A novel set of reduced equations to model perfect layer matched (PML) in FDTD

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Recibido el 8 de junio de 2010; aceptado el 10 de marzo de 2011

We propose a new set of reduced equations describing the Perfectly Matched Layer (PML) boundary condition for the Finite Difference Time Domain Method (FDTD) algorithm. These expressions take into account the main properties of the electromagnetic wave propagation in continuos medias: absorbing, free space and conductive, simplifying the solution of electromagnetic problems as such as the FDTD lattice. A two-dimensional (2-D) transversal electric TE mode Gaussian pulse propagating along free-space is presented as a vehicle of study. The efficiency of this model is validated by a new way to compute the power reflection coefficient of the electromagnetic field arriving at the PML interface at several points. Also a detailed description of the rounding up process to obtain integer values for FDTD equations indexes is discussed.

Keywords: Maxwell equations; FDTD; PML; absorbing boundary conditions; electromagnetic propagation.

Se propone un conjunto de ecuaciones FDTD reducidas que describen la condición de frontera PML y la región de análisis utilizando el método de diferencias finitas en el dominio del tiempo. Para lograr esto el conjunto de ecuaciones describen las propiedades de los principales medios continuos de propagación electromagnética: medio absorbente, espacio libre y conductor, lo que simplifica la implementación y solución de problemas de propagación mediante FDTD. Como caso de estudio se analiza la propagación bidimensional de un pulso gausiano en un modo TE. La eficiencia del modelo propuesto se valida calculando el coeficiente de reflexión en varios puntos de la interfaz entre la región de interés y la región PML. Se presenta también una descripción detallada del proceso de redondeo de los índices fraccionarios del conjunto de ecuaciones FDTD.

Descriptores: Ecuaciones de Maxwell; diferencias finitas en el dominio del tiempo; condiciones de frontera absorbentes; propagación electromagnética.

PACS: 03.50.De; 02.70.Bf; 41.20.Jb; 07.05.Tp; 01.50.H-

1. Introduction

The Finite Difference Time Domain (FDTD) method has recently sparked a wealth of work to solve a large variety of electromagnetic problems. The main advantage of the FDTD method is that it is a straightforward solution of the six-coupled field components of the Maxwells curl equations. It consists in discretizing the time-dependent Maxwell's equations using central-difference approximations for the space and time partial derivatives. The resulting finite-difference equations are solved according to the Yee algorithm [1]. This implies a computational representation of the propagation zone called computation domain that must be limited by the application of boundary conditions. In fact, an important issue in the FDTD research is the requirements of efficient absorbing boundary conditions (ABCs) to truncate open region problems. Berengers perfectly matched layer (PML) has been shown to be one of the most effective FDTD ABCs [2]. Berengers PML consists of a lossy and artificial layer with a thickness varying from 4 to 12 cells and can be placed generally very close to the electromagnetic sources located inside the inner FDTD computational domain. This type of PML is based on splitting each field component into two subcomponents. For large electromagnetic problems PML requires large time computing and memory storage along several lines code. In this paper we propose an original PML model, that is based on a new set of reduced equations that deals -in a simple way- with the main characteristics of the most common media used in electromagnetic problems: absorbing, free space and conductive regions, simplifying the solution field within the FDTD lattice. To test this model we propose a two-dimensional (2-D) transversal electric TE mode Gaussian pulse that propagates within an homogenous medium like free-space, which is a well known boundary problem. The efficiency of the model is validated by calculating at several points of the PML interface, the power reflection coefficient of the electromagnetic field. The rest of the paper is organized as follows: Sec. 2 formulates the equations that describe the studied propagation mode. Sec. 3 includes a discussion on the main features concerning the reduced PML model along a detailed description of the rounding up process to obtain integer values for FDTD equations indexes. Sec. 4

