How can acoustic resonance reduce the average velocity in a falling body?

V. Torres-Zúñiga
Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México,
Cd. Universitaria Apartado Postal 70-186, México, D.F. 04510, México,
e-mail: email:vicentz@gmail.com

Recibido el 15 de abril de 2010; aceptado el 17 de enero de 2011

In this article, a simple experiment is described to overcome the misconception that acoustic pressure and levitation effects are difficult to observe in school laboratories. Analysis of the free fall velocity of a toy parachute inside a vertical tube, driven by sound in a range of frequencies around the resonant condition, exhibits the resonance frequency, the node pressure zones, and the optimal conditions to obtain acoustical levitation of a light body.

Keywords: Resonant tube; acoustic pressure; high school demonstration.

En este artículo se describe detalladamente un experimento sencillo para superar la impresión errónea de que los efectos de presión y levitación acústica son difíciles de observar en laboratorios escolares. Estos fenómenos físicos se pueden estudiar mediante el análisis de la velocidad de un paracaídas de juguete en caída libre, dentro de un tubo transparente y vertical, impulsado por ondas estacionarias sónicas en el intervalo de frecuencias alrededor del quinto modo de resonancia del tubo. Enlazando conceptos de mecánica y acústica, con este experimento se determina la frecuencia de resonancia, las zonas de nodos de presión dentro del tubo, y las condiciones óptimas para obtener levitación acústica de un objeto liviano.

Descriptores: Tubo resonante; presión acústica; levitación acústica; demostraciones físicas escolares.

PACS: 01.50.Lc; 01.50.My; 43.20.Ks; 43.25.Uv

1. Introduction

Many high school students might find that concepts of mechanics and wave physics are not related; this idea can create serious misconceptions in the study of further physics topics. On the other hand, many videos about acoustic levitation that can be found in Internet sites, such as You-Tube, have captured the attention of many students [1,2], but many pupils think that these acoustic experiments are difficult to produce, and to study, in common school laboratories.

From the practical perspective, acoustic levitation has the advantage of being applicable to both liquid and solid materials without inducing chemical reactions. Indeed, either corrosive reactions or very high purity materials can be achieved and studied without the use of containers or holders [3]. Moreover, some extremely fragile materials are suitable for study via acoustic levitation techniques. This is the case of foam, which is very unstable because of the gravity that pulls the liquid compound downward from foam, drying and destroying the bubble system. Foam can be contained, manipulated and studied within acoustic fields. This kind of investigation can lead to a better understanding of how foam could be useful, for instance, for cleaning the ocean water [4].

In this article, we use a variation of the resonance tube experiment to analyze the effect of the acoustic field pressure in the average velocity of a free falling light object. Moreover, from the experimental procedure and the data obtained, it is possible to identify the resonance frequency, the nodes and antinodes in the resonant tube, and the velocity of sound in air. Finally, an acoustic levitation demonstration is also possible using the same simple experimental set-up.

2. Basic model of the velocity in a falling body into a resonance tube

When sound waves are transmitted in air, each tiny volumetric element in the fluid oscillates from an equilibrium position. In fact, only upward or downward displacements are usually considered in a vertical air column that simplifies the model to one dimension. In this work, an upward displacement is considered in the positive y-axis direction.

If $s(y,t)$ represent the displacement of a volume element respect to the equilibrium point in function of the position y and the time t. The equation of longitudinal displacement is, in the case of sinusoidal waves:

$$s = s_m \cos(ky - \omega t), \qquad (1)$$

where s_m is the amplitude, k the propagation constant, and ω the frequency. It is assumed that the wave travels in positive y-axis direction.

Although sound waves can be considered as displacement waves, in the case of wave superposition (*e.g.* sound waves into tubes) the assumption of that conception could lead to serious conceptual mistakes, such as supposed violation in the principle of conservation of energy [5]. In order to avoid those kinds of misconceptions, instead, a description of sound waves as pressure waves would be preferable. Moreover, the pressure changes and not the displacements can be detected both by our ears and by microphones. This pressure approach is very useful in an instrumentation context.

