Useful ratios between two-body nonleptonic and semileptonic decays of B mesons

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We compute important ratios between decay widths of some exclusive two-body nonleptonic and semileptonic B decays, which could be test of factorization hypothesis. We also present a summary of the expressions of the decay widths and differential decay rates of these decays, at tree level, including $l = 0$ (ground state), $l = 1$ (orbitally excited) and $n = 2$ (radially excited) mesons in the final state. From a general point of view, we consider eight transitions, namely $H \to P, V, S, A, A^{'}, T, P(2S), V(2S)$. Our analysis is carried out assuming factorization hypothesis and using the WSB, ISGW and CLFA quark models.

Keywords: B physics; semileptonic decays; nonleptonic decays.

Calculamos varias relaciones importantes entre los anchos de decaimiento de canales exclusivos no leptónicos y semileptónicos del mesón B, las cuales pueden servir como prueba a la hipótesis de factorización. También, presentamos un resumen sobre las expresiones de los anchos de decaimiento y los anchos de decaimiento diferenciales, para estos procesos, a nivel árbol, incluyendo mesones con $l = 0$ (sin excitación orbital), $l = 1$ (excitados orbitalmente) y $n = 2$ (excitados radialmente) en el estado final. Desde un punto de vista general, consideramos ocho transiciones: $H \to P, V, S, A, A^{'}, T, P(2S), V(2S)$. Nuestro análisis se desarrolla asumiendo hipótesis de factorización y utilizando los modelos de quarks WSB, ISGW y CLFA.

Descriptores: Física del B; decaimientos semileptónicos; decaimientos no leptónicos.

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1. Introduction

Exclusive semileptonic and two-body nonleptonic decays of heavy mesons offer a good scenario for studying, at theoretical and experimental levels, CP violation and physics beyond the Standard Model. Some of these channels provide methods for determining the angles of the unitarity triangle, allow to study the role of QCD and test some QCD-motivated models (see for example some recent reviews in Ref 1). These topics are of great interest in particle physics and the knowledge of them will be improved with forthcoming experiments at Large Hadron Collider (LHC) [2].

The purpose of this paper is to compute useful ratios between two-body nonleptonic and semileptonic decays of heavy (H) mesons, at tree level, that could be tested experimentally. Specifically, we work with exclusive B channels although we also consider a couple of B_s processes. We assume naive factorization and use the WSB [3], ISGW [4] and CLFA [5] quark models.

It is expected that naive factorization approach works reasonably well in decays where penguin and weak annihilation contributions are absent or suppressed, such as $B \to DK$ [6], $K^0 \to \pi\pi$, $D^0 \to K^{\pm}\pi^{\mp}$, $D^0 \to K^+K^-$, $\pi^+\pi^-$ and $B_s \to J/\psi\phi$ [7], $D^+ \to \overline{K}_0^{*0}\pi^+$ and $D_s^+ \to f_0\pi^+$ [8] channels. Also, factorization assumption works well in two-body hadronic decays of B_c meson (without considering charmless modes) where the quark-gluon sea is suppressed in the heavy quarkonium [9].

We also present an important summary and a general analysis on the expressions of the decay widths and differential decay rates of two-body nonleptonic and semileptonic decays of heavy mesons, respectively, including $l = 0,\ 1$ and $n = 2$ mesons in the final state. For $l = 0$, we have considered pseudoscalar (P) and vector (V) mesons, for $l = 1$ we have included orbitally excited (p-wave) scalar (S), axial-vector ($A,\ A^{'}$) and tensor (T) mesons, and for $n = 2$, we have studied radially excited $P(2S)$ and $V(2S)$ mesons (see Table I). We have classified eight transitions, namely $H \to P,\ V,\ S,\ A,\ A^{'},\ T,\ P(2S),\ V(2S)$, in three groups. It allows us to manipulate, in an easy way, all these decays.

The paper is organized as follows: In Sec. 2 we present, in a general way, the parametrization of the hadronic matrix element $\langle M|J_\mu|H\rangle$ for eight cases. Sec. 3 contains expressions for $\Gamma(H \to M_1M_2)$ and $d\Gamma(H \to Ml\nu)/dt$ and a brief discussion. In Sec. 4, we analyze vector and axial contributions of the weak interaction to $H \to (P, V, S, A, A^{'}, T)l\nu$ decays assuming a meson dominance model. In Sec. 5, we compute some important ratios between decay widths of exclusive B (and B_s) decays, which allow us to get tests to factorization approach. Concluding remarks are presented in Sec. 6. Finally, in the appendix we briefly mention the quark models used in this work.

TABLE I. Classification of mesons considering the $n\ ^{2s+1}L_J$ and the J^{PC} notations. n is the radial quantum number, l is the orbital angular momentum, s is the spin, and J is the total angular momentum. P and C are parity and charge conjugate operators, respectively.

n	l	s	J	$n\ ^{2s+1}L_J$	J^{PC}	Meson
	0	0	0	$1\ ^1S_0$	0^{-+}	Pseudoscalar (P)
		1	1	$1\ ^3S_1$	1^{--}	Vector (V)
		0	1	$1\ ^1P_1$	1^{+-}	Axial-vector (A')
1	1		0	$1\ ^3P_0$	0^{++}	Scalar (S)
		1	1	$1\ ^3P_1$	1^{++}	Axial-vector (A)
			2	$1\ ^3P_2$	2^{++}	Tensor (T)
2	0	0	0	$2\ ^1S_0$	0^{-+}	$P(2S)$
		1	1	$2\ ^3S_1$	1^{--}	$V(2S)$

2. Hadronic Matrix Elements

In this section, we present the parametrizations of the eight $H \to M$ transitions, where H denotes a pseudoscalar heavy meson and M can be a P, V, S, A, A', T, $P(2S)$, $V(2S)$ meson, classified in three groups[i]. In the first case, the M meson has $J = 0$, in the second, $J = 1$, and in the third group $J = 2$.

2.1. $H \to M(J = 0)$ transition

In this group, there are three transitions if M is a meson with $J = 0$ (see Table I): M can be the pseudoscalar P meson, or the scalar S meson, which is an orbitally excited meson, or the radially excited meson $P(2S)$. The hadronic matrix element $\langle M|J_\mu|H\rangle$ for $M = P,\ S,\ P(2S)$ has the same Lorentz structure and it is defined as follows [4]:

$$\langle M(p_M)|J_\mu|H(p_H)\rangle \equiv F_+(p_H + p_M)_\mu + F_-(p_H - p_M)_\mu, \qquad (1)$$

where J_μ is the $V_\mu - A_\mu$ weak current, $p_{H(M)}$ is the 4 - momentum of the meson $H(M)$, F_+ and F_- are form factors. Following the notation displayed in appendix of the ISGW model [4], these form factors are:

- For $M = P$: $\langle P|J_\mu|H\rangle \equiv \langle P|V_\mu|H\rangle$, $F_+ = f_+$ and $F_- = f_-$.
- For $M = S$: $\langle S|J_\mu|H\rangle \equiv -\langle S|A_\mu|H\rangle$, $F_+ = u_+$ and $F_- = u_-$.
- For $M = P(2S)$: $\langle P(2S)|J_\mu|H\rangle \equiv \langle P(2S)|V_\mu|H\rangle$, $F_+ = f'_+$ and $F_- = f'_-$.