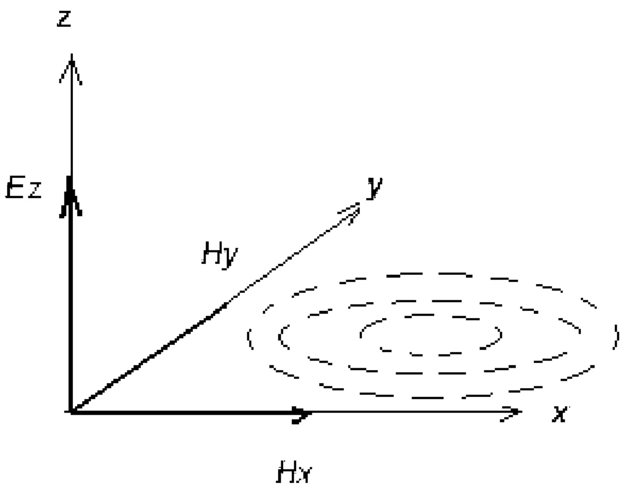


FIGURE 1. Transversal electric - two dimensional mode (TE-2D).

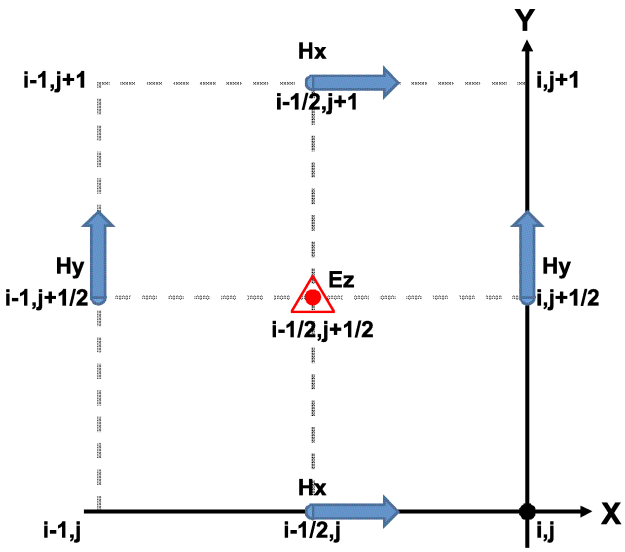


FIGURE 2. E centered base cell.

presents the Modeling setup, while the main simulation results are discussed in Sec. 5, and finally the conclusions of this work are outlined in Sec. 6.

2. The finite difference time-domain method

The FDTD leads to an asymptotic solution of Maxwell's curl equations converting them in difference's equations using the space-time centered scheme with an approximation of second order [3]. To ensure convergence, the standard conditions of stability and numeric dispersion must be applied, see [4] and references therein.

2.1. The two-dimensional transversal electric (2D-TE) Model

In order to obtain PML absorbing boundary equations, a set of symmetric Maxwell equations must be considered by adding the equivalent magnetic conductivity σ^* term into the Faraday's Law. This is an hypothetical consideration required to form an absorbing PML layer for magnetic and electric fields.

From vectorial Maxwell curl equations, TE propagation mode expressions are obtained assuming electromagnetic propagation within the $x-y$ plane [5], based on the following considerations:

1. Both, the physical and electrical properties of propagation region are uniform along the z direction, hence derivatives for the E and H field components in the z direction vanish, leading to define a set of 2-D equations.

2. It is assumed that a TE mode is composed by only one component E_z which is transversal to the propagating $x-y$ plane, leading to $H_z = E_x = E_y = 0$, then the field components that are taken into account on the wave propagation are: E_z, H_x and H_y as depicted in Fig. 1.

Under these considerations, the field equations for the TE-2D mode are written as:

$$\frac{\partial}{\partial t}E_z = \frac{1}{\varepsilon}\left(\frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x\right) - \frac{\sigma}{\varepsilon}E_z \tag{1}$$

$$\frac{\partial}{\partial t}H_y = -\frac{1}{\mu}\left(-\frac{\partial}{\partial x}E_z\right) - \frac{\sigma^*}{\mu}H_y \tag{2}$$

$$\frac{\partial}{\partial t}H_x = -\frac{1}{\mu}\left(\frac{\partial}{\partial y}E_z\right) - \frac{\sigma^*}{\mu}H_x, \tag{3}$$

where σ is the electric conductivity, σ^* is the equivalent magnetic conductivity, μ is the permeability and ε is the permitivity. In the following section discretization of the TE-2D mode is described.