As any mechanical wave, the wave propagation speed depends on the ratio of an elastic property in the medium. In sound waves, this is the response of the medium to the pressure changes with a volume change that is known as bulk modulus B [6], which can be defined as:

$$B = -\frac{\Delta p}{\Delta V/V}, \quad (2)$$

where Δp is the pressure change, ΔV is the volume change of the volume V. The negative sign implies that the increment of pressure ($\Delta p > 0$) produces a reduction in volume ($\Delta V < 0$). We can obtain the wave equation in terms of the pressure variation. From Eq. (2), this pressure variation can be expressed as:

$$\Delta p = -B\frac{\Delta V}{V}, \quad (3)$$

where Δp represents the change from the no-perturbed equilibrium pressure p_0. We want to describe the change of pressure Δp in terms of the position and the time, this is $p(y,t)$. Thus, the real pressure in any point is $p_0 + \Delta p(y,t)$, which can be smaller or larger than p_0 if Δp is negative or positive, respectively, at that point and time.

A fluid layer, at pressure p_0 with thickness Δy and area A, in the transversal section has a volume $V = A\Delta y$. When the pressure changes, the volume changes by $A\Delta s$; where Δs is the change in the thickness in the layer during the air compression and rarefaction. Since $\Delta s \neq \Delta y$ are different numbers, Eq. (3) can be rewritten as:

$$\Delta p = -B\frac{A\Delta s}{A\Delta y}, \quad (4)$$

when the layer is compressed, this is $\Delta y \to 0$ and $\Delta s \to 0$, we get

$$\Delta p = -B\frac{\partial s}{\partial y}. \quad (5)$$

Since s is function of y and t, it is necessary to replace for the partial derivative. From Eq. (1) we find that

$$\frac{\partial s}{\partial x} = -ks_m \sin(kx - \omega t), \quad (6)$$

thus, Eq. (5) can be rewritten as:

$$\Delta p = \Delta p_m \sin(kx - \omega t), \quad (7)$$

Δp represents the change from the pressure p_0. The term $\Delta p_m = Bks_m$ corresponds to the maximum change of pressure and it is known as the pressure amplitude.

On the other hand, propagation of standing sound waves induced by a speaker (which is placed in the edge of a cylinder) is conformed by two pressure waves which travel in appositive directions [6]:

$$\Delta p_1(y,t) = \Delta p_m \sin(ky - \omega t),$$
$$\Delta p_2(y,t) = \Delta p_m \sin(ky + \omega t). \quad (8)$$

The superposition of both waves buildup the pressure standing wave Δp_s. It is easy to demonstrate that

$$\Delta p_s(y,t) = \Delta p_1(y,t) + \Delta p_2(y,t),$$
$$= [2\Delta p_m \sin ky]\cos(\omega t). \quad (9)$$

Therefore, in resonance conditions, the localization of the speaker corresponds to a pressure node $\Delta p_s(0,t) = 0$ and the open part in the tube corresponds to another pressure node $\Delta p_s(L,t) = 0$. Then, the fundamental frequencies of resonance ω_n in this system are

$$\omega_n = n\frac{v}{2L}, \quad n = 1, 2, 3, ... \quad (10)$$

Then, from the mechanical point the view, the forces in this system are the gravity mg ($g = 9.8$ ms^{-2}); the linear drag force F_d, and the acoustical force $A\Delta p_s$ that is opposite to the gravity. The linear drag force is referred to forces in opposite direction relative to the motion of an object through a fluid at relatively slow speeds, without turbulence (*i.e.* low Reynolds number, $\mathrm{R}_e < 1$). In this case, the equation for viscous resistance is [6]:

$$F_d = -bv, \quad (11)$$

where b is a constant that depends on the properties of the fluid and the dimensions of the object, and v is the velocity of the falling object. From the second law of motion and without considering the buoyancy effects, the net force in this system can be written as:

$$ma = mg - bv - A\Delta p_s, \quad (12)$$

where m is the mass of the body, and a is the net acceleration. When the falling object achieves its average terminal velocity v_t the downward force of gravity is equal to the sum of the upward drag force and the acoustical force. In this case the sinusoidal components of $A\Delta p_s$ produce small oscillations which are neglected in the average. This causes the net force on the object to be zero, resulting in zero acceleration. Then, the average terminal velocity can be expressed as:

