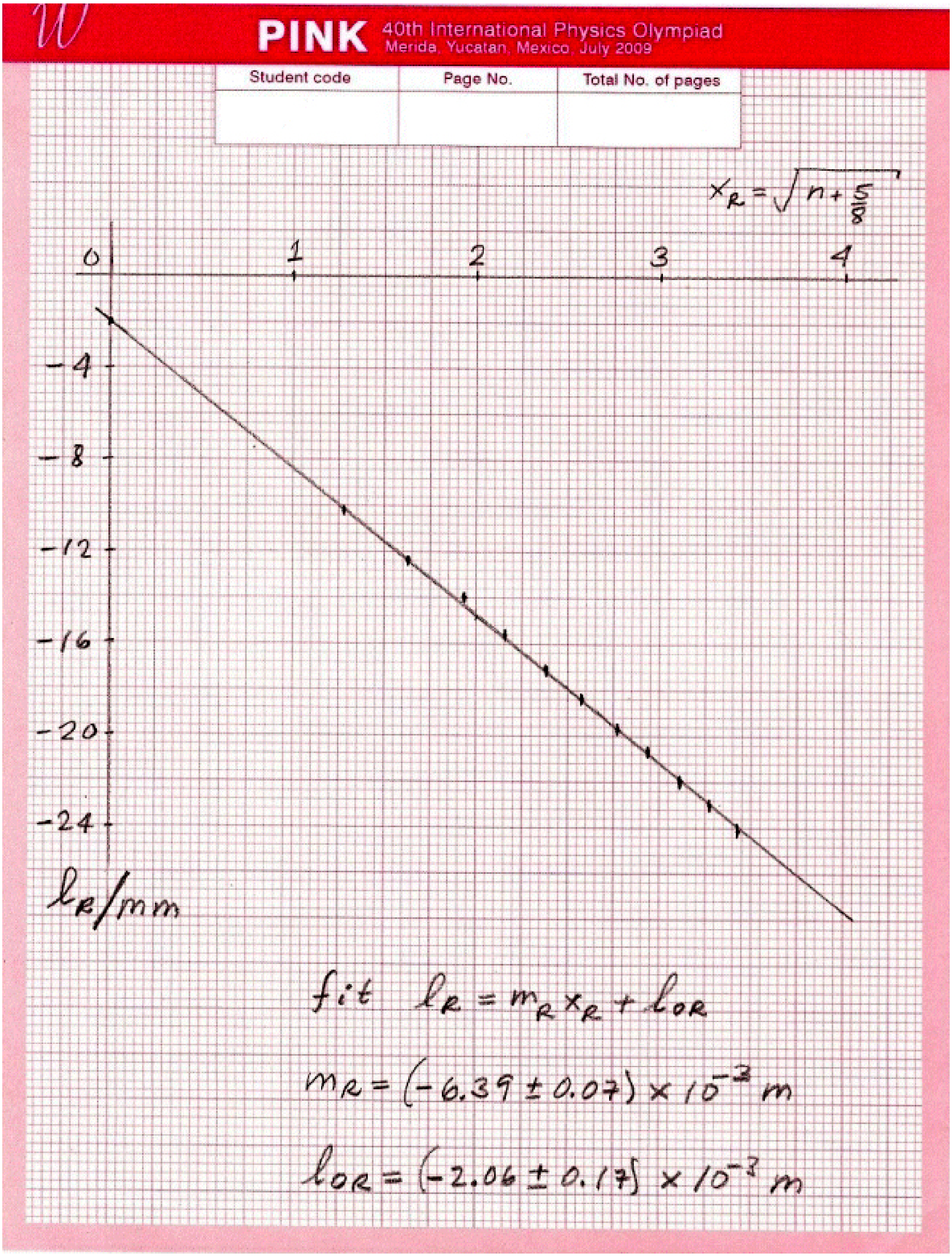

FIGURE 11. Graphical analysis of Case I.