2.2. Discretization of the TE-2D equation set

Figure 2 shows E-centered base cell, surrounded by H field describing their positions depending on the coordinate axis.

From (1)-(3), space and time differences are evaluated obtaining (4)-(6), where the subscript indicates the spatial position, and the superscript represents the temporal position [7].

$$\begin{aligned} Ez^{n+\frac{1}{2}}_{(i-\frac{1}{2},j+\frac{1}{2})} &= \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\right) Ez^{n-\frac{1}{2}}_{(i-\frac{1}{2},j+\frac{1}{2})} \\ &+ \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hy^n_{(i,j+\frac{1}{2})} - Hy^n_{(i-1,j+\frac{1}{2})}}{\Delta x}\right] \\ &- \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hx^n_{(i-\frac{1}{2},j+1)} - Hx^n_{(i-\frac{1}{2},j)}}{\Delta y}\right] \end{aligned} \tag{4}$$

$$Hy^{n+1}_{(i,j+\frac{1}{2})} = \left(\frac{2\mu - \sigma^*_x\Delta t}{2\mu + \sigma^*_x\Delta t}\right) Hy^{n}_{(i,j+\frac{1}{2})} + \left(\frac{2\Delta t}{2\mu + \sigma^*_x\Delta t}\right)\left[\frac{Ez^{n+\frac{1}{2}}_{(i+\frac{1}{2},j+\frac{1}{2})} - Ez^{n+\frac{1}{2}}_{(i-\frac{1}{2},j+\frac{1}{2})}}{\Delta x}\right] \tag{5}$$

$$Hx^{n+1}_{(i-\frac{1}{2},j+1)} = \left(\frac{2\mu - \sigma^*_y\Delta t}{2\mu + \sigma^*_y\Delta t}\right) Hx^{n}_{(i-\frac{1}{2},j+1)} - \left(\frac{2\Delta t}{2\mu + \sigma^*_y\Delta t}\right)\left[\frac{Ez^{n+\frac{1}{2}}_{(i-\frac{1}{2},j+\frac{3}{2})} - Ez^{n+\frac{1}{2}}_{(i-\frac{1}{2},j+\frac{1}{2})}}{\Delta y}\right] \tag{6}$$

3. Reduced equation model from perfectly matched layer equations

The original PML model [2] establishes that an artificial material region surrounding the scatterer has both electrical and magnetic conductivities. Each field component is split into two parts, resulting in a total of 12 field components for a three dimensional region. In this way only normal field components to PML interface are attenuated within the absorption region. Attenuation is properly achieved by a predefined conductivity profile. Practical implementation in a FDTD mesh implies to establish two sets of equations: One set to describe the interest region and the other one to take into account the PML. In contrast with traditional techniques where special conditions must be used to match the main computational domain, the interface and the absorbing boundaries layers, this method assumes that electric and magnetic conductivities are defined in such a way that these simulation regions can be represented by only one set of equations. This leads to only four expressions to completely describe the PML instead the eight expressions proposed by [2] and commonly found in the literature. This procedure allows the simplest solution when compared with standard methods. However, special care must be taken with the round up process of the spatial subscripts whereas the temporal subscripts are omitted because they are implicitly built within the FDTD code [7]. The different computational domains and their corresponding conductivities are schematically represented in Fig. 3. Those regions leads to the reduced set of equations.

The new set of PML equations are obtained from Maxwell's symmetric equations describing a TE-2D propagation according with the following procedure:

1. The transversal electrical component E_z is decomposed into its x and y projections, allowing E_z to be replaced by $(E_{zx} + E_{zy})$.