$$v_t = \frac{mg - A\Delta p_s}{b}. \quad (13)$$

Since the pressure standing wave can produce a force $A\Delta p_s$ on a body of area A, the velocity in a falling body is drastically altered in the node and antinode points, where the acoustical force present its maxima and minima. Consequently, this average terminal velocity is different to the common terminal velocity, which is measured in classical experiments of drag forces [6]. Equation 13 is good enough to observe the changes of terminal velocity in different tube resonance conditions. In addition, the theoretical calculus of the velocity and the trajectory at any time and position of the falling body in the resonant tube implies to solve partial differential equations; such calculus are out of the scope of this work. However, such kind of calculus it will be presented in a future work.

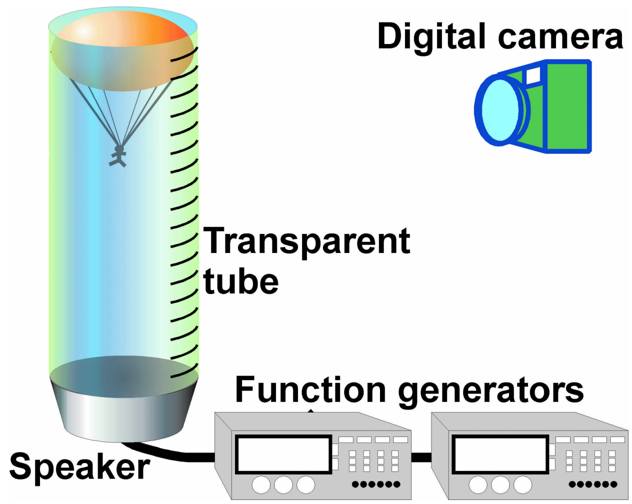

FIGURE 1. Sketch of the experimental set-up. A variation of the acoustic resonance tube experiment allows students to evaluate the frequency resonance, node positions, wavelength, speed of sound in air, and to demonstrate acoustic levitation.

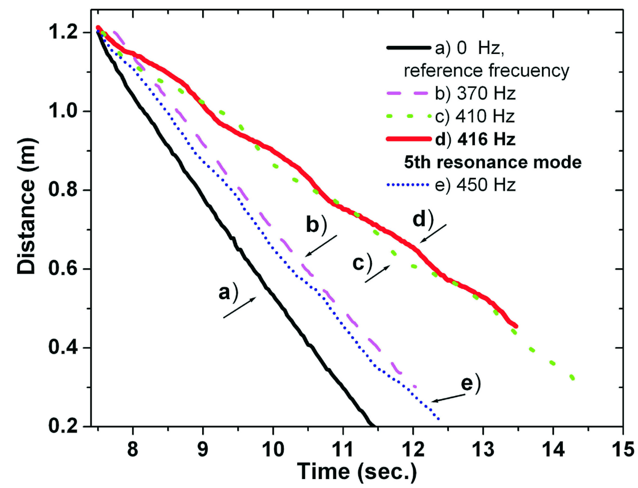

FIGURE 2. Curves of displacement as a function of time for a free-falling light object within a vertical acoustic resonance tube, at frequencies around the fifth resonance, and at 0 Hz as a reference.

3. Experimental methodology

The well-known acoustic resonance tube experiment allows students to evaluate the resonance frequencies, wavelengths, and speed of sound in air [7,8]. We propose enhancing the classical acoustic resonance experiment with the following methodological variation, which smoothly concatenates topics in mechanics with concepts of wave physics, like displacement, average velocity, terminal velocity, and mechanical equilibrium, and to obtain acoustical levitation. Figure 1 shows the proposed experimental set-up. A transparent tube (made of acrylic, 70 mm in diameter, h_r=1.857 m in length) is oriented vertically, with the lower end placed over a loudspeaker cone (400 W, 70 mm in diameter); the speaker is connected in series with two function generators (Pasco, model SF-8101). Both generators are in the maximum amplitude, also synchronized in the same output frequency, and wave form. Thus, the synchronization and serial connection permit to deliver more net voltage into the speaker; the result is the increment of the amplitude in the acoustical wave. However, this set-up configuration also can be changed using only one generator and an amplifier. A piece of common paper (40×40 mm^2) is used as a free falling body, since this object quickly obtains a constant velocity (the terminal velocity in air of a free-falling body). In addition, taking into account that speed of sound in air is 343 m/s at 20°C, and at 1 atm [8], the 5th mode of resonance in the tube, closed at one end, open at the other, is 416.3 Hz. The experiments are performed at frequencies around the 5th mode resonance, and at 0 Hz, in order to allow students the observation of multiple nodes. At each particular frequency, the piece of paper is dropped at the upper end of the tube, and its fall is recorded on a CCD video-camera mounted on a tripod. The video can be analyzed in a personal computer. For the benefit of teachers and students, a video of the experiment, at frequencies of 0, 370, 416, and 450 Hz, is available for download over the Internet [9].