It is important to note that the parity operator requires that $\langle P|A_\mu|H\rangle = 0$ and $\langle S|V_\mu|H\rangle = 0$.

Reference 3 uses a different parametrization for $\langle P|J_\mu|H\rangle$ using dimensionless F_1 and F_0 form factors. It is possible to transform $(F_1,\ F_0) \ \to\ (f_+,\ f_-)$ using the relations showed in the appendix.

2.2. $H \to M(J = 1)$ transition

Considering the M meson with J=1, this group has four transitions (see Table I): M=V, A, A', $V(2S)$. The hadronic matrix element $\langle V(A(1^3P_1), A(1^1P_1), V(2S))|J_\mu|H\rangle$ can be parametrized by means of the following linear combination which is Lorentz-covariant [4]:

$$\begin{aligned}\langle M(p_M,\epsilon)|J_\mu|H(p_H)\rangle \equiv & iG\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p_H+p_M)^\rho(p_H-p_M)^\sigma \\ & +F\epsilon^*_\mu+A_+(\epsilon^*.p_H)(p_H+p_M)_\mu \\ & +A_-(\epsilon^*.p_H)(p_H-p_M)_\mu, \qquad (2)\end{aligned}$$

where G, F, and $A_\pm$ are form factors, ϵ is the polarization vector of meson M and $p_{H(M)}$ is the 4-momentum of the meson $H(M)$. Following the notation used in the appendix of the ISGW model [4], these form factors are:

- For $M = V$: $G = g$, $F = -f$, $A_+ = -a_+$ and $A_- = -a_-$.
- For $M = A(1\ ^3P_1) \equiv A$: $G = -q$, $F = l$, $A_+ = c_+$ and $A_- = c_-$.
- For $M = A(1\ ^1P_1) \equiv A'$: $G = -v$, $F = r$, $A_+ = s_+$ and $A_- = s_-$.
- For $M = V(2S)$: $G = g'$, $F = -f'$, $A_+ = -a'_+$ and $A_- = -a'_-$.

The parametrization of the matrix element for the $H \to A$ transition has the same structure that the matrix element of the $H \to V$ transition just interchanging the role of vector and axial currents: $\langle V|V_\mu(A_\mu)|H\rangle \leftrightarrow \langle A|A_\mu(V_\mu)|H\rangle$.

Reference 3 ([5]) works with another parametrization for the $H \to V\ (A)$ transition, which is very useful because it allows to write the decay width of two-body nonleptonic decays of heavy mesons as a function of helicity form factors (see for example the Refs. 3 and 10). It is easy to transform the parametrization given by the Eq. (2) into the parametrization given in the Refs. 3 and 5 by using the relations between form factors showed in the appendix.

2.3. $H \to M(J = 2)$ transition

This group contains only one transition (see Table I): when M is a tensor meson (T), which is a p-wave. The Lorentz-covariant parametrization of the hadronic matrix element $\langle T|J^\mu|H\rangle$ given in the ISGW model is [4]:

$$\begin{aligned}&\langle T(p_T,\epsilon)|J^\mu|H(p_H)\rangle = ih(q^2)\varepsilon^{\mu\nu\rho\sigma} \\ &\times \epsilon_{\nu\alpha}p_H^\alpha(p_H+p_T)_\rho(p_H-p_T)_\sigma - k(q^2)\epsilon^{*\mu\nu}(p_H)_\nu + \\ &\epsilon^*_{\alpha\beta}p_H^\alpha p_H^\beta\left[b_+(q^2)(p_H+p_T)^\mu + b_-(q^2)(p_H-p_T)^\mu\right], \qquad (3)\end{aligned}$$

TABLE II. Differential decay widths of $H \to (P, V, S, A, A^{'}, T, P(2S), V(2S))l\nu_l$.

$H \to Ml\nu_l$	$d\Gamma(H \to Ml\nu_l)/dt$
$H \to (P,\ S,\ P(2S))l\nu_l$	$\zeta\left[A(t)\lvert F_1^{HM}(t)\rvert^2\lambda^{3/2} + B(t)\lvert F_0^{HM}(t)\rvert^2\lambda^{1/2}\right]$
	$\zeta\mathcal{G}(t)$
$H \to (V,\ A,\ A^{'},\ V(2S))l\nu_l$	$\zeta t\lambda^{1/2}\left[\lvert H_+(t)\rvert^2 + \lvert H_-(t)\rvert^2 + \lvert H_0(t)\rvert^2\right]$
	$\zeta\left\{\varphi(t)\lambda^{5/2} + \rho(t)\lambda^{3/2} + \theta(t)\lambda^{1/2}\right\}$
$H \to Tl\nu_l$	$\zeta\left\{\alpha(t)\lambda^{7/2} + \beta(t)\lambda^{5/2} + \gamma(t)\lambda^{3/2}\right\}$

where $\epsilon_{\nu\alpha}$ is the polarization tensor of the tensor meson, $p_{H(T)}$ is the momentum of the heavy meson $H(T)$, and $h, k, b_{\pm}$ are form factors. k is dimensionless and $h, b_{\pm}$ have dimensions of GeV^{-2}.

In the literature [11, 12], there is another parametrization of $\langle T|J^{\mu}|H\rangle$, which is constructed in analogy with the parametrization of $\langle V|J^{\mu}|H\rangle$ given in Ref. 3, using the tensor polarization $\epsilon_{\mu\nu}$ of the T meson.

3. $d\Gamma(H \to Ml\nu)/dt$ and $\Gamma(H \to M_1M_2)$

In this section we collect, in a compact form, using the classification of the last section, the expressions, at tree level, of the differential decay rate of $H \to Ml\nu_l$ (see Table II) and the decay width of $H \to MM^{'}$ (see Table III), where H is a heavy meson (D, D_s, B, B_s or B_c), and M $(M^{'})$ can be any of the eight mesons $P,\ V,\ S, A,\ A^{'},\ T, P(2S),\ V(2S)$.

In the first row of Table II, we display the differential decay rate of the semileptonic $H \to Ml\nu_l$ decay, where M is a meson with $J = 0$, *i.e*, $M = P,\ S,\ P(2S)$, using the parametrization given in the WSB model [3]. The second row shows the differential decay rate of $H \to Ml\nu_l$, where M is a meson with $J = 1$, *i.e*, $M = V,\ A,\ A^{'},\ V(2S)$, using parametrizations given in the WSB [3] and ISGW [4] quark models, and in the last row we give the differential decay rate for $H \to T(J = 2)l\nu_l$ using the parametrization of the ISGW model [4].