2. An artificial decomposition of conductivities is also performed according to its axis projection, leading to $\sigma = \sigma_x = \sigma_y$ and $\sigma^* = \sigma^*_x = \sigma^*_y$, resulting into (7)-(10).

$$Ezx_{(i-\frac{1}{2},j+\frac{1}{2})} = \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\right) Ezx_{(i-\frac{1}{2},j+\frac{1}{2})} + \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hy_{(i,j+\frac{1}{2})} - Hy_{(i-1,j+\frac{1}{2})}}{\Delta x}\right] \tag{7}$$

$$Ezy_{(i-\frac{1}{2},j+\frac{1}{2})} = \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\right) Ezy_{(i-\frac{1}{2},j+\frac{1}{2})} - \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hx_{(i-\frac{1}{2},j+1)} - Hx_{(i-\frac{1}{2},j)}}{\Delta y}\right] \tag{8}$$

$$\begin{aligned} Hy_{(i,j+\frac{1}{2})} &= \left(\frac{2\mu - \sigma^*_x\Delta t}{2\mu + \sigma^*_x\Delta t}\right) Hy_{(i,j+\frac{1}{2})} \\ &\quad + \left(\frac{2\Delta t}{2\mu + \sigma^*_x\Delta t}\right)\left[\frac{(Ezx+Ezy)_{(i+\frac{1}{2},j+\frac{1}{2})}}{\Delta x}\right] - \left(\frac{2\Delta t}{2\mu + \sigma^*_x\Delta t}\right)\left[\frac{(Ezx+Ezy)_{(i-\frac{1}{2},j+\frac{1}{2})}}{\Delta x}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} Hx_{(i-\frac{1}{2},j+1)} &= \left(\frac{2\mu - \sigma^*_y\Delta t}{2\mu + \sigma^*_y\Delta t}\right) Hy_{(i-\frac{1}{2},j+1)} \\ &\quad - \left(\frac{2\Delta t}{2\mu + \sigma^*_y\Delta t}\right)\left[\frac{(Ezx+Ezy)_{(i-\frac{1}{2},j+\frac{3}{2})}}{\Delta y}\right] + \left(\frac{2\Delta t}{2\mu + \sigma^*_y\Delta t}\right)\left[\frac{(Ezx+Ezy)_{(i-\frac{1}{2},j+\frac{1}{2})}}{\Delta y}\right] \end{aligned} \tag{10}$$

An specific computational domain can be modeled by (7)-(10) trhough the following steps:

1. Absorbing region is defined by considering $\sigma{=}\sigma_x{=}\sigma_y$ and $\sigma^* = \sigma^*_x = \sigma^*_y$ besides $\sigma/\varepsilon_0 = \sigma^*/\mu_0$ is accomplished as a matching condition.

2. Free space is defined when $\sigma_x = \sigma_y = \sigma^*_x = \sigma^*_y = 0$ which leads to Maxwell's equations in free space.

3. The conductive region is defined by σ_x and $\sigma_y \neq 0$ and $\sigma^*_x = \sigma^*_y = 0$.

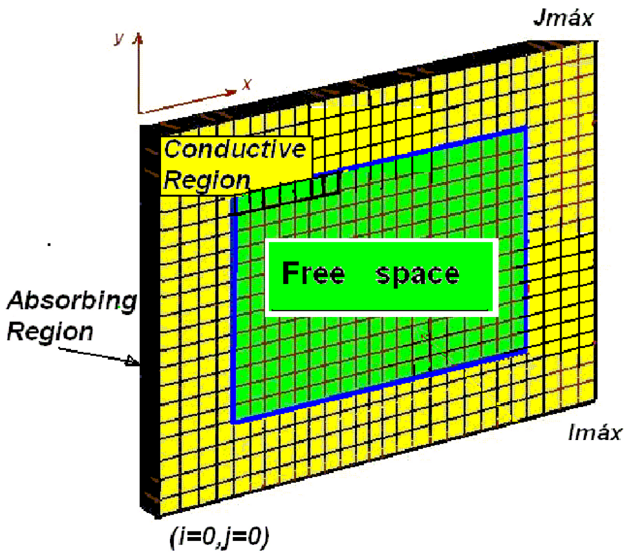


FIGURE 3. Free space, absorbing and conductive regions.

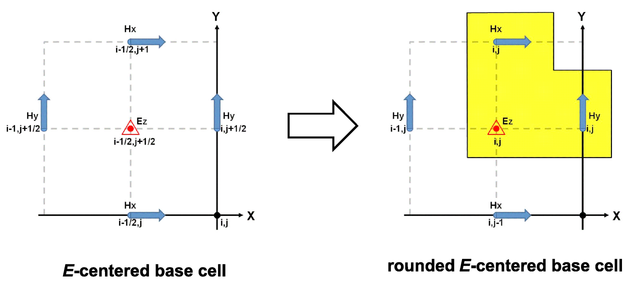


FIGURE 4. Location of the base cell PML projected on to the mesh of the computational region.