Digital videos of the experiment were analyzed using the computer program Logger-Pro 3.4.6^{TM}. The cursor in this program allows setting up a general referential length-scale, and also to spot the position of the falling paper frame by frame, obtaining in this way, more than 180 points per video/experiment. The program generates data of position as a function of time; this time vector is obtained automatically from the video information, since the time interval is constant frame by frame. From this data, graphics and a linear fit were obtained for numerical analysis.

Finally, as a demonstration of acoustic levitation this is when the sample's weight is equal to the upward force due to interaction with the acoustic field, a 100×100 mm^2 foamy plate (2.10 g) is carefully placed above the top of the tube, when driven at the resonance condition. The plate remains in stable suspension until the frequency is changed somewhat above, or below resonance.

4. Results and discussion

Figure 2 shows distance-time curves from several experiments. Only some frequencies are presented, in order to avoid cluttering the figure. In Fig. 2, curve a) corresponds to the paper falling without sound (at a frequency of 0 Hz), while curve d) is with sound at the 5th resonance mode, with a frequency of 416 Hz; finally, the curves obtained at 370 and 450 Hz exhibit similar slopes, but lying in between the slopes obtained at 0 Hz, and 416 Hz. Linear fitting was done via the linear least-squares method. The physical interpretation of the fitting coefficients is as follows: the slope m is the average velocity, and the intercept b is the initial height of the object.

Figure 3 shows the average terminal velocity as a function of frequency. The curve presents a clear peak close to

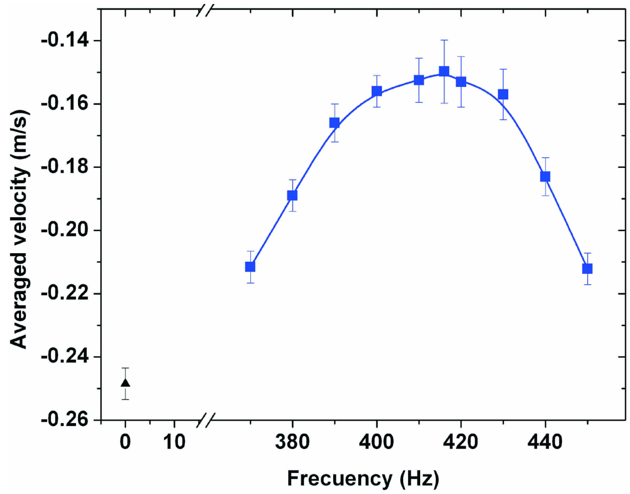

FIGURE 3. The average velocity of the falling body, as a function of frequency, exhibits the acoustic resonance in a vertical resonance tube.

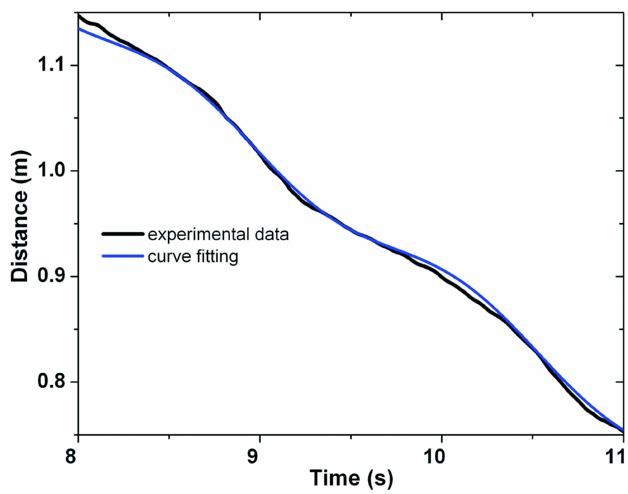

FIGURE 4. Curves of displacement as a function of time: linear-trigonometric fit, and experimental data at the frequency of the 5^{th} resonance mode.