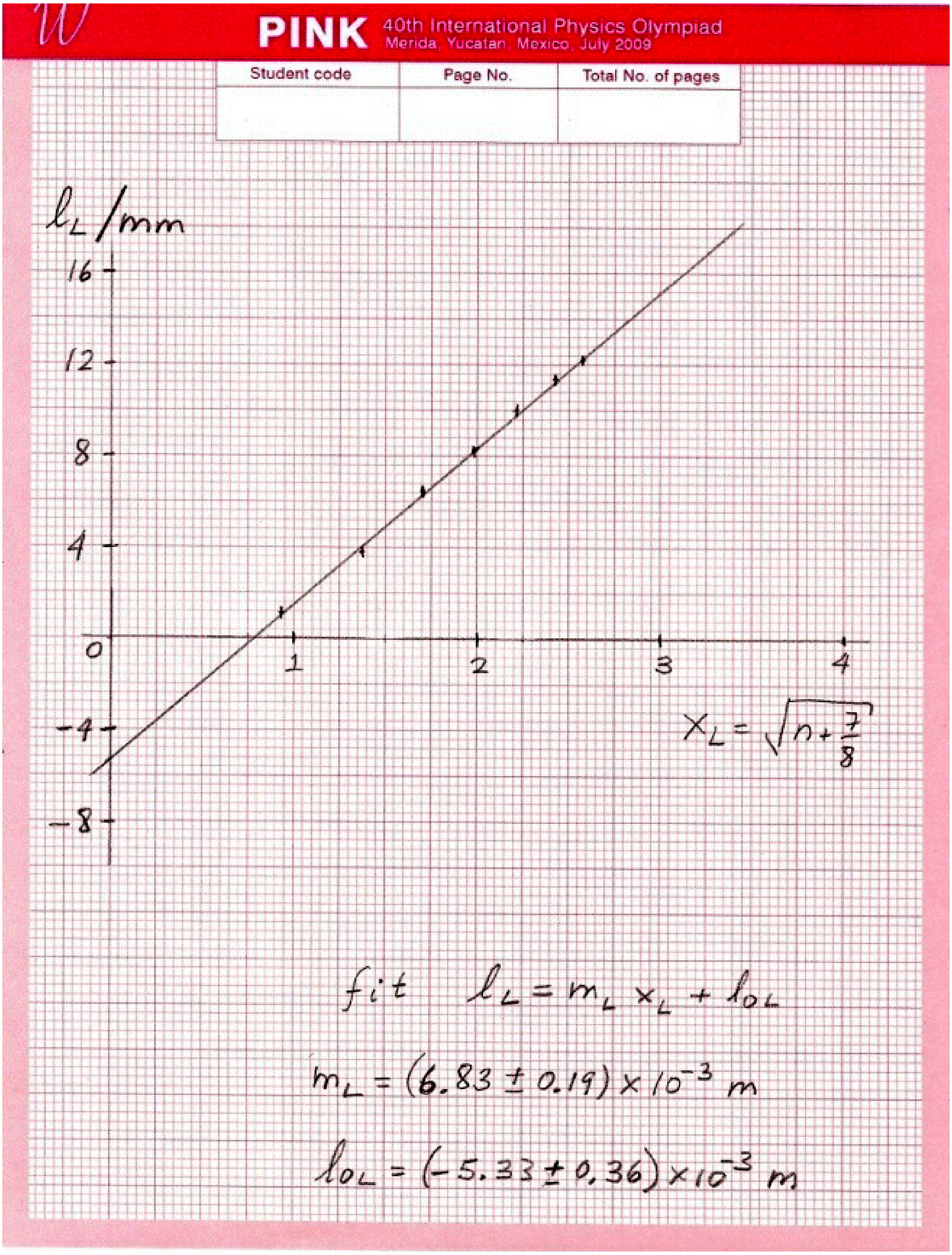

FIGURE 12. Graphical analysis of Case II.

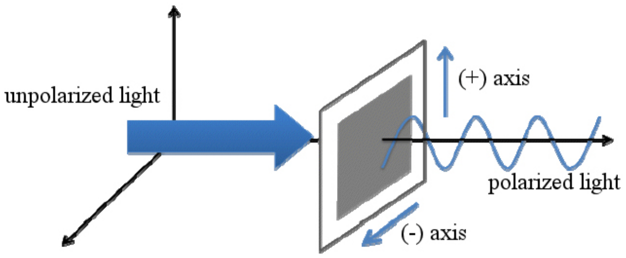

FIGURE 13. Unpolarized light normally incident on a polarizer. Transmitted light is polarized in the (+) direction of the polarizer.

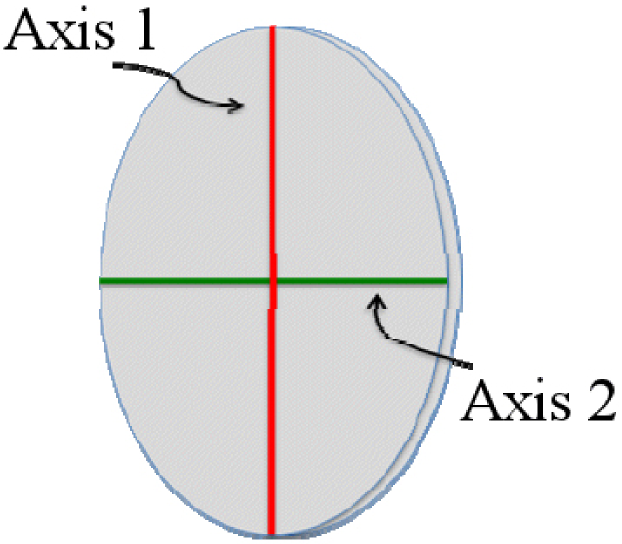

FIGURE 14. Thin slab of mica with its two optical axes, Axis 1 and Axis 2.

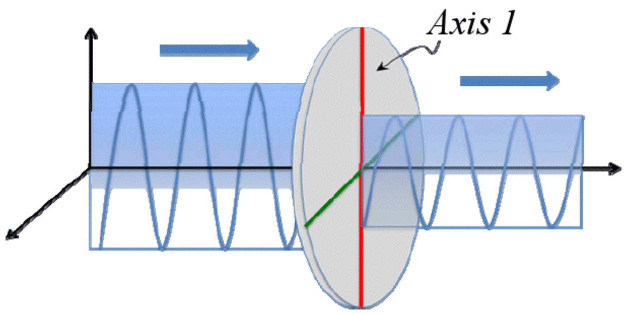

FIGURE 15. Axis 1 is parallel to polarization of incident wave. Refractive index is n_1.

