Students' strategies for solving partially specified physics problems

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In this study we present a pilot-investigation of high-school and university students' abilities to solve partially specified physics problem and ways which they handle the task. Students are asked to answer the question whether the given problem situation is physically possible or not in real-life without an explicit advice on what to calculate and how to judge. We used a combination of individual interview and written test methods. According to the type of the problem-solving approach the respondents were divided into the five categories. We found the majority belong to categories with preferably quantitative approach regardless of curriculum, level of education or research method. Our results indicate that standard numerical exercises, usually used in teaching, do not develop sufficiently critical thinking and real-life problem-solving abilities. We think that students could be given also partially specified problems to help them in preparing for real-life problem-solving situations. To our knowledge, this study is the first one to explore students' reactions to this type of a problem and we are motivated to continue research.

Keywords: Student strategy; problem solving; partially specified gravity problem.

En este trabajo hemos presentado una investigación-prueba sobre la capacidad de los estudiantes universitarios y de secundaria para resolver problemas físicos parcialmente especificados y la manera cómo manejan la tarea. Hemos pedido a los estudiantes que respondieran si el problema planteado es posible o no en la vida real, sin consejos explícitos qué hay que calcular y cómo resolverlo. Hemos combinado el método de la entrevista individual con los métodos de exámenes escritos. Según la manera de resolver el problema, los estudiantes han sido divididos en cinco categorías. La mayoría pertenece a las categorías del enfoque preferentemente cuantitativo, independientemente del currículo, nivel de educación o método de investigación. Nuestros resultados indican que los clásicos ejercicios numéricos, que suelen ser usados en la enseñanza, no desarrollan suficiente el pensamiento crítico y la capacidad para resolver los problemas en la vida real. Creemos que hay que enfrentar a los estudiantes a los problemas parcialmente especificados, que les ayudarían en desarrollar las capacidades para resolver los problemas en la vida real. Según las informaciones que tenemos, este estudio es el primero en investigar las reacciones de los estudiantes a este tipo de problema, por lo que estamos motivados para continuar con la investigación.

Descriptores: Estrategia estudiantil; resolver el problema; problema de gravitación parcialmente especificado.

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1. Introduction

In the past few decades, the research community has given great attention to developing the ability of problem solving as an important part of teaching and learning physics [1]. The aim is to prepare students for life-long learning in highly changing personal and working worlds. Competences to be developed in terms of achieving the set goal are: recognizing questions and problems that can be considered scientific as well as key features of scientific research; applying the knowledge of physics and science in general to describe or to interpret phenomena in a scientific manner and to predict their changes; interpreting evidence for drawing and explaining the conclusions, for recognizing assumptions and the sequence of reasoning in which these conclusions are based, and for reflection on their implications [2].

One of the most important results of researching problemsolving ability in physics is the fact that there is a significant difference in strategies between experts and students. Reif and Heller [3] found that this difference is reflected in the way of organizing and using knowledge. Experts first describe the problem and plan its solution using qualitative arguments, and then use elaborate mathematical details. They organize their knowledge in a very structured way and therefore it can be applied in different situations. Unlike physicists, students try to reach a solution as soon as possible using a series of mathematical equations, whereby they encounter difficulties very quickly. Because their knowledge consists of disconnected facts and equations that have little conceptual meaning, they do not have the necessary knowledge [4], structured in order to enable them complex real-life problem solving [5-9].

What type of tasks should be provided to students to develop all of the required competencies or how to help them in becoming experts that can apply their knowledge and understanding in real-life situations?

In 1960s, Crane had tried to provide an initial answer to these questions [10-12]. He argued that students should be given problems where they could participate in the selection of objectives, criteria, options, solving methods, and in which they could evaluate the order of magnitude and the statistical validity of data or find the conditions for optimal result. This new generation of problems, if applied on a daily basis in the physics teaching and learning, would require a fuller engagement of students and an explicit connection with the real world. Such a change would allow the application of acquired knowledge and ways of thinking to the problems from their own out-of-classroom world.

The recent approach to this issue is developed through several different physics problem designs that include a greater or lesser engagement and prior knowledge of students to solve problems.