the expected resonance frequency. However, the error in the velocity is maximum at the resonance frequency, since the distance time curve of the falling paper is not completely linear, as can be observed from curves c) and d) in Fig. 2. The reduction of the average velocity at the resonance condition is a result of the increment of pressure at the nodes, which correspond to the velocity antinodes of the standing acoustic wave [10]. In fact, the falling paper minifies its velocity when the vertical acoustic force reaches a maximum. On the other hand, the body does not stop at the pressure node, because the inertia of the body is large enough, as it keeps falling after this zone.

Table I shows linear fit coefficients (slope, and intercept) and regression error (percent correlation factor) at four representative frequencies: 0 Hz as reference condition without sound, 370 Hz below resonance, 416 Hz the resonance frequency, 450 Hz above resonance. From Fig. 2, and Table I,

TABLE I. Linear fit parameters and regression correlation factor at four representative frequencies.

Frequency (Hz)	Slope (m/s)	Intercept (m)	Percent correlation factor
0	-0.248	1.78	99.8%
370	-0.212	1.79	99.9%
416	**-0.125**	**1.81**	**99.8%**
450	-0.212	1.79	99.8%

it can be observed that the linear intercept is in good agreement with the physical length of the tube; *i.e.* the average height h_a is 1.8 m. On the other hand, it can be observed that the maximum velocity occurs at 0 Hz, with the loudspeaker turned off, producing no sound; in fact, the velocity observed in this case, might correspond with the terminal velocity of the falling paper, given its geometry which promotes high friction with the surrounding air. At frequencies below resonance, the average velocity decreases as a result of the wave/paper interaction, neglecting that the paper changes the length of the resonant-tube. The average velocity is minimal at the resonance frequency, where well defined nodes of pressure are responsible for the slower motion of the falling paper. Finally, an increment in the average velocity is again observed at frequencies above resonance.

Moreover, the curve obtained at resonance presents clear maxima and minima around the linear fit line. In fact, this curve can be represented approximately as the sum of linear and sinusoidal functions. It is possible then, to find the nodes using the following mathematical model:

$$y = A\sin(\omega t + \phi) + mt + c. \tag{14}$$

The m and c parameters have been calculated via linear least-squares fitting technique. Thus, subtraction of this part allows us to find the frequency ω, phase φ, and amplitude A via simple calculus. In such way, a best fit curve is obtained as follows:

$$y = 0.015\sin(4t + 5) - 01252t + 2.146, \tag{15}$$

where the correlation factor improves from 0.9982 for the linear fitting, to 0.9995 for the sine-plus-linear fit. Figure 4 shows a detailed view of the experimental data, and the proposed sine-plus-linear fit.

Thus, the wavelength can be calculated using Eq. (2), from the intersections between the linear and the oscillating parts. The wavelength for the 5^{th} resonance mode turns out to be 0.785 m. From here, the product of the resonance frequency and the wavelength gives an estimated speed of sound in air of 326.6 m/s.

Finally, Fig. 5 shows two photographs of acoustical levitation of the foamy plate above the resonant tube. Photographs Fig. 5(a) and Fig. 5(b) correspond to 20 s and 2 min, after the foamy plate was left up over the tube. The acoustic