Conductivity values at the absorbing region are assigned by the conductivity profile, making the possibility of wave absorption within the PML layer. For this case a polynomial conductivity function profile is used and it will be introduced later on this paper.

3.1. Rounding up equation's fractional indexes procedure to implement the computer code

It has been found in FDTD literature a lack of information about the process of writing code from FDTD fractional equations. To the authors knowledge, this is the first time that a detailed description of the rounding up process to obtain integer values for the FDTD equation indexes is presented. This procedure is described as follows:

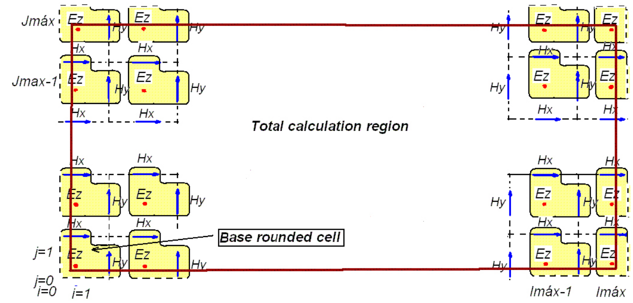


FIGURE 5. Rounding of the cell base subscripts.

1. Considering the centered E based cell depicted in Fig. 2, the positions along the X and Y directions are identified as $(i-1)$ and $(j-1)$ respectively whereas subsequent positions along X and Y direction are identified as $(i+1)$ and $(j+1)$ respectively.

2. When the position of equation's fractional indexes are related to Fig. 4, then (7), (8), (9) and (10) can be easily rewritten in terms of integer indexes.

3. To conform a computational region (FDTD mesh) it is usual to replicate the E centered base cell as in Fig. 5. The region is over-dimensioned onto the upper and right side in order to maintain symmetry and to apply uniform perfect conductor conditions even around the outer side. In this case, the added elements in X and Y directions are identified as I_{max} and J_{max} indexes.

4. Following this procedure, it is found for example, that H_x can be evaluated in X direction from 1 to I_{max}, however in Y direction it is evaluated from $(1 : J_{max}-1)$ so index range for Hx in Eq. (10) is $(1 : I_{max}, 1{:}J_{max}-1)$.

After this process, a rounded-up PML equation system can be written by:

$$Ezx_{(2:Imax-1,2:Jmax-1)} = \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\right) Ezx_{(2:Imax-1,2:Jmax-1)} + \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hy_{(2:Imax-1,2:Jmax-1)}}{\Delta y}\right] - \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hy_{(1:Imax-2,2:Jmax-1)}}{\Delta y}\right] \quad (11)$$

$$Ezy_{(2:Imax-1,2:Jmax-1)} = \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\right) Ezy_{(2:Imax-1,2:Jmax-1)} - \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hx_{(2:Imax,2:Jmax-1)}}{\Delta x}\right] + \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\right)\left[\frac{Hx_{(2:Imax,1:Jmax-2)}}{\Delta x}\right] \quad (12)$$

$$Hx_{(1:Imax,1:Jmax-1)} = \left(\frac{2\varepsilon - \sigma_y^* \Delta t}{2\varepsilon + \sigma_y^* \Delta t}\right) Hx_{(1:Imax,1:Jmax-1)} - \left(\frac{2\Delta t}{2\varepsilon + \sigma_y^* \Delta t}\right) \left[\frac{(Ezx + Ezy)_{(1:Imax,1:Jmax)}}{\Delta y}\right] + \left(\frac{2\Delta t}{2\varepsilon + \sigma_y^* \Delta t}\right) \left[\frac{(Ezx + Ezy)_{(1:Imax,1:Jmax-1)}}{\Delta y}\right] \tag{13}$$

$$Hy_{(1:Imax-1,1:Jmax)} = \left(\frac{2\varepsilon - \sigma_x * \Delta t}{2\varepsilon + \sigma_x^* \Delta t}\right) Hy_{(1:Imax-1,1:Jmax)} + \left(\frac{2\Delta t}{2\varepsilon + \sigma_x^* \Delta t}\right) \left[\frac{(Ezx + Ezy)_{(2:Imax-1,1:Jmax)}}{\Delta x}\right] - \left(\frac{2\Delta t}{2\varepsilon + \sigma_x^* \Delta t}\right) \left[\frac{(Ezx + Ezy)_{(1:Imax-1,1:Jmax)}}{\Delta x}\right] \tag{14}$$

those equations can be directly entered into a computer code.