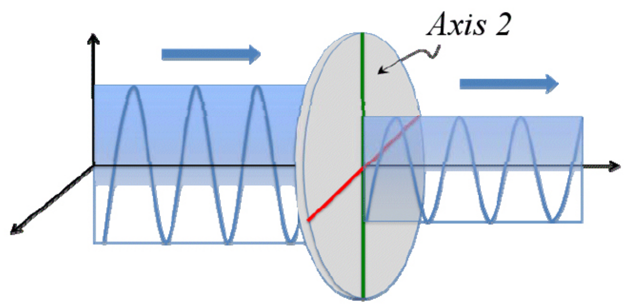

FIGURE 16. Axis 2 is parallel to polarization of incident wave. Refractive index is n_2.

depend on the direction and/or polarization of the incident wave. The analysis of arbitrary propagation in a birefringent crystal is quite complicated [4], but the basic phenomenon can be understood in terms of a polarized beam normally incident on a thin plate.

To guide the students to prepare their experiment, and partly because birefringence is not part of the official syllabus of the Physics Olympiads, we first gave a brief explanation of the use of a polarizing film as follows: A polarizing film, or simply, a polarizer, is a material with a privileged axis parallel to its surface, such that, transmitted light emerges polarized along the axis of the polarizer. Call (+) the privileged axis and (-) the perpendicular one.

Then, a slab of birefringent material properly cut or cleaved as we specify below, may be seen as having two characteristic orthogonal axes, which we will call Axis 1 and Axis 2. As a matter of fact, one of the axes is the so-called optical axis but this experiment neither distinguishes it needs to be specified.

One can then analyze two simple cases to exemplify the birefringence. Assume that a wave **polarized in the vertical direction** is normally incident on one of the surfaces of the thin slab of mica.

$Case\ 1)$ Axis 1 or Axis 2 is parallel to the polarization of the incident wave. The transmitted wave passes without changing its polarization state, but the propagation is characterized as if the material had either a refractive index n_1 or n_2. See Figs. 15 and 16.

$Case\ 2)$ Axis 1 makes an angle θ with the direction of polarization of the incident wave. See Fig. 17. The transmitted light has a more complicated polarization state. This wave, however, can be seen as the *superposition* of two waves, one that has polarization **parallel** to the polarization of the incident wave (*i.e.* "vertical") and another that has polarization **perpendicular** to the polarization of the incident wave (*i.e.* "horizontal").

For purposes of the analysis, we define I_P as the *intensity* of the wave transmitted *parallel* to the polarization of the incident wave, and I_O as the *intensity* of the wave transmitted *perpendicular* to polarization of the incident wave. These intensities depend on the angle θ, on the wavelength λ of the light source, on the thickness L of the thin plate, and on the absolute value of the difference of the refractive indices, $|n_1 - n_2|$. This last quantity is called the *birefringence* of the material. The measurement of the intensity of the light had to be done with a photodetector and a multimeter that we describe below, and it is important to recall that such measurements are independent of the polarization of the light incident on the detector.

Before presenting the theory necessary to analyze the intensities I_P and I_O as a function of the angle θ, we describe the parts that the contestants had to use to devise an appropriate experimental setup.

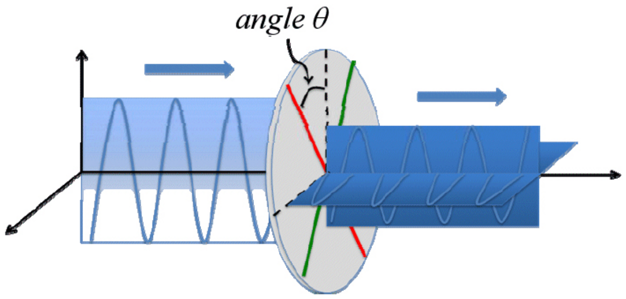

FIGURE 17. Axis 1 makes and angle θ with polarization of incident wave.

FIGURE 18. Polarizer mounted in slide holder with acrylic support (LABEL J).

The students were given a) two polarizing films (Rainbow Symphony Inc.) mounted in slide holders, each with an additional acrylic support (LABEL J), see Fig. 18; b) A thin mica plate mounted in a plastic cylinder with a scale with no numbers; acrylic support for the cylinder (LABEL K). The mica is mounted so that its axis can be rotated with respect to the polarization direction of the incoming beam, see Fig. 19. We used squares of mica of about 12.5 mm per side and thickness ranging from 35 to 160 ± 1 microns (they were measured by Mitutoyo digital mircometer). The fabrication of the acrylic supports and mounts, as well as the cleavage of the mica plates and its micrometric thickness measurement, were made at the Instituto de Fisica, UNAM. We used optical grade Muscovite mica (VENDOR).

