Validating quantum storage and state transference based on spin systems through elimination of exchange degeneracy

M. Ávila
Centro Universitario UAEM Valle de Chalco, Universidad Autónoma del Estado de México,
Hermengildo Galeana 3, María Isabel, CP 56615, Valle de Chalco, Estado de México, México.

Received 28 November 2014; accepted 26 January 2015

A quantum storage and state transference machine based on spin systems is considered. The process described cannot be regarded a a quantum teleportation because it does not involve any measurement. In previous work on quantum storage and state transference based on spin systems, *exchange degeneracy* was not taken into account and this is important because the initial and final states can become indistinguishable from each other and so the state transference may loose its meaning. It is shown that such a failure can be corrected by symmetrization. We conclude that in a consistent state transference and storage process, the parity of the initial state is not necessarily conserved.

Keywords: Exchange degeneracy; symmetrization; quantum information storage; quantum information transference.

PACS: 05.30.-d; 03.67.-a

1. Introduction

If Alice and Bob each posses an identical particle and they are distant enough for the wave function of the particles do not overlap during the time of observation of the system, then we can keep track of the particles when they are distinguishables. However, if the wave functions overlap it is not possible to know which particle is initial and which the final one. In such a case it is impossible to track each particle and the particles become indistinguishable having arbitrary labels (not just Alice and Bob) each one. Then we conclude that the description of the system becomes ambiguous because of the *exchange degeneracy* phenomenon [1]. Such a situation may happen in protocols implementing the physical process of quantum information storage and state transfer. For instance, in Ref. 2 a quantum spin system in the form of a ferromagnetic Heisenberg spin chain or an isotropic antiferromagnetic spin ladder system was employed for transferring coherently a quantum state. However, in Ref. 2 it was not taking into account the fact that if the wave function of the input state and the output state overlap, there is a confusion between the state initially to be transferred and the final state transferred (storage system). In Fig. 1 it is depicted such a situation. In order that the process be successful it is necessary that the initial and final state are distant enough for their respective wave functions do not overlap. But if the quantum bus for transferring the information is a long spin chain there appears the unwelcome decoherence effects. This effect was not considered in Ref. 2. To avoid decoherence effects, one is forced to employ a short distance quantum bus for transferring and storage the information with the risk that the wave functions overlap, appearing with this the *exchange degeneracy* problem. According to the principles of Quantum Mechanics the solution to the problem of *exchange degeneracy* is to symmetrize the states either boson (symmetric wave function) or fermion (antisymmetric wave function) to distinguish the states. In the present work, we pinpoint the way in which form the wave function of the system S can be constructed for preventing such a problem.

2. Transference of a state: Data bus model

In the past there have been attempts for transferring coherently and store quantum information [3–6]. An interesting approach was introduced in Ref. 2 where it is explored the possibility of implementing quantum information storage and state transfer by using quantum spin systems. In such a work, it is shown that quantum state transfer can be seen as a generalized quantum storage with three subsystems, the input with a Hilbert space S^A, the data bus with D and output with S^B. For this, we assume that the subsystems A and B are very distant one of each other in such a way that they are not coupled and the wave function of the total system can be separated. It is worth noting that in such a situation the data bus is a very long spin chain with which the decoherence effects increase deteriorating with this the perfect state transfer. The Hilbert space of the total system should be

$$S_T = S^A \otimes D \otimes S^B \equiv S^A \otimes M, \tag{1}$$

where $M = D \otimes S^B$ is the generalized quantum memory with the memory space spanned by the orthonormal vectors $|M_n\rangle = |D\rangle \otimes U_B|S_n^B\rangle$. In the above $|D\rangle$ is the data bus state and U_B is a local unitary transformation with respect to B. The data bus state $|D\rangle$ and the transformation U_B are independent of the initial state for a legitimate quantum bus. The independence of U_B with respect to the state to be transferred A will be important for the present approach and such a property will be employed lines below. At $t = 0$ the total state is