Authors, such as a Blickensderfer [13], state that students of introductory courses should deal with traditional, **fully specified problem situation**. It arises through the formulation of numerical problems which explicitly show what students need to calculate whereby they are given all the necessary numerical data to be inserted into a formula and give an accurate result. This type of a problem that basically consists of searching for the right formula requires algorithmic approach to problem solving.

More complex problems of this type, often found in textbooks and specified in an abstract form, are called by Yerushalmi and Magen **context-poor problems** [14]. They usually contain an associated diagram and are divided into sub-problems which guide students through the solving process step by step. In this way, the development of students' ability to analyze problem situations and planning is neglected.

Gil-Perez and his collaborators [15] try to avoid purely mathematical approach. They argue that learners should solve the so-called **unspecified problem situations**, *i.e.* tasks in which numerical data are completely omitted. In this way, students would develop a research-like approach to problem solving, because they are forced to ask questions and generate hypotheses by adopting a problem solving strategy that is similar to those in scientific research, and in which the solution validity depends on the extent to which the supposed solving models and the data correspond to real values. Because most students lack the necessary knowledge and skills to transform an unspecified problem situation into a simple conceptual and numerical exercise, they easily give up and often feel discouraged when dealing with this type of problems.

The paper [16] suggests a way out by proposing the **partially specified problem**, which lies between the standard and radical formulations. This type of tasks contains some physical values. Main determinants of this problem situation are: (i) Avoidance of proposing ways of calculating certain physical values; (ii) Providing a problem in such a way that evaluation the result is a necessary step in reaching the final answer. In other words, the formulation of the task should not provide an explicit advice on what should be calculated and how to judge whether the result or the default situation is possible or not. In this way, the important features of the scientific process, such as decision making and analysis of results, are promoted. By encouraging learners to make decisions about what should be calculated in order to judge the reality of problem situations and their solutions, the problems of this type should develop the necessary competencies.

It can be said that the **unreasonable-result problems** were on the trail of a partially specified problems. They are promoted by Urone *et al.* [17-19]. Urone argues that students should solve these problems because they help in testing different concepts and problem solving techniques [19]. He points out that properly applied physics must accurately describe nature, and it's not just an equation solving process. Such problems are in his university physics textbook at the end of the chapter. A further step in improving this type of a problem can be achieved by providing appropriate advices or guidance in order to help students self-conclude what is wrong with the set problem in their physics textbook [17,18].

The **context-rich problems** were studied by many authors [14,20-22]. They deal with real-life situations in which the unknown variable does not have to be explicitly provided, they may contain information unnecessary for their solution, some of the information needed for solving may be missing, and they could require reasonable assumptions to simplify the problem situation and enabling meaningful solution. As such, context-rich problems require students' involvement in terms of planning and analysis of the problem-solving process, and can be difficult and frustrating even for the best learners. These tasks are better carried out by students working in cooperative groups [21]. However, research that was conducted by Walsh *et al.* [23] shows that most students cannot independently solve them, which confirms that the skills of problem solving should be an explicit element of teaching.

Kariž Merhar [24] considers that students should deal with **untraditional problems** occasionally. These are tasks with unrealistic solutions, with inconsistent data, with more than one solution, and insignificant data. Use of such problems reduces the probability of obtaining accurate solution without understanding the phenomenon or problem.

In this study, we present the results of initial research of students' abilities to solve one partially specified problem proposed in the paper [16], and students thinking during the problem-solving task. The research was done on a heterogeneous sample to examine the impact that the educational level and curriculum can have on a problem-solving approach. We combined the two different methods of examination, individual interviews and written tests, to study the examiner influence on the participants. Having in mind that all the abovementioned types of problem-solving approaches have some disadvantages, we find our approach helpful for developing the desired students' competencies.

2. Research

Students were asked to solve the following partially specified problem situation:

The centres of two equal spheres are at 1 m distance. Can the gravitational force between them be 1 N?