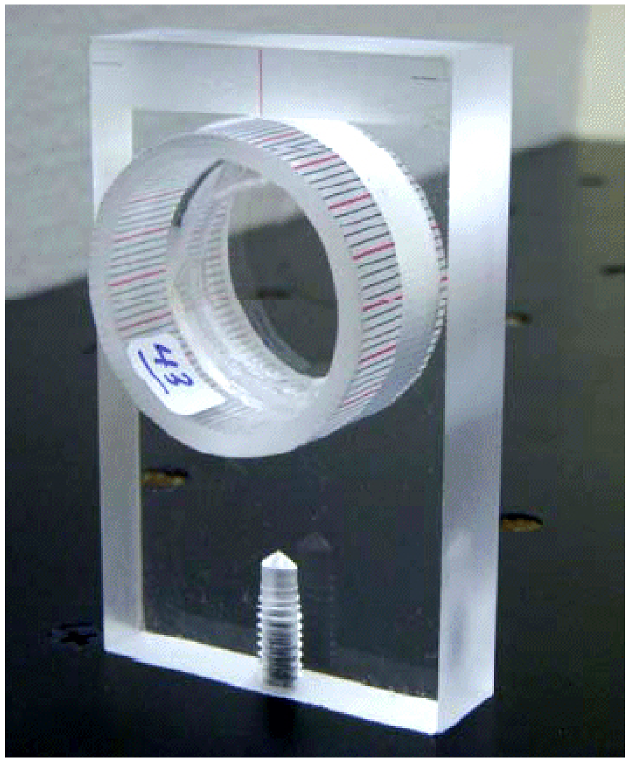

FIGURE 19. Thin mica plate mounted in cylinder with a scale with no numbers, and acrylic support (LABEL K).

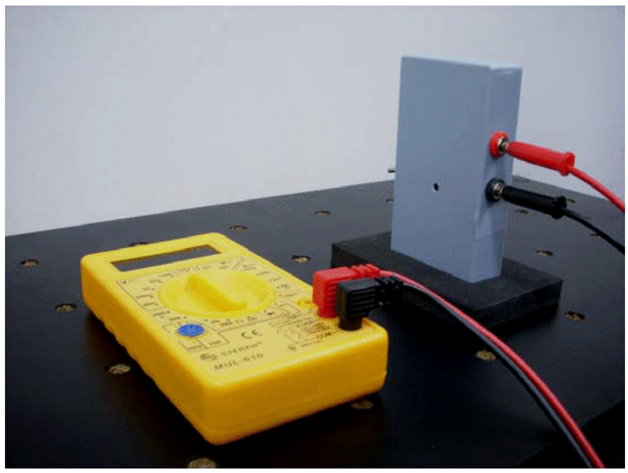

FIGURE 20. Light detector and multimeter.

The polarizer and the cylinder mount all had threaded holes at the bottom so that they could be fixed to the optical table with standard 1/4 - 20 screws. The heights were also chosen to have the centers of the polarizers and the cylinder axis all aligned with the laser beam. In our design all centers were 5 cm above the surface of the optical table.

The light detector, see Fig. 20, was a simple biased photodiode (Vishay BPV10) with a low-pass RC filter and a load resistor. The circuit used is shown in Fig. 21 where we also indicate typical values used for the components. A 1 mW laser beam saturates this circuit so a simple filter made of a black plastic trash bag was put in front of the photodiode. The resistor value and/or the filter thickness could be adjusted according to the laser intensity. Care must be taken so that at the maximum measured intensity the detector does not saturate. The circuit was feed with a regular 9 V battery and was built inside a standard electronic box with a hole on the side at the height of the laser beam (in our case 5 cm). The light intensity is linearly proportional to the voltage drop across the load resistor. Any laboratory voltmeter can be used to measure this voltage.

The experiment then consisted in making a polarized beam of laser light impinge on the thin mica plate and measure its transmitted intensity both parallel and perpendicular to the incident polarization, as a function of the angle setting of the mica plate. The polarized light was obtained by placing a polarizer before the mica plate, and for the measurement of the transmitted beam, another polarizer had to be placed after it. Thus, if the polarizers were parallel, the parallel transmitted intensity could be measured, and for crossed polarizers, the perpendicular intensity was obtained. A typical setup is shown in Figs. 22 and 23. Care had to be taken to ensure a good polarized incident beam due to the fact that the laser beam is already partially polarized. Thus, the first polarizer had to be place with either its (+) or (–) axes vertically in such a way to obtain the maximum transmitted intensity in the absence of any other optical device.

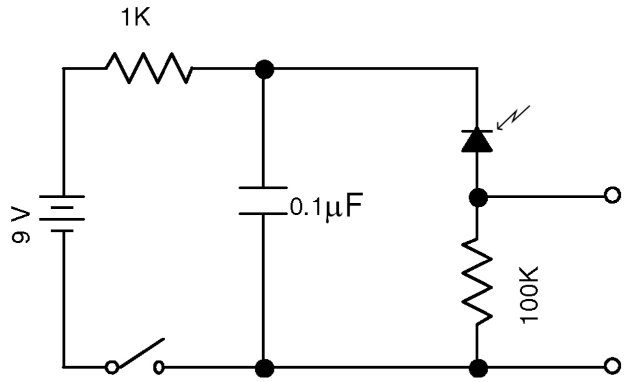

FIGURE 21. Circuit for the light detector.