In Table II, $\lambda = \lambda(m_H^2, m_M^2, t)$, where

$$\lambda = \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

is the triangular function, $t = (p_H - p_M)^2$ is the momentum transfer and $H_{\pm,0}$ are helicity form factors [3]. The factor ζ and functions $A(t)$, $B(t)$, $\mathcal{G}(t)$, $\varphi(t)$, $\rho(t)$, $\theta(t)$, $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are defined by:

$$\zeta = \frac{G_F^2|V_{q'q}|^2}{192\pi^3 m_H^3}, \tag{4}$$

$$A(t) = \left(\frac{t - m_l^2}{t}\right)^2\left(\frac{2t + m_l^2}{2t}\right), \tag{5}$$

$$B(t) = \frac{3}{2}m_l^2\left(\frac{t - m_l^2}{t}\right)^2\frac{(m_H^2 - m_P^2)^2}{t}, \tag{6}$$

$$\begin{aligned}\mathcal{G}(t) = &\left[\frac{2t|V(t)|^2}{(m_H + m_V)^2} + \frac{(m_H + m_V)^2|A_1(t)|^2}{4m_V^2}\right.\\ &\left.-\frac{(m_H^2 - m_V^2 - t)A_1(t)A_2(t)}{2m_V^2}\right]\lambda^{3/2}\\ &+\frac{|A_2(t)|^2}{4m_V^2(m_H + m_V)^2}\lambda^{5/2}\\ &+3t(m_H + m_V)^2|A_1(t)|^2\lambda^{1/2},\end{aligned} \tag{7}$$

$$\varphi(t) = \frac{s_+^2}{4m_A^2}, \tag{8}$$

$$\rho(t)=\frac{1}{4m_A^2}\left[r^2+8m_A^2tv^2+2(m_H^2-m_A^2-t)rs_+\right], \tag{9}$$

$$\theta(t) = 3t\,r^2, \tag{10}$$

$$\alpha(t) = \frac{b_+^2}{24m_T^4}, \tag{11}$$

$$\beta(t)=\frac{1}{24m_T^4}\left[k^2+6m_T^2th^2+2(m_H^2 - m_T^2-t)kb_+\right], \tag{12}$$

$$\gamma(t) = \frac{5tk^2}{12m_T^2}, \tag{13}$$

where G_F is the Fermi constant, $m_{H(P,\ V,\ A,\ T)}$ is the mass of the $H(P,\ V,\ A,\ T)$ meson, m_l is the mass of the lepton, $V(t)$ and $A_{1,2}(t)$ are form factors [3], $\varphi(t)$, $\rho(t)$ and $\theta(t)$ are quadratic functions of the form factors s_+, r and v (c_+, l and q) for $H \to A(^1P_1)l\nu$ ($H \to A(^3P_1)l\nu$), $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are quadratic functions [13] of the form factors k, b_+ and h. All these form factors are explicitly given in the appendix B of the Ref. 4.

The dependence of $d\Gamma(H \to Ml\nu)/dt$ with

$$\lambda(|\overrightarrow{p}|=\lambda^{1/2}/2m_H,$$

where $\overrightarrow{p}$ is the three-momentum of the M meson in the H meson rest frame) is given by,

$$d\Gamma/dt \sim \lambda^{l+\frac{1}{2}},$$

where l is the orbital angular momentum of the wave at which the particles in the final state can be coupled. Assuming conservation of total angular momentum J and a meson dominance model we can find specific values for l in each exclusive $H \to Ml\nu$ decay. Thus, in $H \to M(J = 0)l\nu$ the particles in the final state are coupled to $l = 0, 1$ waves (see the first row in Table II). When $m_l \approx 0$ ($l = e, \mu$), the coefficient $B(t)$ vanishes, so the contribution of the s-wave is negligible; in $H \to M(J = 1)l\nu$ the particles in the final state can be coupled to $l = 0, 1, 2$ waves (see the second row in Table II); and in $H \to T(J = 2)l\nu$ to $l = 1, 2, 3$ waves (see the last row in Table II).

It is also possible to write in a compact expression the differential decay rate of the semileptonic $H \to Ml\nu_l$ decay, where M is a p-wave (orbitally excited) meson: scalar, vector-axial or tensor meson, in terms of helicity amplitudes (see Ref. 14).

As for two-body nonleptonic decays of heavy mesons, the effective weak Hamiltonian $\mathcal{H}_{eff}$ has contributions from current-current (tree), QCD penguin and electroweak penguin operators [15]. In general, $\mathcal{H}_{eff} \approx \sum_i C_i(\mu)\mathcal{O}_i$, where $C_i(\mu)$ are the Wilson coefficients and $\mathcal{O}_i$ are local operators. The amplitude for the $H \to M_1 M_2$ decay is

$$\mathcal{M}(H \to M_1 M_2) \approx \sum_i C_i(\mu)\langle\mathcal{O}\rangle_i. \qquad (14)$$

In the scenario of naive factorization, it is assumed that

$$\mathcal{M}(H \to M_1 M_2) \approx C_i(\mu)\langle M_2|(J_{1i})_\mu|0\rangle \times \langle M_1|(J_{2i})^\mu|H\rangle + (M_1 \leftrightarrow M_2), \qquad (15)$$

where J_μ is the $V_\mu - A_\mu$ weak current and the hadronic matrix element of a four-quark operator is written as the product of a decay constant and form factors [16].

This factorization presents a difficulty because the Wilson coefficients are μ scale and renormalization scheme dependent while $\langle\mathcal{O}\rangle_i$ are μ scale and renormalization scheme independent, so clearly the physical amplitude depends on the μ scale. The naive factorization disentangles the short-distance effects from the long-distance sector assuming that $\langle\mathcal{O}\rangle_i$, at μ scale, contain nonfactorizable contributions in order to cancel the μ dependence and the scheme dependence of $C_i(\mu)$. Thus, the naive factorization is an approximation because it does not consider possible QCD interactions between the meson M_2 and the H and M_1 mesons. In general, it does not work in all two-body heavy meson decays [16].

Assuming naive factorization, we have considered only those decays which are produced by the color-allowed external W-emission tree diagram or the color-suppressed internal W-emission diagram. It is expected that naive factorization works reasonably well in decays where penguin and weak annihilation contributions are absent or negligible, as for example in $B \to DK$ [6], $K^0 \to \pi\pi$, $D^0 \to K^\pm\pi^\mp, K^+K^-, \pi^+\pi^-$ and $B_s \to J/\psi\phi$ [7], $D^+ \to \overline{K}_0^{*0}\pi^+$ and $D_s^+ \to f_0\pi^+$ [8] channels. Also, factorization assumption works well in two-body hadronic decays of B_c meson (except in charmless processes, because they are produced only by annihilation contributions) where the quark-gluon sea is suppressed in the heavy quarkonium [9].

We have used the notation $H \to M_1, M_2$ [17] to mean that M_2 is factorized out under factorization approximation, *i.e.*, M_2 arises from the vacuum. For $H \to TM$ decays there is not any possibility to produce the T meson from the vacuum with the $V - A$ current, because $\langle T|(V - A)_\mu|0\rangle \equiv 0$. So, this decay has only the contribution $H \to T, M$. Recently, it has been reported that it is possible to produce tensor mesons from the vacuum involving covariant derivatives [12, 18].

Using the parametrizations given in Sec. 2 for eight transitions, namely $H \to M(J = 0,\ 1,\ 2)$, we display, in Table III, expressions of decay widths for 40 different types of $H(q_H\bar{q}') \to M_1(q\bar{q}')M_2(q_i\bar{q}_j)$ decays, which are produced by the $q_H \to q\bar{q}_j q_i$ transition.