In order to obtain the correct solution, participants were first expected to calculate the mass of each sphere $(m=1.224 \cdot 10^5 \text{kg})$. However, this result is not sufficient to answer the problem question. To find out if it is possible that the spheres have the calculated mass they had to calculate the density of a sphere. Because the radius of a sphere cannot be greater than 0.5 m, the minimum density should be $\rho = 2.34 \cdot 10^5 \text{kg/m}^3$. In order to determine whether spheres with such densities are possible in real world, students had to find out the upper limit of density for known materials on the Earth. With the help of the examiner, the students were able to determine that the osmium in normal conditions has the highest density, $\rho_{\text{osmium}} = 2.3 \cdot 10^4 \text{kg/m}^3$. If the gravitational force between the spheres was 1 N, the spheres would have a density more than 10 times greater than osmium. Therefore, the assumed problem situation is not possible in normal conditions on the Earth.

The aim of our research was not to explore students' knowledge of appropriate formulas and mathematical apparatus testing, but rather to investigate their strategies to solve one partially specified problem. We used the following research methods: (i) individual interviews; (ii) written tests.

2.1. Testing students by individual interviews

Individual interviews were conducted with specific questions which the examiner asked the participants, but any unexpected student thinking during the problem solving was followed as well.

Students were not allowed to take notes independently in order to encourage them to verbally express their thoughts. The examinees were first asked to express the first ideas regarding their understanding of the problem, and to describe their way of solving the problem qualitatively. In order to help students focus only on their problem-solving thinking, all that they wanted to write (*e.g.* known and unknown physical variables and formulas), to draw (*e.g.* diagrams of forces), or to calculate (*e.g.* the required physical variables), the examiner did for them following their detailed instructions. Examiner also helped them to recall the appropriate formula, the value of a physical constant, or for mathematical support.

If students needed some encouragement, they were asked the following questions: *How did you make that conclusion?*; *Why do you think so?; How would you check your own conclusion?* The interviews were not time limited. They ended when the student could not continue solving the task even after the examiners support. Each interview was between 6 and 16 minutes long, with the average time of about 10 minutes.

At the beginning of every interview, the examiner read the task out loudly to avoid any misunderstanding and to provide approximately equal initial conditions. The students had a time for independent reading. All interviews were recorded on a voice recorder to enable sub-sequential analysis. At the end of interview, students were asked to express their opinion about the task.

Using this method we examined twenty-four, third or fourth grade, high-school students from Rijeka (Croatia); twelve from the gymnasium and twelve from the vocational school. Participants were randomly selected among the students who agreed to participate. All of them were taught the law of gravity in the first grade of high school.

Interviews were transcribed from audio-recordings and analysed together with the notes made by the examiner. Significant statements were selected, and then compared and grouped accordingly.

The similarities and the differences between the students' strategies were analysed. We formed four groups that included all the data obtained by the method of individual interviews, and we made the classification of the results regarding the school:

1) gymnasium students (GS)

2) vocational-school students (VSS)

and the accomplished problem solving steps:

- a) the first part of the task (obtaining result for the mass)
- b) the second part of the task (the answer to the question whether the obtained results is possible).

The corresponding groups are denoted with marks 1a), 1b), 2a), and 2b).

All students in groups 1b) and 2b) are associated with abbreviations in parentheses (ScA), (SA), (UA), (MA), (NA) that signify the membership in the appropriate problem solving approach category described in detail in the paper [23]. These are: scientific approach, structured approach, unstructured approach, memory-based approach, no clear approach.

Students in ScA category solved the task correctly and independently. They qualitatively evaluated the physical situation by applying the relevant physical concepts necessary to solve the problem, and then identified the variables needed to give the answers. They sometimes drew sketches to help them self in evaluation of the problem and have no problems with formulas and mathematical background. Since they have the necessary real-life knowledge, they self-critically evaluate the result. They are aware that the correct mathematical solution may not be possible in the real world.

Students in SA category had solving plan based on the given variables. They initially estimated the formulas they needed and identified the variables that were not given but were necessary for finding the solution. Then they related the concepts and appropriate variables. Using this approach, students faced obstacles, but they still managed to solve the problem using guidance of the examiner who helped them by asking suggestive questions. They are aware that some mathematically correct results are physically impossible.

Students in UA category were able to solve only the first part of the task. During the problem evaluation, they were concentrated only on identifying the relevant variables. The known variables were properly associated with the corresponding formula. Since these students had difficulty with the mathematical background, they needed help from the examiner. They accept the obtained result as the correct one, avoiding any further evaluation, because they cannot relate the concepts and the variables.