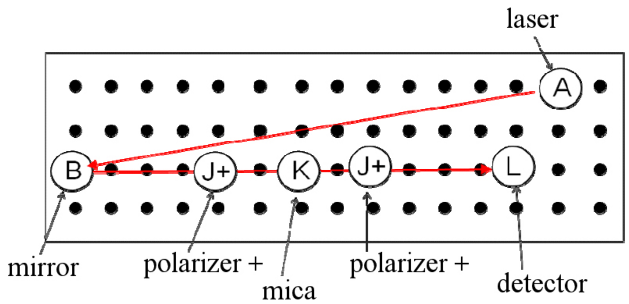

FIGURE 22. Experimental setup for I_P.

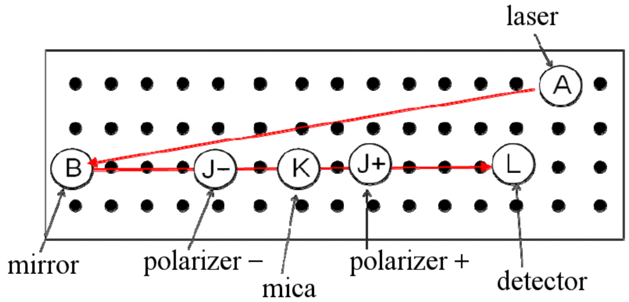

FIGURE 23. Experimental setup for I_O.

A delicate problem has to be solved to find the actual position of the optical axis of the mica. This is because the regular graduation outside the cylinder holding the mica plate was arbitrarily put, and there was no insurance that the one of the settings coincided with the axis. Thus, it was suggested to find the closest setting to the axis and use it as a provisional origin for the measured angles. Since there were 100 lines on the cylinder, the interval between two consecutive lines corresponded to 3.6 degrees. We suggested to call $\bar{\theta}$ the angles measured from the provisional origin and later to correct for the actual angle. The next step was to make as many measurements of the parallel and perpendicular intensities, I_P and I_O, as a function of the angle $\bar{\theta}$, and collect them in a table, as shown below. The students had to realize that it was enough to measure an interval around 90 degrees due to the periodicity of the intensities.

Now the students has to correct for the actual zero of the angles. It turned out that the optical axis 1 can be identified as that in which the perpendicular transmitted intensity was a minimum. Theoretically, it had to be zero, but the background light put it around 0.2 to 0.4 mV. In Table II one can see that the minimum is between 0.2 and 0.6 mV corresponding to the settings $\bar{\theta}$ equal to 0 and 3.6 degrees, respectively. Therefore, the actual location of the optical axis was between those settings, for this particular experiment. The students were left free to find the zero. Two possibilities are to make a graphical analysis of few points around the zero and another was to to fit a parabola to three points in the neighborhood of the zero. Thus, one should find a shift $\delta\bar{\theta}$, such that the actual angle is $\theta = \bar{\theta} + \delta\bar{\theta}$. In the example given, from Table II, we can choose the first three points of $\bar{\theta}$ and $I_O(\bar{\theta})$ (intensities in millivolts)

$$(x_1, y_1) = (-3.6, 1.1)\,, \quad (x_2, y_2) = (0, 0.2)\,,$$
$$(x_3, y_3) = (3.6, 0.6)$$

We want to fit $y = ax^2 + bx + c$. This gives three equations:

$$1.1 = a(3.6)^2 - b(3.6) + c$$
$$0.2 = c$$
$$0.6 = a(3.6)^2 + b(3.6) + c$$

whose solution is the location of the minimum of the parabola,

TABLE II.

$\bar{\theta}$ (degrees)	$(I_P \pm 1) \times 10^{-3}$ V	$(I_O \pm 1) \times 10^{-3}$ V
-3.6	46.4	1.1
0	48.1	0.2
3.6	47.0	0.6
7.2	46.0	2.0
10.8	42.3	4.9
14.4	38.2	9.0
18.0	33.9	12.5
21.6	27.7	17.9
25.2	23.4	22.0
28.8	17.8	27.0
32.4	12.5	31.7
36.0	8.8	34.8
39.6	5.2	38.0
43.2	3.6	39.4
46.8	3.2	39.6
50.4	4.5	38.7
54.0	6.9	36.6
57.6	10.3	33.6
61.2	14.7	29.4
64.8	20.1	24.7
68.4	25.4	19.7
72.0	30.5	14.7
75.6	36.6	10.2
79.2	40.7	6.1
82.8	44.3	3.2
86.4	46.9	1.0
90.0	47.8	0.2
93.6	47.0	0.4
97.2	45.7	2.0

$$\bar{\theta}_{\min} = -\frac{b}{2a} \approx 0.7 \text{ degrees}$$

and , therefore, $\delta\bar{\theta} = -0.7$ degrees.