3.2. Conductivity profile in the PML layer

A smooth transition from inner region to PML layer is required for the correct performance of the absorption boundary condition. Abrupt changes can generate undesirable reflections or PML mismatching, then an adequate incremental conductivity evolution trough PML layer is desired. This can be accomplished by the implementation of a conductivity profile function. To this end, we use a polynomial function as in Ref. 2:

$$\sigma_{(\rho)} = \sigma_{max} \left[\frac{\rho}{\delta}\right]^m . \tag{15}$$

Where $\sigma(\rho)$ is the conductivity value at a distance ρ from interface to an inner point into the PML layer, σ_{max} is the maximum conductivity value at the outer cell of the PML layer, δ is the total thickness of the PML layer and m is the degree of the polynomial profile.

Optimal values reported for polynomial profile in our previous work [7] remain between 3 and 4 for m and 1.0 for σ_{max}. When the conductivity profile is implemented an average value around a selected point L is performed as in Ref. 6:

$$\sigma_\rho(L) = \frac{1}{\Delta\rho} \int_{\rho(L)-\frac{\Delta\rho}{2}}^{\rho(L)+\frac{\Delta\rho}{2}} \sigma_p(u) du, \tag{16}$$

where $\Delta\rho$ is the space step and $\sigma_\rho(u)$ is a general conductivity distribution around the point L. By using (15) in (16), an L-based algorithm is conformed to assign a conductivity value for every L point into the PML layer. Hence for $L = 0$ reads:

$$\sigma_\rho(0) = \frac{1}{\Delta\rho} \int_0^{\frac{\Delta\rho}{2}} \sigma_{max} \left(\frac{\rho}{N\Delta\rho}\right)^m d\rho = \frac{\sigma_{max}}{(m+1)2^{(m+1)} N^m} \tag{17}$$

And for $L \neq 0$:

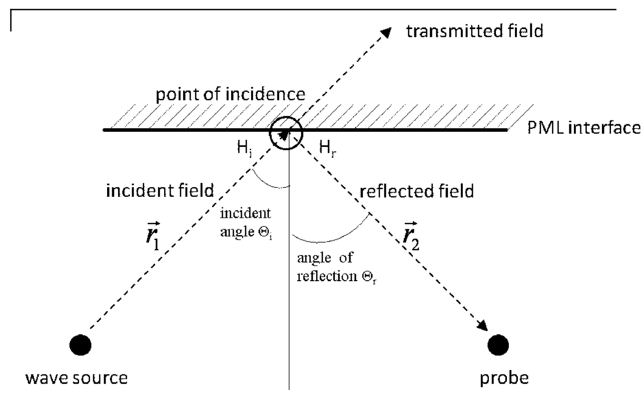


FIGURE 6. Schematic setup for the calculation of the reflection coefficient.

$$\sigma_\rho(L > 0) = \frac{1}{\Delta\rho} \int_{\rho(L)-\frac{\Delta\rho}{2}}^{\rho(L)+\frac{\Delta\rho}{2}} \sigma_{max} \left(\frac{\rho}{N\Delta\rho}\right)^m d\rho = \frac{\sigma_{max}}{(m+1)2^{(m+1)} N^m} \times \left[(2L+1)^{(m+1)} - (2L-1)^{(m+1)}\right] \tag{18}$$

4. Modelling setup

As is known, Berenger has established the reflection coefficient for the PML as:

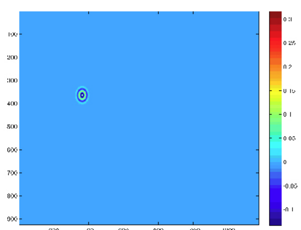
$$R(\Theta) = e^{-\frac{2}{(n+1)} \frac{\sigma_m \delta}{\epsilon_0 c}} \tag{19}$$