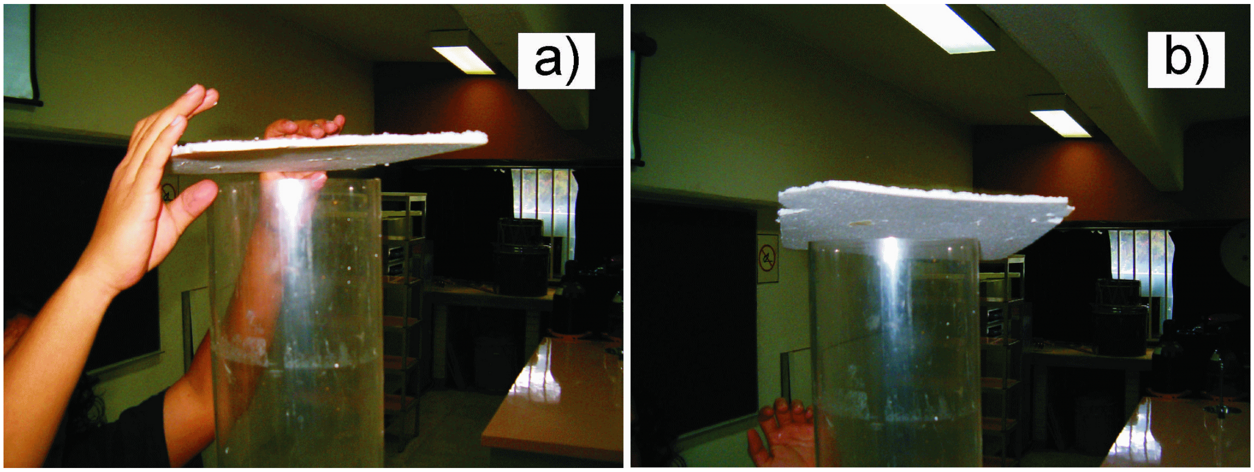

FIGURE 5. Acoustic levitation of a foamy plate when the tube is driven by sound at the 5^{th} resonance mode. Photographs correspond to a) 20 s, and b) 2 min., after placing the foamy plate above the tube.

levitation effect is only present at resonance frequencies, at others frequencies the plate felt down.

In this experiment, we used a simple calculus to determinate the average pressure $\langle P\rangle$, which is the average force $\langle F\rangle$ over the area A. Considering stable and equilibrium in the foamy plate, the average force is the weight. So the average pressure to levitate the plate is

$$\langle P\rangle = \langle F\rangle / A = mg/A \tag{16}$$

This is a pressure 2058 Pa, in the top of the tube to maintain is levitating a foamy plate.

5. Conclusions

A simple variation of the classical acoustic resonance tube experiment has been presented. This proposal allows the opportunity to discuss mechanical and wave physics concepts; for example, terminal velocity and resonance. This experiment allows the quantification of sound wave parameters, such as wavelength and speed of wave propagation, much as in the typical experiment. However, driving at different frequencies, and further data analysis to obtain the average velocity of a free falling light object within the tube, leads to the concept, and the demonstration of acoustic levitation. Most of the equipment used in this demonstration can normally be found in many high-school laboratories; thus, a demonstration of acoustic levitation can be readily performed [11]. In further work, we are planning to present low-cost variations of this demonstration of acoustic levitation.

Acknowledgments

The author acknowledges financial support from the ICyT-DF grant 2010, and Dr. Felipe Orduña Bustamante for his very useful comments and suggestions.

1. Acoustic Levitation Chamber http://www.youtube.com/watch?v=94KzmB2bI7srevised 03/19/2010 [last view: 11/18/2010].
2. Ultrasonic Levitation http://www.youtube.com/watch?v=S4exO4CuoSU [last view: 11/18/2010].
3. J.R. Gao, C.D. Cao, and B. Wei, *Adv. Space Res*. **24** (1999) 1293.
4. R.G. Holt, C.C. Clarke, and J. Gregory McDaniel, *J. Acoust. Soc. Am.* **105** (1999) 1146.
5. C.T. Tindle, *Am. J. Phys*, **54** (1986) 749.
6. R. Resnick, D. Halliday, and K.S. Krane , *Física* Vol. 1 (John Wiley &. Sons, New York, 1992).
7. R.W. Peterson *Am. J. Phys*. **63** (1995) 489.
8. I. Carl Romer, Jr. and Preston E. Koentop, *Am. J. Phys.* **33** (1965) 803.
9. Experimento de paracaidas en tubo resonante http://www.youtube.com/watch?v=MkdPCzfxlgs [last view: 11/18/2010]
10. P. Gluck, *Phys. Educ.* **41** (2006) 117.
11. Demostración de levitación acústica con paracaídas de juguete http://www.youtube.com/watch?v=TpNMBHXmNew [last view: 11/18/2010]