In the first row of Table III, we show the decay width for six different types of channels: $H \to P, P'$; $P, P'(2S)$; S, P'; $S, P'(2S)$; $P(2S), P'$; $P(2S), P'(2S)$. They are produced by the $H \to M(J = 0)$ transition. The hadronic matrix elements $\langle P(S,\ P(2S))|J_\mu|H\rangle$, which are neccesary in order to calculate the decay width, have the same parametrization. In this case, we have used the parametrization presented in Ref. 3. In these decays the particles in the final state are coupled to a s- wave because $\Gamma \sim \lambda^{0+1/2}$. In a similar way, in the second row of Table III, we display the decay width of nine different modes: $H \to P, V$; P, A; $P, V(2S)$; S, V; S, A; $S, V(2S)$; $P(2S), V$; $P(2S), A$; $P(2S), V(2S)$. These nine channels have in common the $H \to M(J = 0)$ transition. In these decays, the particles in the final state are coupled to a p-wave ($l = 1$).

In the third row of Table III, we present the decay width for eight different types of decays: $H \to V, P$; $V,\ P(2S)$; A, P; $A,\ P(2S)$; A', P; $A',\ P(2S)$; $V(2S), P$; $V(2S),\ P(2S)$. The hadronic matrix elements $\langle V(A, A'; V(2S))|J_\mu|H\rangle$, which correspond to the $H \to M(J = 1)$ transition, have a similar parametrization. The particles in the final state in these decays are coupled to a p-wave ($l = 1$). In the fourth row of Table III, we display the decay width for twelve different decays: $H \to V_1, V_2$; V_1, A_2; $V_1, V_2(2S)$; A_1, V_2; A_1, A_2; $A_1, V_2(2S)$; A_1', V_2; A_1', A_2; $A_1', V_2(2S)$; $V_1(2S), V_2$; $V_1(2S), A_2$; $V_1(2S), V_2(2S)$. They also arise from the $H \to M(J = 1)$ transition[ii]. The two $J = 1$ particles in the final state can be coupled to $l = 0, 1, 2$ waves.

In the fifth row of Table III, we show the decay widht for the $H \to T, P(P(2S))$ channels, which are produced by the $H \to T$ transition. We have used the parametrization for $\langle T|J_\mu|H\rangle$ given in the Ref. 4. In this case, the particles in the final state can be coupled to a $l = 2$ wave. Using the same parametrization, we present in the last row of Table III, the decay width for three different modes: $H \to T, V$ ($A,\ V(2S)$). In this case, the particles in the final state can be coupled to $l = 1, 2, 3$ waves.

In Table III, all form factors and the function λ are evaluated in $m_{M_2}^2$ because the momentum transfer $t{=}(p_H{-}p_1)^2 = p_2^2 = m_{M_2}^2.\xi^{(M_2)}$, $\mathcal{F}^{H\to T}$ and the decay constants are given by

TABLE III. Decay widths of $H \to M_1 M_2$, where $M_{1,2} = P, V, S, A, A^{'}, T, P(2S), V(2S)$

$H \to M_1, M_2$	$\Gamma(H \to M_1, M_2)$
$H \to (P_1, S_1, P_1(2S)),\ (P_2, P_2(2S))$	$\xi^{(M_2)}(m_H^2 - m_{M_1}^2)^2 \lvert F_0^{HM_1}(m_{M_2}^2)\rvert^2 \lambda^{1/2}$
$H \to (P_1, S_1, P_1(2S)),\ (V, A, V(2S))$	$\xi^{(M_2)} \lvert F_1^{HM_1}(m_{M_2}^2)\rvert^2 \lambda^{3/2}$
$H \to (V, A, A^{'}, V(2S)),\ (P, P(2S))$	$\xi^{(M_2)} \lvert A_0^{HM_1}(m_{M_2}^2)\rvert^2 \lambda^{3/2}$
$H \to (V_1, A_1, A_1^{'}, V_1(2S)),\ (V_2, A_2, V_2(2S))$	$\xi^{(M_2)} \mathcal{G}(t = m_{V_2}^2)$
	$\xi^{(M_2)} m_2^2 \lambda^{1/2} \left[\lvert H_+(m_{M_2}^2)\rvert^2 + \lvert H_-(m_{M_2}^2)\rvert^2 + \lvert H_0(m_{M_2}^2)\rvert^2\right]$
$H \to T,\ (P, P(2S))$	$\xi^{(M_2)}(1/24 m_T^4) \lvert \mathcal{F}^{H\to T}(m_{M_2}^2)\rvert^2 \lambda^{5/2}$
$H \to T,\ (V, A, V(2S))$	$\xi^{(M_2)} \left[\alpha(m_{M_2}^2)\lambda^{7/2} + \beta(m_{M_2}^2)\lambda^{5/2} + \gamma(m_{M_2}^2)\lambda^{3/2}\right]$

TABLE IV. Vector and axial contributions to semileptonic $H \to (P, V, S, A(^3P_1 \text{or} ^1P_1), T)l\nu$ decays.

Contribution	J^P of W^*	$H \to Pl\nu$	$H \to Vl\nu$	$H \to Sl\nu$	$H \to Al\nu$	$H \to Tl\nu$
Vector	0^+	$l = 0$			$l = 1$	
	1^-	$l = 1$	$l = 1$		$l = 0, l = 2$	$l = 2$
Axial	0^-		$l = 1$	$l = 0$		$l = 2$
	1^+		$l = 0, l = 2$	$l = 1$	$l = 1$	$l = 1, l = 3$

$$\xi^{(M_2)} = \frac{G_F^2 \lvert V_{qq_H}\rvert^2 \lvert V_{q_i q_j}\rvert^2 a_{1(2)}^2 f_{M_2}^2}{32\pi m_H^3}, \tag{16}$$

$$\mathcal{F}^{H\to T}(m_P^2) = k + (m_H^2 - m_T^2)b_+ + m_P^2 b_-, \tag{17}$$

$$\langle M(p)\lvert J_\mu \rvert 0\rangle = i f_M p_\mu, \quad M = P,\ P(2S), \tag{18}$$

$$\langle M(p,\epsilon)\lvert J_\mu \rvert 0\rangle = f_M m_M \epsilon_\mu, \quad M = V,\ A,\ V(2S), \tag{19}$$

where k and $b_\pm$ are form factors given in the ISGW model [4], evaluated at $t = m_P^2$, $a_{1(2)}$ are the QCD factors, and $\lvert V_{qq_H}\rvert$ and $\lvert V_{q_i q_j}\rvert$ are the appropriate CKM factors.

Finally, we do not consider decays where a tensor meson, or a scalar meson or an axial-vector meson $1\ ^1P_1$ arises from the vacuum. In the first case, as we mentioned before, $\langle T\lvert J_\mu\rvert 0\rangle \equiv 0$; in the second case, the decay constant of the scalar mesons, defined as $\langle S\lvert J_\mu\rvert 0\rangle = f_S p_\mu$ vanishes or is small (of the order of $m_d - m_u$, $m_s - m_{u,d}$); and in the last case, the decay constant of the $1\ ^1P_1$ meson vanishes in the SU(3) limit by G- parity [19].