To analyze the data, the students were given a little bit of theoretical considerations. For normal polarized incidence, and assuming that the system is truly birefringent with no absorption at all, one can find out exact expressions for the transmitted intensities in terms of the incident intensity [4]. As it turns out, those expressions are quite formidable and, in addition, there are absorption effects as well as other technical complications that prevent a simple direct analysis. Nevertheless, we found out if the one considers normalized intensities, angle by angle, defined as

$$\bar{I}_P(\theta) = \frac{I_P(\theta)}{I_P(\theta) + I_O(\theta)} \quad \text{and} \quad \bar{I}_O(\theta) = \frac{I_O(\theta)}{I_P(\theta) + I_O(\theta)}$$

simple approximated expressions can be obtained, and these are,

$$\bar{I}_P(\theta) = 1 - \frac{1}{2}(1 - \cos\Delta\phi)\sin^2(2\theta) \tag{4}$$

and

$$\bar{I}_O(\theta) = \frac{1}{2}(1 - \cos\Delta\phi)\sin^2(2\theta) \tag{5}$$

where $\Delta\phi$ is the difference of phases of the parallel and perpendicular transmitted waves. This quantity is given by,

$$\Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2|$$

where L is the thickness of the thin plate of mica, λ the wavelength of the incident radiation and $|n_1 - n_2|$ the birefringence. Thus, the problem reduces to determine the phase difference $\Delta\phi$, and if the thickness of the plate is known, as well as the light wavelength, the birefringence is obtained. As we

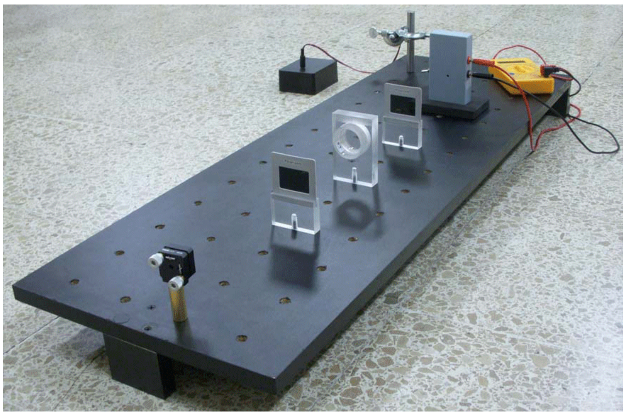

FIGURE 24. Experimental setup for measurement of birefringence of mica.

TABLE III.

$\bar{\theta}$(degrees)	$x = \sin^2(2\theta)$	$y = \bar{I}_O \pm 0.018$
2.9	0.010	0.013
6.5	0.051	0.042
10.1	0.119	0.104
13.7	0.212	0.191
17.3	0.322	0.269
20.9	0.444	0.392
24.5	0.569	0.484
28.1	0.690	0.603
31.7	0.799	0.717
35.3	0.890	0.798
38.9	0.955	0.880
42.5	0.992	0.916