Students in MA category failed to solve even the first part of the task independently. They were unsuccessfully trying to connect the given problem with some familiar problem. These students tried to recall the formula that should have been used, and then to connect known variables with this formula not including the concepts.

Students in NA category did not have any strategy to solve the problem. They analyze the situation in terms of given variables by discussing them as unrelated quantities or letters, not as concepts. In an attempt to manipulate given quantities, they obtain random and mostly wrong solutions. They don't support any particular opinion. This is concluded from the situations when the examiner asked questions, which resulted in their change of mind and trying to give an answer that they thought was expected of them. Heaving no confidence in their problem solving approach, they weren't trying to obtain a solution or give a final answer.

1) Gymnasium students

1a) Solving the first part of the task by GS

Almost all GS started solving the problem inserting data for:

mass

 $m_1 = m_2 = m$,

 $G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$

 $d = 1 \, {\rm m},$

distance between the centres of the spheres gravitational constant and gravitational force $F_G = 1 \text{ N}$

in Newton's law of gravity

$$F_G = G \cdot \frac{m_1 \cdot m_2}{d^2}$$

to calculate the mass.

Two GS started trying to use the gravitational force near the Earth's surface $F_g = m \cdot g$. Since they could not insert a distance d they decided to use the gravitational force formula instead.

After obtaining the result for the mass, their thinking steps differed as it follows.

1b) Solving the second part of the task by GS

Two GS (ScA) solved the task correctly, without being prompted by the examiner, thinking as follows. Because the radius of each sphere can be r < 0.5 m, they inserted r = 0.5m into the formula $V = 4/3r^3\pi$ and obtained the volume V of a sphere. Then they calculated the density $\rho = m/V$. The resulting density was compared with the highest density in the tables. Since the calculated density is ten times higher than density of osmium, the element with

highest density, the conclusion was that the situation is not possible. If r < 0.5 m, the density of the sphere in the task would be even higher, that confirms the conclusion.

Three GS (SA) solved the problem with a different way of thinking and with the help of the examiner who asked some guiding sub-questions. For example, one of them believed intuitively that the calculated mass is possible. In order to prove that, he calculated the density of each sphere. The resulting density was compared with the density of neutron star which was much higher. But the neutron star must have a much larger volume to be formed and the student concluded that the situation was not possible. For normal conditions on the Earth, if the calculated density is compared with the highest density of the element in the periodic table of elements, the same conclusion follows.

Three GS (UA) put back the calculated mass in the law of gravity, and then calculated the gravitational force. They obtained $F_G = 1$ N, and concluded that the situation is possible. After that, they were asked to justify their conclusion. One of them believed that the situation is possible, because the calculated mass was smaller than the mass of the Earth. He tried to conclude something from the calculation of gravitational force near the Earth's surface on each sphere to be sure of his claim, but he gave up.

Three GS (NA) compared the weight of the sphere, whose mass was calculated, with the gravitational force of 1 N. They concluded that the mass is too large. For example, one of them said that the situation is impossible, because he had never heard of such a large mass.

One GS (MA) made the conclusion using solely a mathematical argument. According to her, there was nothing illogical in the calculated result, e.g. negative mass, so she concluded that the problem situation was "normal".

2) Vocational-school students

2a) Solving the first part of the task by VSS

All twelveVSS calculated the mass, either independently (four students) or with the examiner's guidance (eight students), using the gravitational force formula. For example, one of them didn't know how to begin, because he had never seen such a task. He asked for a formula and was guided to calculate the mass.

Their further thinking steps also differed as follows.

2b) Solving the second part of the task by VSS

One of the two VSS (ScA) who solved the task correctly intuitively concluded that it would not be possible to have such spheres. To prove it, he needed information about the density of spheres obtained after the calculation of their volume for r = 0.5 m. Next, he sought the highest density in tables, and concluded that the problem situation is impossible.

Two VSS (UA) put back the calculated mass into the law of gravity, obtained $F_G = 1$ N, and concluded that the situation in the task is possible.

One of the two VSS (MA) believed that such a mass is not possible, because the gravitational force on the sphere near the Earth's surface is too large compared to the gravitational force of 1 N.