4. Contributions of the vector and axial couplings

In this section, we illustrate how the particles in the final state of $H \to Ml\nu$ and $H \to M_1 M_2$ decays can be coupled to specific waves, obtain the quantum numbers of the poles that appear in the monopolar form factors, and explain the correspondence between the form factors and the respective waves in the final state. We show that these numbers depend on the vector and axial couplings of the weak interaction. Let us consider the decay chain $H \to MM^* \to MW^* \to Ml\nu(MM')$, where W^* is the off-shell intermediate boson of the weak interaction. We need to combine parity and total angular momentum conservations in the strong $H \to MM^*$ process.

In Table IV, we show the specific waves in which particles in the final state of $H \to (P, V, S, A, T)l\nu$ decays can be coupled and determine if they come from the vector or axial contributions. We must keep in mind that the off-shell W^* boson has spin 0 or 1. Thus, in the vector coupling there are two possibilities: $S_{W^*} = 0$ with $P_{W^*} = +1$, and $S_{W^*} = 1$ with $P_{W^*} = -1$ (S_{W^*} and P_{W^*} denote spin and parity of W^*, respectively). In a similar way, in the axial coupling there are two options: $S_{W^*} = 0$ with $P_{W^*} = -1$ and $S_{W^*} = 1$ with $P_{W^*} = +1$. Thus, there are four cases for the W^* boson: $J^P = 0^+$, 1^-, 0^- and 1^+. They are displayed in the second column of Table IV. Assuming total angular momentum and parity conservations of the strong $H \to MM^*$ process, we obtain the values of the orbital angular momentum l of the particles in the final state of $H \to Ml\nu$ (see Table IV). These values can be verified with the exponent $l + (1/2)$ of λ in the expressions for $d\Gamma/dt$ in Table II. We can see in the third (fourth) and the fifth (sixth) columns in Table IV, that the vector and axial contributions interchange their roles in $H \to Pl\nu$ ($H \to Vl\nu$) and $H \to Sl\nu$ ($H \to Al\nu$), respectively.

In Table V, we show the respective form factors with the corresponding poles in $H \to P(V)l\nu$ decays. In the second column, we list the quantum numbers J^P of poles, which are the same J^P options for the off-shell W^* boson (see the second column in Table IV). In this case, we must check the form factors that appear in the parametrization of the hadronic matrix elements $\langle M\lvert V_\mu\rvert H\rangle$ and $\langle M\lvert A_\mu\rvert H\rangle$ for $M = P, V$. Following this idea, we obtain the quantum numbers of the poles for $H \to Ml\nu$ where M is a p-wave meson: for $H \to Sl\nu$ the poles are 0^- and 1^+; for $H \to Al\nu$, the poles are 0^+, 1^-

TABLE V. Form factors and the vector and axial contributions of the weak interaction to $H \to (P,V)l\nu$ decays.

Contribution	J^P of Pole	$H \to Pl\nu$	$H \to Vl\nu$
Vector	0^+	$F_0(t)$	
	1^-	$F_1(t)$	$V(t)$
Axial	0^-		$A_0(t)$
	1^+		$A_1(t)$, $A_2(t)$, $A_3(t)$

and 1^+; and for $B \to Tl\nu$ the poles are 1^-, 0^- and 1^+. These values are important if we are interested in constructing a quark model with monopolar form factors for $H \to S,\ A,\ T$ transitions.

Let us illustrate, as an example, the situation on $H{\to}Pl\nu$ from Tables IV and V. This decay has two contributions: $l = 0$ and $l = 1$ (see exponents of λ in Table II) which arise from the vector coupling of the weak current (see Table IV). The respective poles have quantum numbers 0^+ and 1^- and the form factors are F_0 and F_1 (see Table V).

5. Useful ratios

In this section, we present some ratios between exclusive semileptonic and two-body nonleptonic decays of B and B_s mesons, using the expressions for $d\Gamma(H \to Ml\nu)/dt$ and $\Gamma(H \to M_1, M_2)$ (see Tables II and III, respectively), that could be a test of factorization hypothesis with forthcoming measurements at LHC. We have worked with decays where it is expected that naive factorization works well. In order to obtain the numerical values presented in this section, we have evaluated the form factors in the WSB [3] and CLFA [5] quark models and taken from the Particle Data Group [20] the values of the CKM factors, branching ratios, masses and mean lifetime of mesons.

5.1 $B \to P, M(c\bar{c})$ *decays:* Let us consider exclusive two-body nonleptonic B decays with orbitally or radially excited charmonium mesons in the final state, which are produced by the color suppressed $b \to c\bar{c}s(d)$ transition. The following ratio

$$\frac{\Gamma(B^+ \to P^+, M_1(c\bar{c}))}{\Gamma(B^+ \to P^+, M_2(c\bar{c}))} = (\text{kinematical factor}) \left(\frac{f_{M_1}}{f_{M_2}}\right)^2 \left|\frac{F_{0(1)}^{B\to P}(m_{M_1}^2)}{F_{0(1)}^{B\to P}(m_{M_2}^2)}\right|^2,$$

allows to obtain decay constants of charmonium mesons. The form factor $F_{0(1)}$ corresponds when M_1 and M_2 are $J = 0(1)$ mesons.

Evaluating the form factors in the CLFA model [5], we obtain

$$\frac{f_{J/\psi}}{f_{\psi(2S)}} = 1.15 \pm 0.07\ (1.29 \pm 0.17),$$

$$\frac{f_{J/\psi}}{f_{\chi_{c1}(1P)}} = 1.41 \pm 0.13\ (1.51 \pm 0.32),$$

$$\frac{f_{\eta_c}}{f_{\eta_c(2S)}} = 1.65 \pm 1.27,$$

$$\frac{f_{\eta_c}}{f_{\chi_{c0}(1P)}} = 2.63 \pm 0.52,$$

taking ($M_1{=}J/\psi,\ M_2{=}\psi(2S),\ P{=}K(\pi)$), ($M_1 = J/\psi$, $M_2 = \chi_{c1}(1P)$, $P = K(\pi)$), ($M_1 = \eta_c$, $M_2 = \eta_c(2S)$, $P = K$), and ($M_1 = \eta_c,\ M_2 = \chi_{c0}(1P),\ P = K$), respectively. The most important sources of uncertainties come from experimental values of branching ratios and form factors. However, the error in the last ratios is dominated by the uncertainty in the branching ratios. These quotients between decay constants with orbitally and radially excited charmonium states are a good test of the factorization hypothesis.

On the other hand, taking $f_{J/\psi} = (416.3 \pm 5.3)$ MeV [21,22] and $f_{\eta_c} = (335 \pm 75)$ MeV [23], we obtain:

$$\begin{aligned} f_{\psi(2S)} &= 361.7 \pm 22.5\ (322.7 \pm 42.7) \quad \text{MeV}, \\ f_{\chi_{c1}(1P)} &= 295.24 \pm 27.48\ (275.7 \pm 58.5) \quad \text{MeV}, \\ f_{\eta_c(2S)} &= 203.03 \pm 102.12 \quad \text{MeV}, \\ f_{\chi_{c0}(1P)} &= 127.4 \pm 38.1 \quad \text{MeV}. \end{aligned} \tag{20}$$

From these values we obtain $f_{\eta_c}/f_{\eta_c(2S)} = 1.65 \pm 0.9$ and $f_{J/\psi}/f_{\psi(2S)} = 1.15 \pm 0.07(1.29 \pm 0.17)$ while in the Ref. 21 is obtained $f_{\eta_c}/f_{\eta_c(2S)} = f_{J/\psi}/f_{\psi(2S)} = 1.41$.