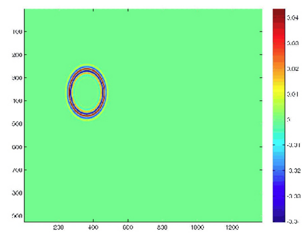
Being Θ the incidence angle over the PML. The procedure defines a test domain that is centered within a benchmark domain, simulating an almost infinite domain for the reflection layer. The difference between measurements perfomed inside the test domain and the benchmark domain provides a measure of the spurious reflection caused by PML (see Fig. 6).

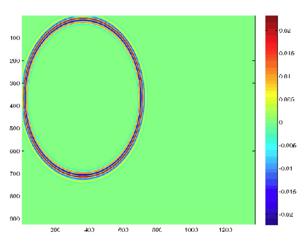
A different way of power reflection measurement is proposed in this paper. We assume a Gaussian pulse propagating over the domain, the incident and reflected waves over the

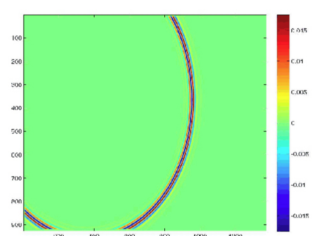
TABLE I. Main parameters of the experiment to model a TE-2D with ABC-PML, 12 cells.

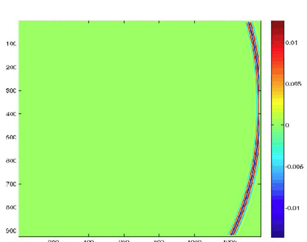
Calculation region (FDTD)	Imáx:924, Jmáx:1374
Number of cells (layer PML)	12
Power source	Gaussian pulse
Dispersion parameter	$s = 0.2 \times 10^{-9}$
Location - source (FDTD)	I:362, J:562
Frequency	3 GHz
Degree of polynomial profile	4
Initial conductivity	$\sigma_0 = 2 \times 10^{-6}$ S/m
Numerical stability	$t = 8.33 \times 10^{-12}$ sec.
Numerical dispersion	0.005 m.
Relative permittivity	1
Relative permeability	1


(a) Ez component for 300 iterations


(b) Ez component for 500 iterations


(c) Ez component for 1000 iterations


(d) Ez component for 1500 iterations


(e) Ez component for 2340 iterations

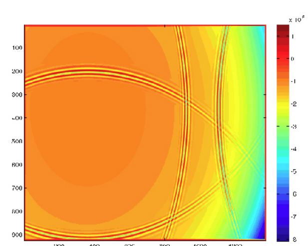

(f) Ez component for 2880 iterations

FIGURE 7. Propagation of the E_z field component.

PML boundary are measured at test points (probes), located inside the domain as shows Fig. 8, then we calculate reflection coefficient to estimate PML absorption as:

$$\Gamma = 20 \log \frac{H_i}{H_r} \tag{20}$$

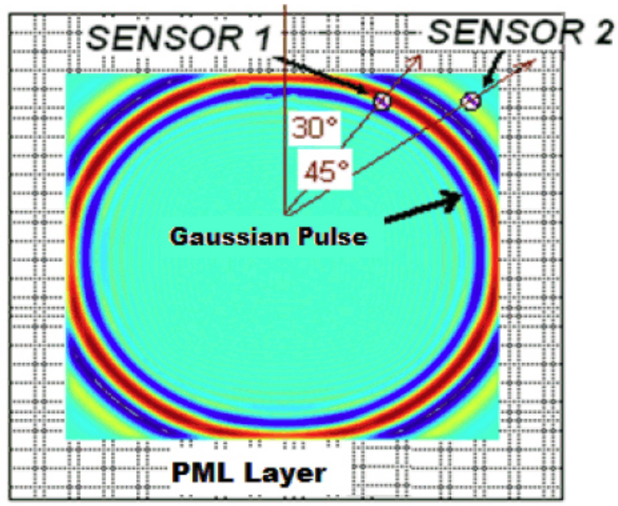


FIGURE 8. Position of probe sensors located at 30° and 40° incidence angles.