One of the remaining **six VSS** (**NA**) initially considered that the mass is not possible, because it seemed too big. When he was asked to be more specific and to compare it to other objects, he changed his mind saying that "everything was possible".

Some students wished to comment the task by comparing it with familiar problem situations. Here are two comments:

-GS: The task is not so difficult, but it is logical. It is not common except, perhaps, in competitions. I like this task because it encourages me to think. It's not that we have a formula and that's it. We have to see whether it is realistic, and whether it can be done. If we do not perceive the situation in reality, formulas and mathematical knowledge will mean nothing. This way of thinking is not developed enough in school.

-VSS: The task is difficult and strange, because I had never seen it before. Therefore, it is interesting, but I do not like it because I cannot solve it. This type of tasks should be given in school because it would be useful.

2.2. Testing students by written tests

We examined fifty students from the gymnasium in Split who attended the II, III or IV grade, and fifty fourth-yearuniversity students from the Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture of the University of Split, using written tests.

Each student was given the problem in written form. They were asked to write down all their thoughts and calculations. When the student needed a formula, he had to write down his query, and request assistance from the examiner by raising hands. The examiner would have come and offered him a written response, without any suggestions about how to solve the problem. In this way, we have obtained more detailed and objective insights into students' problem solving strategies as well as information about students' attitudes toward the task. Participants had 30 minutes for finding the required answer.

Significant statements were sought among the students' writings, which were then mutually compared with regard to the accomplished problem solving steps. Accordingly, we classified all research data, obtained by the method of written tests, into four strategy categories.

All participants are associated with abbreviations in brackets to signify membership in the appropriate problem solving approach category mentioned above.

The first group includes five GS and eight university students (US) who gave a completely correct answer. All of these participants solved the task in a manner equal to that used by GS from Rijeka described above. Two GS (ScA) and three US (ScA) solved the task independently and accurately, while three GS (SA) and five US (SA) required help from the examiner. The second group includes three GS (UA) and sixteen US (UA) who obtained the exact mass using the Newton's law of gravity. For example, one of those GS set a calculating density problem, but he did not find the density. One of the US said that if the formula conditions had been accomplished, the force of 1 N would have been possible, but he was not able to recognize these conditions.

The third group includes twenty-four GS (MA) and seven US (MA) who mistakenly approached the problem, using the gravitational force near the Earth's surface formula $F_g = m \cdot g$ to calculate the mass of the sphere. They obtained m = 0.101 kg, and then followed different thinking steps. For example, seven GS and three US inserted the obtained mass into the gravitational-force formula. They showed by calculation that the obtained force was much less than 1 N, and then concluded that the given problem is impossible.

The fourth group includes eighteen GS (NA) and nineteen US (NA) who approached the problem in a very special way that revealed their misconceptions. For example, one of those GS observed the spheres as pendulums, considering the possibility of vibration supported by the gravitational force near the Earth's surface. Three of the US stated that the same gravitational force was acting on all bodies with equal masses, and that it was directed downwards. Therefore, they concluded that there was no force between the two spheres mentioned in the task. According to them, the force could only exist, if the spheres were electrically charged.

Here are two comments of students who were tested by written tests:

-GS: This task is neither common nor standard. It requires much more thinking and finding connections between the given and possible facts than standard tasks. I like this task because, although it provides a small quantity of information and is seemingly simple, it leads to an interesting and unexpected conclusion.

-US: I was excited when I tried to solve this interesting and logical task, but I feel ashamed for not knowing some physical formulas.

3. Discussion

Although our investigation has been performed on a relatively small sample of very different groups of students, problem solving approach categories are found in all of them. The percentages of students classified to the particular category are given as follows.

ScA is used by 17% of students tested by individual interviews (17% of GS, and 17% of VSS), and 5% of students tested by written tests (4% of GS, and 6% of US).

SA is used by 12% of students tested by individual interviews (25% of GS, and 0% of VSS), and 8% of students tested by written tests (6% of GS, and 10% of US).

UA is used by 21% of students tested by individual interviews (25% of GS, and 17% of VSS), and 19% of students tested by written tests (6% of GS, and 32% of US).



FIGURE 1. Distribution of Gymnasium and Vocational-school students from Rijeka tested by individual interviews according to categories of problem solving approaches.