5.2 $B^+ \to K^+(\pi^+), J/\psi$ *decays*: An important test to naive factorization is given by

$$\frac{\Gamma(B^+ \to K^+, J/\psi)}{\Gamma(B^+ \to \pi^+, J/\psi)} = (18.31 \pm 1.51) \left|\frac{F_1^{B\to K}(m_{J/\psi}^2)}{F_1^{B\to \pi}(m_{J/\psi}^2)}\right|^2 = 33.21 \pm 5.14,$$

where the errors come from the numerical values of the CKM and form factors (which are evaluated in the CLFA model [5]). The experimental value of this ratio is 20.7 ± 1.8 [20]. This sizable difference means that these exclusive channels have large nonfactorizable contributions [24]. Some authors have explored the possibility of new physics in these decays [25].

5.3 $\overline{B_s^0} \to D_s^+(K^+), K^-(D_s^-)$ *decays:* The ratio between the branching ratios of $\overline{B_s^0} \to D_s^+, K^-$ (mediated by the $b \to c\bar{u}s$ transition) and $\overline{B_s^0} \to K^+, D_s^-$ decays (mediated by the $b \to u\bar{c}s$ transition), which are color favored, is

$$\mathcal{R} = \frac{\mathcal{B}(\overline{B_s^0} \to D_s^+, K^-)}{\mathcal{B}(\overline{B_s^0} \to K^+, D_s^-)} = (3.94) \left(\frac{f_{K^-}}{f_{D_s^-}}\right)^2 \times \left|\frac{F_0^{B_s\to D_s}(m_{K^-}^2)}{F_0^{B_s\to K}(m_{D_s^-}^2)}\right|^2.$$

This ratio is sensitive to the value of the decay constant $f_{D_s^-}$. Evaluating the form factors in the CLFA model [5], we obtain $\mathcal{R} = 9.82 \pm 1.27$ (11.34 ± 1.43) with $f_{D_s^-} = 259 \pm 7$ [26] $(241 \pm 3$ [27]) MeV. The sources of the uncertainty come from the CKM factors, the decay constants and the form factors. The dominant error comes from the value of V_{ub}. From the experimental value $\mathcal{B}(\overline{B_s^0} \to D_s^{\pm} K^{\mp})$ $= (3.0 \pm 0.7) \times 10^{-4}$ [20] it is obtained $\mathcal{R} = 1$ while we compute $\mathcal{R} \approx 10$. Thus, with improved measurements, this ratio is a good test to the numerical inputs for V_{ub} and $f_{D_s^-}$.

5.4 *$H \to P'P$ and $P \to l\nu_l$ decays:* Let us compare the two-body nonleptonic $H(\overline{q}q_1) \to P'(\overline{q}q_2), P(\overline{q_3}q_4)$ and the leptonic $P(\overline{q_3}q_4) \to l\nu_l$ decays. It is well known that the decay rate of $P(\overline{q_3}q_4) \to l\nu_l$ is

$$\Gamma(P \to l\nu_l) = \frac{G_F^2 |V_{q_3q_4}|^2 f_P^2 m_P m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_P^2}\right)^2.$$

The ratio between $\Gamma(H \to P', P)$ and the last expression is given by

$$\frac{\Gamma(H \to P', P)}{\Gamma(P \to l\nu_l)} = \frac{|V_{q_1q_2}|^2 a_1^2}{4} \times \frac{(m_H^2 - m_{P'}^2)^2 \lambda^{\frac{1}{2}}(m_H^2, m_{P'}^2, m_P^2)}{m_H^3 m_l^2 m_P (1 - \frac{m_l^2}{m_P^2})^2} \times \left| F_0^{H \to P'}(m_P^2) \right|^2. \quad (21)$$

This quotient is independent of the decay constant f_P, and could be used as a test for the form factor $F_0^{H \to P'}(m_P^2)$. For some exclusive channels, we obtain

$$\left| F_0^{B^- \to D^0}(m_{D_s^-}^2) \right|^2 = 0.301 \pm 0.037 \; (0.293 \pm 0.053),$$

$$\left| F_0^{B_s^0 \to K^+}(m_{D_s^-}^2) \right|^2 = 0.765 \pm 0.216 \; (0.681 \pm 0.197),$$

with $l = \tau^-$ (μ^-). The error comes basically from the experimental value of the branching ratios. We can see that the value of $|F_0^{B \to D}(m_{D_s}^2)|^2$ is approximately equal when the lepton l is τ or μ. The situation for $|F_0^{B_s \to K}(m_{D_s}^2)|^2$ is different because the value of V_{ub} also is a source of uncertainty. On the other hand, the value of $F_0^{B_s \to K}$ in $q^2 = 0$ depends strongly on phenomenological models, ranges from 0.23 to 0.31 [28]. Thus, the improvement of these experimental ratios in future experiments, as LHCb, will be a test of the respective form factors.

5.5 *$H \to P_1, P_2(V')$ decays:* Another important ratio is given by the decay widths of $H \to P_1, P_2$ and $H \to P_1, V'$, where P_2 and V' have the same quark content with $P_1 = P$, S, $P(2S)$, $P_2 = P$, $P(2S)$ and $V' = V$, $A(^3P_1)$, $V(2S)$. Using the expressions given in Table III and monopolar form factors with the fact that $F_0^{H \to P_1}(0) = F_1^{H \to P_1}(0)$ [3], we obtain:

$$\frac{\Gamma(H \to P_1, P_2)}{\Gamma(H \to P_1, V')} = \left(\frac{f_{P_2}}{f_{V'}}\right)^2 \left[\frac{1 - m_{V'}^2/m_{1^-}^2}{1 - m_{P_2}^2/m_{0^+}^2}\right]^2 \times \frac{\left[\lambda(m_H^2, m_{P_1}^2, m_{P_2}^2)\right]^{1/2}}{\left[\lambda(m_H^2, m_{P_1}^2, m_{V'}^2)\right]^{3/2}} (m_H^2 - m_{P_1}^2)^2. \quad (22)$$

This ratio provides information on the quotient $f_{P_2}/f_{V'}$. As an example, we obtain $(f_{\pi^+}/f_{\rho^+}) = 0.631 \pm 0.045$ using the $B^0 \to D^-, \pi^+$ and $B^0 \to D^-, \rho^+$ decays which branching ratios are $(2.68 \pm 0.13) \times 10^{-3}$ and $(7.6 \pm 1.3) \times 10^{-3}$, respectively [20]. The main uncertainty arises from these experimental values. On the other hand, taking $f_{\pi^+} = (130.7 \pm 0.4)$ MeV and $f_{\rho^+} = (216 \pm 2)$ MeV [5] it is obtained $(f_{\pi^+}/f_{\rho^+}) = 0.605 \pm 0.006$. So, in this case factorization assumption gives a good approximation to the value of this quotient.