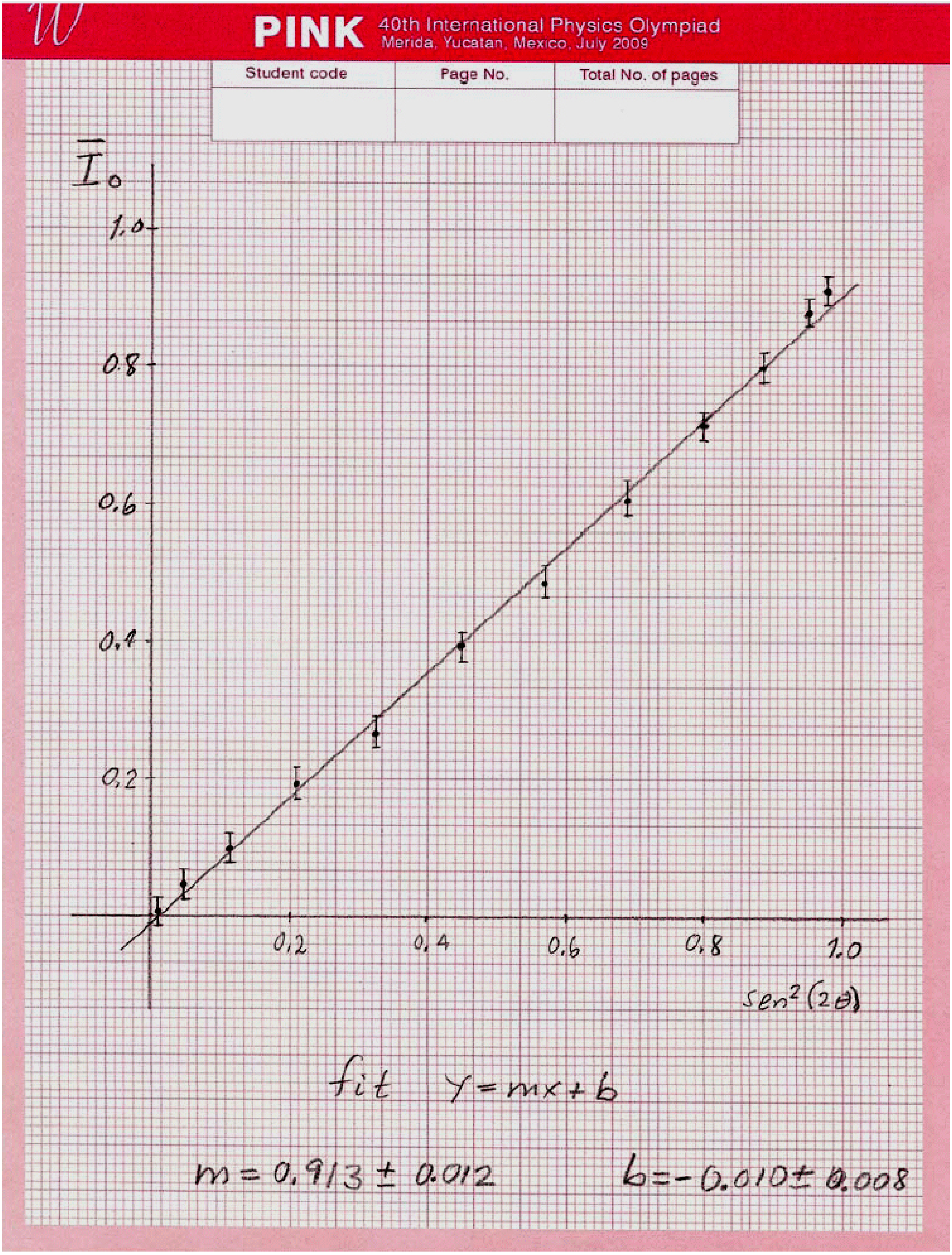

FIGURE 25. Plot of the normalized perpendicular intensity $\bar{I}_O$ versus $\sin^2 2\theta$.

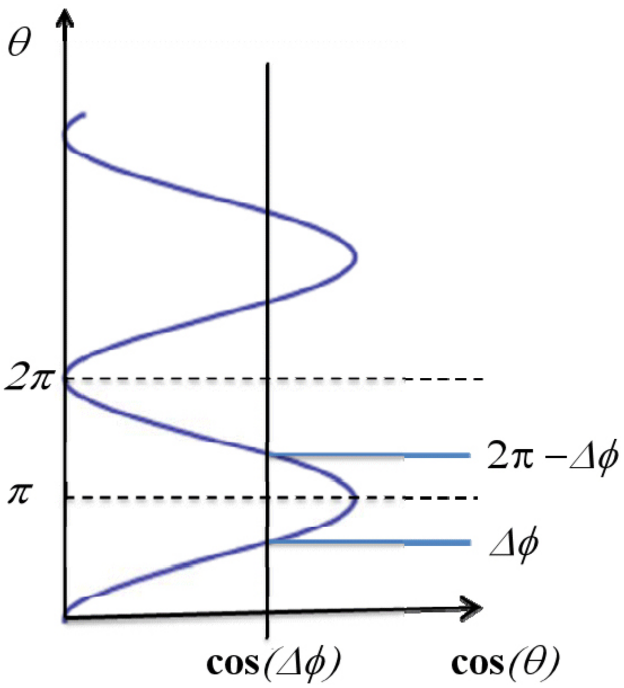

FIGURE 26. The phase angle θ as a function of $\cos\theta$, to show that it is a multiple valued function.

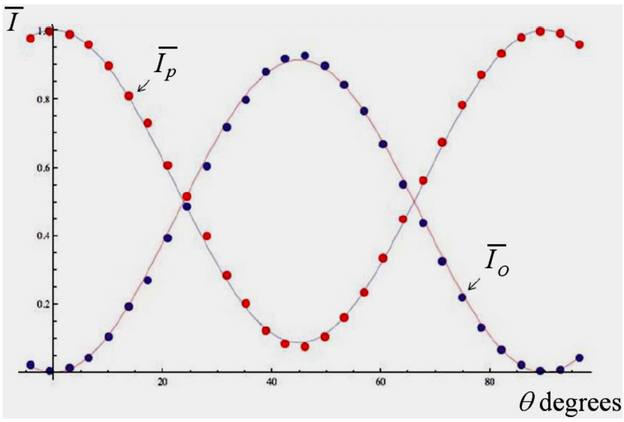

FIGURE 27. Comparison of experimental data for the normalized intensities $\bar{I}_P$ and $\bar{I}_O$, see Table III, with fitting equations (4) and (5) using the calculated value of the phase difference $\Delta\phi$.

shall see below, the fitting of the data by those formulas, once one has found the phase difference $\Delta\phi$, is remarkably good.

Thus, the problem reduces to find the phase difference $\Delta\phi$ first. The contestants were free to choose the appropriate variable to make their statistical analysis, but it appears as a simple solution to cast the perpendicular intensity, see equation (4), as a straight line $y = mx + b$, with

$$y = \bar{I}_O(\theta), \quad x = \sin^2(2\theta) \quad \text{and} \quad m = \frac{1}{2}(1 - \cos\Delta\phi)$$

from which the phase may be obtained. One may notice at this point that, actually, only angles θ from 0 to 45 degrees are needed. Table III shows the data needed for the analysis and Fig. 24 shows a plot of the corresponding values.