FIGURE 2. Distribution of Gymnasium and University students from Split tested by written tests according to categories of problem solving approaches.



FIGURE 3. Distribution of all students according to categories of problem solving approaches.

MA is used by 12% of students tested by individual interviews (8% of GS, and 17% of VSS), and 31% of students tested by written tests (48% of GS, and 14% of US).

NA is used by 38% of students tested by individual interviews (25% of GS, and 49% of VSS), and 37% of students tested by written tests (36% of GS, and 38% of US).

Distributions of students tested using individual interviews (GS – Rijeka; VSS - Rijeka), and students tested us-

ing written tests (GS – Split; US – Split) according to categories of problem solving approaches are shown graphically in Figs. 1 and 2. Distribution of all students according to the same categories is shown graphically in Fig. 3.

Comparing students at different education levels and attending different physics educational programs, it is evident that almost all categories of problem solving approaches are represented in all examined groups (exception is that none of the VSS had the SA). However, there are significant differences in category distributions within certain group.

The distribution of students tested using individual interviews (GS – Rijeka; VSS - Rijeka), in Fig. 1, shows that a higher percentage of GS in comparison to VSS belong to the categories of ScA and SA, *i.e.* they manage to solve a problem completely, either working independently or with the guidance of the examiner. GS also had fewer difficulties than VSS in the first part of the problem task when they needed to calculate the mass of the sphere. It is evident from the data which show they are highly represented in the category of UA, unlike the categories of MA and NA. The abovementioned results were expected with regard to the type of physics curriculum in these institutions. Up to 71% of students tested by individual interviews failed to reach the final answer.

The distribution of students tested using written tests (GS – Split; US – Split), in Fig. 2, shows that a higher percentage of US in comparison to GS belong to the categories of ScA, SA, and UA, and smaller percentage belong to the category of MA. This was expected, taking into account that US are at a higher educational level than GS. When compared to GS, it is surprising that a higher percentage of US use NA, which means there are certain gaps in students' knowledge and skills that remain undiscovered by using the traditional education tasks.

The different results for individual interviews and for written tests are caused by the testing method itself. The students tested by written tests were forced to use their own knowledge and skills for problem solving, as well as for asking a help via written questions. The students tested by interviews had the examiners interactive guidance and stimulus which significantly contributed to their better result. This resulted in a higher percentage of students in the categories of ScA, SA and UA.

This study confirms that most participants (93%) who do not have a ScA, the largest number have NA (see Fig. 3), use quantitative approach to physical problems although many researchers like Van Heuvelen [4] and Meltzer [25] emphasize the importance of qualitative approach. It is common that students try to solve a problem by referring to formulas which could lead to the result, instead of using appropriate concepts. Even those students or experts, who chose a qualitative approach to complex problems, tend to solve a simple algorithmic problem in this way [23,26]. Consequently the difficulties also occur in checking the validity of the result or finding the final answer in partially specified problems.

4. Conclusion

In this research students were asked to solve one partially specified physics problem in order to investigate their problem solving strategies. The partially specified problem is one in which student has to answer the question whether something is possible or not in real-life. To answer this question, student has to use the physical laws to calculate the desired physical quantity, but this is not enough. They also have to take into account real-world conditions and to judge the numerical result with respect to that. To our knowledge, this study is the first to explore students' reaction to this type of a problem. We combined two methods of investigation, individual interviews and written tests. Participants were divided into five categories according to their problem solving approach. The results showed that the majority of students belong to categories preferring a quantitative approach, regardless of curriculum, educational level or research method.

Based on our results of investigation and the student comments, we consider that the standard numerical exercises do not sufficiently develop students' critical thinking ability, one of the most important aims of physics teaching. Students are not sufficiently trained to solve real-life problems that are structured to have great freedom of parameters, several alternative solving options, and different criteria for evaluating solution. We suggest the partially specified problems to be provided to students and also included in education of future physics teachers. Such tasks could help in acquiring different concepts and problem solving techniques, and train students and teachers to solve real-life problems using physical principles and assumptions. We believe that the partially specified problems could encourage learners to make decisions about what should be calculated and how to judge the solutions and reality of problem situations. Thus, they could develop competencies necessary for life-long learning in highly changing personal and working worlds.

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