5.6 *$H \to P', V_{1(2)}$ decays:* In order to obtain f_{V_1}/f_{V_2}, we can consider the ratio between the decay rates of $H \to P'$, $V_1(q_i\overline{q_j})$ and $H \to P'$, $V_2(q_i\overline{q_j})$, where $P' = P$, S, $P(2S)$ and $V_{1,2} = V$, $A(^3P_1)$, $V(2S)$:

$$\frac{\Gamma(H \to P', V_1)}{\Gamma(H \to P', V_2)} = \left(\frac{f_{V_1}}{f_{V_2}}\right)^2 \times \left|\frac{F_1^{H \to P'}(m_{V_1}^2)}{F_1^{H \to P'}(m_{V_2}^2)}\right|^2 \left[\frac{\lambda(m_H^2, m_{P'}^2, m_{V_1}^2)}{\lambda(m_H^2, m_{P'}^2, m_{V_2}^2)}\right]^{3/2}. \quad (23)$$

Let us choose, as an application, the $B \to P, V$ and $B \to P, A$ processes. From the expressions in Table III and using monopolar form factors [3] we obtain:

$$\frac{\Gamma(B \to P, V)}{\Gamma(B \to P, A)} = \left(\frac{f_V}{f_A}\right)^2 \times \left[\frac{1 - m_A^2/m_{1^-}^2}{1 - m_V^2/m_{1^-}^2}\right]^2 \left[\frac{\lambda(m_B^2, m_P^2, m_V^2)}{\lambda(m_B^2, m_P^2, m_A^2)}\right]^{3/2}. \quad (24)$$

Taking the $B^0 \to D^-, \rho^+$ and $B^0 \to D^-, a_1^+$ decays we get $(f_\rho/f_{a_1}) = 1.06 \pm 0.31$. The dominant error comes from the experimental value $\mathcal{B}(B^0 \to D^- a_1^+)$ $= (6.0 \pm 3.3) \times 10^{-3}$. With $f_\rho = (216 \pm 2)$ MeV [5] it is obtained $f_{a_1} = (0.203 \pm 0.059)$ GeV. This value is smaller than the one reported in the literature. For example, in the Ref. 29, $f_{a_1} = 0.238 \pm 0.010$ GeV while the Ref. 8 gives $f_{a_1} = 0.229$ GeV (extracted from the $\tau^- \to M^-\nu_\tau$ decay) and $f_{a_1} = 0.256$ GeV (from the $\overline{B^0} \to D^{*+}, a_1^-$ and $\overline{B^0} \to D^{*+}, \rho^-$ decays). On the other hand, in Ref. 30 obtained $f_{a_1} = 0.215$ (0.223) GeV for $\theta = 32^\circ$ (58°), where θ is the mixing angle between the K_{1A} and K_{1B} mesons. As the error in $\mathcal{B}(B^0 \to D^- a_1^+)$ is too big, it is important to get a more precise estimation of this branching in future experiments in order to test hypothesis factorization with these exclusive decays.

It is also possible to obtain the quotient (f_ρ/f_{a_1}) from $\mathcal{B}(\overline{B_s^0} \to D_s^+, \rho^-)/\mathcal{B}(\overline{B_s^0} \to D_s^+, a_1^-)$ and

$\mathcal{B}(B_c^- \to \eta_c, \rho^-)/\mathcal{B}(B_c^- \to \eta_c, a_1^-)$. At present, there are not experimental values of these branchings. So, in the future these decays will be a test of naive factorization by means of the ratio (f_ρ/f_{a_1}).

5.7 $H \to V_1, V_{2(3)}$ *decays:* Another important ratio in order to compute the quotient f_{V_1}/f_{V_2} comes from the $H \to V_1, V_2(q_i\overline{q_j})$ and $H \to V_1, V_3(q_i\overline{q_j})$ processes, where $V_1 = V,\ A(^1P_1),\ A(^3P_1),\ V(2S)$ and $V_{2,3} = V,\ A(^3P_1),\ V(2S)$. As an example, we consider the $B \to V, V'$ and $B \to V, A$ decays. From expressions displayed in Table III we obtain:

$$\frac{\Gamma(B \to V, V')}{\Gamma(B \to V, A)} = \left(\frac{f_{V'}}{f_A}\right)^2 \frac{\mathcal{G}(m^2_{V'})}{\mathcal{G}(m^2_A)}. \qquad (25)$$

Taking the $B^0 \to D^{*-}, \rho^+$ and $B^0 \to D^{*-}, a_1^+$ decays and evaluating $\mathcal{G}$ with appropriate monopolar form factors [3], we get $(f_\rho/f_{a_1}) = 0.81 \pm 0.07$, where the source of uncertainty are the form factors. This value agrees with the one reported in Ref. 29, although is smaller than the value obtained in previous subsection.

f_ρ/f_{a_1} can also be obtained from

$$\Gamma(\overline{B^0_s} \to D_s^{*+}, \rho^-)/\Gamma(\overline{B^0_s} \to D_s^{*+}, a_1^-)$$

and $\Gamma(B_c^- \to J/\psi, \rho^-)/\Gamma(B_c^- \to J/\psi, a_1^-)$. At present, there is not experimental information of these decays in order to test the factorization hypothesis.

5.8 $B \to M_1, M_2$ *and* $B \to M_1 l\nu_l$ *decays:* It is well known that the ratio

$$R = \Gamma(B \to M_1, M_2)/[d\Gamma(B \to M_1 l\nu_l)/dt|_{t=m^2_{M_2}}]$$

provides a method to test factorization hypothesis and may be used to determine some unknown decay constants [29, 31]. Also, it is possible combining exclusive semileptonic and hadronic B decays to measure CKM matrix elements (see for example Ref. 32). If M_1 is any of the eight mesons showed in Table I, $M_2(q_i\overline{q_j})$ is a $J = 1$ meson and $m_l \approx 0$, we obtain

$$R_{V'} = \frac{\Gamma(H \to M, V')}{d\Gamma(H \to M l\nu_l)/dt|_{t=m^2_{V'}}} = \frac{\xi^{(V')}}{\zeta} = 6\pi^2|V_{ij}|^2(a_1^H)^2 f^2_{V'}, \qquad (26)$$

where $V' = V,\ A(^3P_1),\ V(2S)$. Thus, $R_{V'}$, which is model-independent, is a clean and direct test of factorization hypothesis. On the other hand, assuming the validity of the factorization with a fixed value for a_1, it provides an alternative use: it may be used for determination of unknown decay constants. For example, f_ρ can be obtained from

$$R_{\rho^-} \equiv \frac{\Gamma(B^- \to D^0, \rho^-)}{d\Gamma(B^- \to D^0 l\nu_l)/dt|_{t=m^2_\rho}} = \frac{\Gamma(B^0_s \to D^+_s, \rho^-)}{d\Gamma(B^0_s \to D^+_s l\nu_l)/dt|_{t=m^2_\rho}} = \frac{\Gamma(B_c^- \to \eta_c, \rho^-)}{d\Gamma(B_c^- \to \eta_c l\nu_l)/dt|_{t=m^2_\rho}}, \qquad (27)$$

where $R_{\rho^-} = 6\pi^2|V_{ud}|^2(a_1^H)^2 f^2_{\rho^-}$.