TABLE II. Reflection coefficient for: (TE-2D), FDTD, ABC-PML, 12 cells.

Incidence angle	30°	45°
Incident wave amplitude	.023	.017
Reflected wave amplitude	1.09×10^{-8}	6.74×10^{-9}
Maximum value (Reflected wave)	-5.5×10^{-8}	-7×10^{-8}
Minimum value (Reflected wave)	-7.7×10^{-8}	-8.4×10^{-8}
Power Reflection Coefficient (dB)	-126.5	-128.1

In order to evaluate the absorption performance of this algorithm a simulation is performed by considering a 3 GHz TE wave propagating in vacuum with a magnitude of H=0.2982 A/m; the 2D computational domain dimensions are 1374 $\times$ 924 cells, with space and time increments of 5×10^{-3} mts. and 8.33×10^{-12} sec. respectively to satisfy stability conditions. Such quantities ensure that the waves can fully propagate and achieve the PML interface. Numerical results are obtained for many iterations that represent the propagation of the E_z component. Its complete evolution has been modeled from the source up to the walls of the computation domain.

As a source we use a Gaussian Pulse given by:

$$E_z(t) = e^{\frac{-1}{2s^2}(\Delta t(n_0-n))^2} \cos(\omega \Delta t(n_0 - n)) \tag{21}$$

Where s is the scattering parameter, n is the time step, n_0 is the reference time-step and ω the angular frequency. We measure the power reflection coefficient Γ just at the domain interface with the PML medium in order to have an estimate of PML absorption effectiveness at several measurement points. The main parameters employed for the simulation are listed in Table I.

5. Results

Figure 7 illustrates the evolution of the E_z field component from 300 up to 2880 iterations. Figure 7a and Fig. 7b illustrate the onset of the electric field propagation. Figure 7c shows the E_z filed component for 1000 iterations, arriving over the PML layer, with values of field ranging from -0.02 up to 0.02 V/m. At 1500 iterations (Fig. 7d), the filed completely arrives over the inferior wall . A significant wave absorption from the PML layer can be observed, at this moment the field values fluctuate from -0.15 to 0.15 V/m. As the wave goes on 2340 iterations (Fig. 7e) it arrives to the opposite right wall of the PML layer, with values from -0.01 to 0.01 V/m. Finally, the field component E_z has concluded its propagation (Fig. 7f), presenting some reflections with values from -8×10^{-8} to 1×10^{-8} V/m, which are minimum comparing with precedent values.

Several sensors are located within the domain, at different angles ranging from $0°$ to $45°$, in such a way that incident and reflected waves can be measured without interference between them. To avoid interference between both waves the s parameter in (21) is established to vanish the incident wave, before the reflected wave impinges the probe. As we assume that propagation is over free space, the wavefront energy is uniformly scattered over a circumference area given by πr^2, then is possible to calculate the magnitude of the incident field at the interface. After measuring the reflected field at the probe, the reflected field at the interface can be calculated in the same way. The relationship between both fields defines the power reflection coefficient. In this paper we present results only for $30°$ and $45°$, as shown in Fig. 8. Calculated power reflection coefficients are listed in Table II.

6. Conclusions

An electromagnetic TE-2D wave is modeled by the FDTD method with PML model containing a reduced number of equations. These expression can define the propagating zone, the interface and the PML layers through the electric and magnetic conductivities. The round up of the subscripts of the whole model equations is discussed in detail. A experiment is carried out by considering a polynomial conductivity profile with $m = 4$ in order to achieve a smooth transition from one medium to other. As a result, the distribution of the electric field E_z component propagating over the periphery of the computation domain is illustrated. Finally the data are reported for 30^o and 40^o incidence angles, to calculate the power reflection coefficient, reaching values as low as -126.5 and -128.1 dB respectively, in contrast power reflection coefficient values reported in Ref. 2 are around -70 dB, this clearly illustrates the correct performance of the PML model based on the proposed expressions.

Acknowledgements

The financial support of SIP-IPN, COFAA-IPN, CONACyT 106698 project is gratefully acknowledged.

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