A least squares analysis yields the following results for the slope and the y-intercept,

$$m \pm \Delta m = 0.913 \pm 0.012$$

$$b \pm \Delta b = -0.010 \pm 0.008$$

As shown above, one needs the thickness of the slab, which for this particular experiment was $L \pm \Delta L = (100 \pm 1) \times 10^{-6}$ m and the wavelength of the laser beam. We use the value found in the first Question, $\lambda \pm \Delta\lambda = (663 \pm 25) \times 10^{-9}$ m. A direct calculation of the phase $\Delta\phi$ in radians in the interval $[0, \pi]$ yields, from the slope $m = 1/2\,(1 - \cos\Delta\phi)$,

$$\Delta\phi \pm \Delta(\Delta\phi) = 2.54 \pm 0.04,$$

whose uncertainty was calculated from

$$\Delta m = \left|\frac{\partial m}{\partial \Delta\phi}\right| \Delta(\Delta\phi) = \frac{1}{2}\sin(\Delta\phi)\Delta(\Delta\phi),$$

and therefore,

$$\Delta(\Delta\phi) = \frac{2\Delta m}{\sin(\Delta\phi)}.$$

Here, there is an additional consideration. One notes that adding $2N\pi$ to the phase $\Delta\phi$, with N any integer, or changing the sign of the phase, the values of the intensities and the slope m are unchanged. However, the value of the birefringence $|n_1 - n_2|$ would change, but this cannot be so. In other words, the phase is multivalued for a given value of $\cos\Delta\phi$, as shown in Fig. 26.

As we carefully measured, and we discuss this below, the value of the birefringence for the particular mica we used was $|n_1 - n_2|$ between 0.003 and 0.005, and we considered as its nominal value $|n_1 - n_2| = 0.004$. Therefore, the phase difference became $\Delta\phi = \pi$ for a plate thickness $L \approx 82 \times 10^{-6}$m. Therefore, we can conclude that

$$\Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2| \quad \text{if} \quad L < 82 \times 10^{-6}\text{ m}$$

or

$$2\pi - \Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2|$$
$$\text{if} \quad 82 \times 10^{-6}\text{ m} \le L < 164 \times 10^{-6}\text{ m}.$$

Thus, for our sample $L \pm \Delta L = (100 \pm 1) \times 10^{-6}$ m, and we used the second expression above, yielding a value for the birefringence,

$$|n_1 - n_2| \pm \Delta|n_1 - n_2| = (3.94 \pm 0.16) \times 10^{-3}.$$

The error was calculated using,

$$\Delta\left|n_1-n_2\right|=\sqrt{\left(\frac{\partial|n_1-n_2|}{\partial\lambda}\right)^2\Delta\lambda^2+\left(\frac{\partial|n_1-n_2|}{\partial L}\right)^2\Delta L^2+\left(\frac{\partial|n_1-n_2|}{\partial\Delta\phi}\right)^2\Delta(\Delta\phi)^2}$$

and thus,

$$\Delta\left|n_1-n_2\right|=\sqrt{\left(\frac{|n_1-n_2|}{\lambda}\right)^2\Delta\lambda^2+\left(\frac{|n_1-n_2|}{L}\right)^2\Delta L^2+\left(\frac{\lambda}{2\pi L}\right)^2\Delta(\Delta\phi)^2}.$$

As mentioned above, this experiment is fairly good to determine the phase difference $\Delta\phi$, as can be seen in Fig. 27, where we plot the actual data for the normalized intensities $\bar{I}_P$ and $\bar{I}_O$ versus θ, and the corresponding calculated values given by equations (4) and (5) using the phase $\Delta\phi = 2.54$. The agreement is quite remarkable given the simplicity of the experimental apparatus.

To conclude the discussion of this second question, we want to point out that we determined quite carefully the birefringence of the Muscovite mica we used. For this, we actually measured all the samples given to the students and much more, that is, of the order of 400 samples. The main difference was that the samples were obtained from cleaving thick pieces of mica as delivered by S&J Trading Inc, NY. These pieces had thickness of the order of one millimeter and we cleaved them to thickness ranging from 4 to several hundred micrometers. We gave the students samples with less than 150 micrometers, so the above formulas could be used. From all these measurements we arrived to our nominal value $|n_1 - n_2| = (4 \pm 1) \times 10^{-3}$. We were surprised to find out that this measurement is actually very good in comparison with reported values in the literature.

4. Final comments

The experimental exam of IPhO40 was well accepted by the leaders of the teams that participated in the Olympiad. In opinion of some, see Ref. 5 for instance, “The exam was fantastic…They had to think.”, and this is certainly a reason for joy and pride for the committee that prepared the problems. We believe the examination was actually hard and demanding for the students. Certainly only a handful of participants were able to tackle both problems satisfactorily, but many of them were capable of solving at least one of them in the five hours allowed. One must remember that the Physics Olympiads are a competition to find out the best among the best, and the exams are expected to be very difficult. We certainly believe that most of the participants, given enough time, would have been able to solve both problems.

But in addition to have provided a good and acceptable set of problems for an Olympiad, the committee believes that this exercise should and could be done as a regular activity to design interesting and useful experiments to be performed not only at competitions, such as an Olympiad, but as part of the regular laboratory classes at high schools.

Acknowledgments

The authors wish to thank the enthusiastic collaboration of INAOE and IFUNAM personnel who help solving several problems and tasks during the design of the experiments, in particular we gratefully acknowledge their shops for the careful construction of most of the the equipment parts.

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