We also can use the equation (26) in order to obtain ratios between decay constants of $J = 1$ mesons:

$$\frac{R_{V_1'}}{R_{V_2'}} = \left(\frac{f_{V_1'}}{f_{V_2'}}\right)^2, \quad V'_{1,2} = V,\ A(^3P_1),\ V(2S). \qquad (28)$$

5.9 $\overline{B^0_{(s)}} \to D^+_{(s)}, \pi^-(K^-)$ *decays:* Taking pairs of decays that are U-spin[iii] partners, we get

$$\mathcal{R}_{\pi/K} = \frac{\mathcal{B}(\overline{B^0_s} \to D^+_s, \pi^-)}{\mathcal{B}(\overline{B^0} \to D^+, K^-)} = (12.45)\left|\frac{F_0^{B_s \to D_s}(m^2_\pi)}{F_0^{B \to D}(m^2_K)}\right|^2 = 13.07 \pm 0.32$$

and $\mathcal{R}_{K/\pi} = 0.082 \pm 0.002$, evaluating the form factors in the CLFA model [5]. The dominant source of error comes from these form factors. In the first (second) case, the ratio between the experimental values of the branching ratios [20] is 16.0 ± 5.4 (0.112 ± 0.027). In both cases, the experimental ratio is bigger than the theoretical one. Therefore, with improved measurements at future experiments as LHCb, these ratios will be a good test of the breaking of U-spin symmetry through the ratio of the form factors. On the other hand, they provide an alternative strategy in order to determine $f_K/f\pi$ and compare with other methods (see for example in Ref. 34).

6. Summary

We computed several useful ratios between decay widths of two-body nonleptonic and semileptonic B and B_s decays, which with improved measurements in forthcoming experiments as LHCb, could be test of factorization approach by means of quotients between form factors or decay constants. The ratios with B decays considering charmonium states and light mesons in final state (see subsection 5.1) could be the more likely scenario to test the factorization scheme. It is important to mention that divergences from the results obtained assuming the current approximations do not imply a failure of the QCD itself or the factorization approach alone. It would be required a more exhaustive and comprehensive analysis for getting more conclusions on these and possible new physics effects in these decays. We also presented a summary of the expressions for $\Gamma(H \to M_1, M_2)$ and $d\Gamma(H \to M_1 l\nu)/dt$, at tree level, including eight types of mesons in final state: $M_{1,2}$ can be a ground state meson ($l = 0$), or an orbitally excited meson ($l = 1$) or a radially excited meson ($n = 2$), assuming factorization hypothesis and using the parametrizations of $\langle M|J_\mu|B\rangle$ given in the WSB and the ISGW quark models. The form factors were evaluated in the WSB and CLFA quark models. We classified in three groups the $H \to M_{1,2}$ transitions and explained some aspects related with the dynamics of these processes.

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Appendix

In this appendix, we briefly mention the quark models and their form factors that are used in this work.

1. The ISGW model [4]: it is a hybrid model that combines a nonrelativistic quark potential model with a phenomenological ansatz. It is consistent with heavy quark symmetry at maximum recoil t_m. Their form factors are modeled by a gaussian and normalized at q^2_{max}. All the form factors in this model are in function of

$$F_n^{H\to M}(q^2) = \left(\frac{\tilde{m}_M}{\tilde{m}_H}\right)^{\frac{1}{2}} \left(\frac{\beta_H\beta_M}{\beta^2_{HM}}\right)^{\frac{n}{2}} e^{-\Lambda(t_m - q^2)}, \quad (29)$$

where $\Lambda = m_d^2/(4\kappa^2\tilde{m}_H\tilde{m}_M\beta^2_{HM})$. $\tilde{m}_{H(M)}$ is the mock mass of the $H(M)$ meson, β is a variational parameter and $\kappa = 0.7$ is a relativistic compensation factor of the model. The appendix B of the Ref. [4] has all the required inputs for evaluating the form factors for the $H \to M(J = 0, 1, 2)$ transition.

2. *The WSB model [3]:* It gives the form factors in terms of relativistic bound state wave functions taking the solutions from a relativistic harmonic oscillator potential. The form factors are calculated as wave function overlaps in the infinite momentum frame at $q^2 = 0$. The mononopolar form factors in this model present a vector meson dominance form of the q^2 dependence and are given by

$$F^{H\to M}(q^2) = \frac{F^{H\to M}(0)}{1 - q^2/m^2_{J^P}}, \quad (30)$$

where m_{J^P} is the mass of the pole. The Ref. 3 provides the values of $F_n^{H\to M}(0)$ and m_{J^P} for the $H \to M$ transition. We use these form factors in order to compute the numerical values showed in subsections 5.5, 5.6 and 5.7.

We can obtain the form factors of the WSB model [3] in function of the form factors of the ISGW model [4] comparing the parametrizations given in both models for the $H\to P(V)$ transition. Making $\langle P|J_\mu|H\rangle_{WSB}=\langle P|J_\mu|H\rangle_{ISGW}$ we obtain:

$$F_0(t) = \frac{t}{(m_H^2 - m_P^2)}\, f_-(t) \; + f_+(t), \quad (31)$$

$$F_1(t) = f_+(t), \quad (32)$$

and from $\langle V|J_\mu|H\rangle_{WSB}$ = $\langle V|J_\mu|H\rangle_{ISGW}$ it is obtained:

$$A_0(t) = \frac{i}{2m_V} \times \left[f(t) + ta_-(t) + (m_H^2 - m_V^2)a_+(t)\right], \quad (33)$$

$$A_1(t) = \frac{if(t)}{(m_H + m_V)}, \quad (34)$$

$$A_2(t) = -i(m_H + m_V)\; a_+(t), \quad (35)$$

$$V(t) = -i(m_H + m_V)\; g(t). \quad (36)$$

Using these relations it is straightforward to get $d\Gamma(H\to P(V)l\nu)/dt$ or $\Gamma(H \to P(V), M)$ with the parametrization of the WSB model from respective expressions in the ISGW model, and viceversa.

3. *The CLFA model [5]:* The relativistic light-front quark model gives a fully treatment of quark spin and the center-of-mass motion of the hadron. In a covariant approach of this model the decay constants and the form factors are calculated by means of Feynman momentum loop integrals which are manifestly covariant [5]. The form factors in the spacelike region are given by the three-parameter form

$$F^{H\to M}(q^2) = \frac{F^{H\to M}(0)}{1 - a(q^2/m_H^2) + b(q^2/m_H^2)^2}. \quad (37)$$

We have taken from the Ref. [5] the values of $F^{H\to M}(0)$, a and b for obtaining the numerical values presented in Subsecs. 5.1, 5.2, 5.3 and 5.9.

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i. We use the ISGW model [4] because it provides all the parametrizations considered in this work.

ii. For the $H \to A, A'$ transitions it is required to interchange the role of vector and axial currents in order to obtain the specific expressions displayed in Tables II and III.

iii. The U-spin symmetry is a SU(2) subgroup of the SU(3) flavor symmetry group, in which quarks d and s form a doublet [24,33].

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