$$|\psi(0)\rangle = \sum_n c_n |S_n^A\rangle \otimes |M\rangle, \tag{2}$$

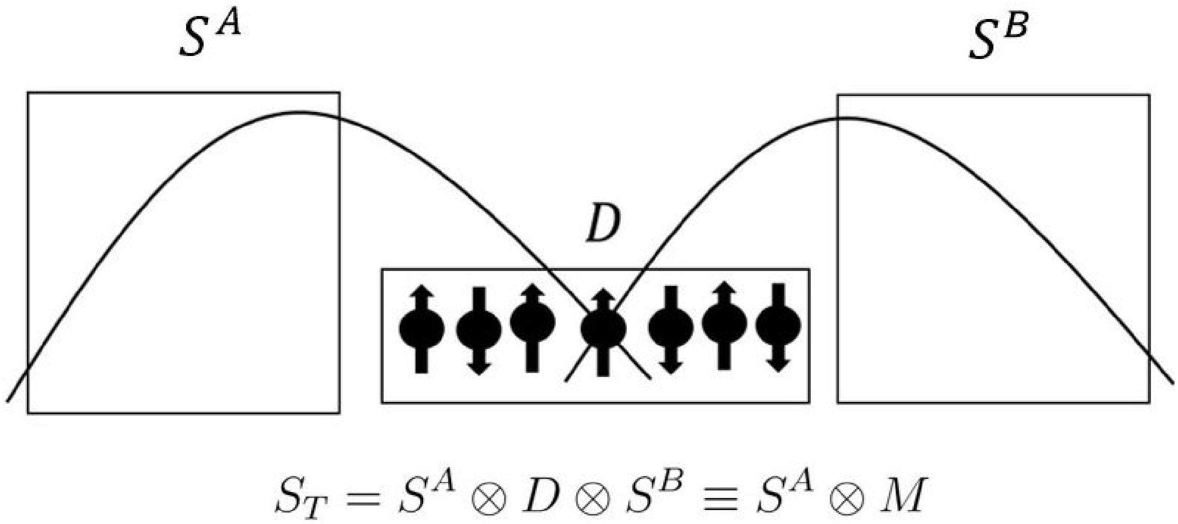

FIGURE 1. A quantum spin system in the form of a ferromagnetic Heisenberg spin chain or an isotropic antiferromagnetic spin ladder system is employed for transferring coherently a quantum state. There appears the *exchange degeneracy* if the input state S^A and its associated transferred state S^B overlap.

where $|M\rangle = |D\rangle \otimes |S^B\rangle$. Thus, the quantum state transfer can be described after a period of time $t = T_f$ as

$$|\psi(T_f)\rangle = |S\rangle \otimes |D\rangle \otimes \sum_n c_n U_B \left|S_n^B\right\rangle$$
$$= |S\rangle \otimes \sum_n c_n |M_n\rangle, \quad (3)$$

where $|S\rangle = \sum_n c_n' |S_n^A\rangle$. The separation of the above wave function has been already explained lines above. Equation (3) indicates that quantum state transfer can be thought of as a generalized quantum memory. In order that the effects of decoherence do not intervene in the transferring process, the extension of the data bus spin chain through which the transferred state transits, must be short enough. However, in this case *exchange degeneracy* appears making indistinguishable the systems A and B and creating confusion between the original state $|\psi(0)\rangle$ and the transferred state $|\psi(T_f)\rangle$. We should realize that the storage process is not teleportation because the later leads to an output state that corresponds to the input with a fidelity $F = 1$. In quantum teleportation there is a measure (collapse) of the Bell state while in quantum storage the system is permanently in evolution without no measurement never requiring of any measurement whatsoever.

If we assume a spatial reflection symmetry under the parity operator P then $[H, P] = 0$ where H is the Hamiltonian of the system. It is straightforward to prove according to the approach of Ref. 2 that the transferred wave function is

$$|\psi\Big(\mathbf{r}, t = \frac{\pi}{E_0}\Big)\rangle = \sum_n C_n (-1)^{N_n} |\phi_n(\mathbf{r})\rangle$$
$$= \pm P|\psi(\mathbf{r})\rangle = \pm|\psi(-\mathbf{r})\rangle, \quad (4)$$

where the state to be transferred is $|\psi(\mathbf{r}, t = 0)\rangle = |\psi(\mathbf{r})\rangle$. The interpretation of Eq. (4) is that the transference process through a quantum bus changes up to a phase the probability amplitude of the wave function. The later means that a probability (*e.g.* measurement) of occurrence of the transferred state does not change. A justification for that is that the quantum bus along the spin chain is prepared in such a way that in any time the initial and final state are not interacting (*i.e.* absence of dechoherence). Equation (4) follows immediately if one notes that the wave function evolves in time according to

$$|\psi(\mathbf{r}, t)\rangle = e^{-iHt}|\psi(\mathbf{r})\rangle = \sum_n C_n e^{-iN_n E_0 t}|\phi_n(\mathbf{r})\rangle, \quad (5)$$

where $C_n = \langle\phi_n|\psi\rangle$. With the above two equations one can conclude that if the eigenvalues $\epsilon_n = N_n E_0$ of a $1 - D$ Hamiltonian H with spatial reflection symmetry are odd-number spaced ($N - n - N_{n-1}$ always odd), any initial state $|\psi(x)\rangle$ evolves into $\pm|\psi(-x)\rangle$ at time $t = \pi/E_0$. However, when the quantum bus composed by a spin chain is short enough then there appears *exchange degeneracy* between the initial state to be transferred $|\psi(x)\rangle$ and the transferred state $\pm|\psi(-x)\rangle$. By the above reason, the initial and final states become indistinguishable. According with the principles of Quantum Mechanics the solution to the indistinguishability of the states is the symmetrization postulate of the wave function that describe them [1, 7].

Symmetrization Postulate - *In a system containing indistinguishable particles, the only possible states of the system are: (i) either completely symmetrical with respect to permutation (bosons); (ii) either completely antisymmetrical with respect to permutation (fermions).*

By assuming that the quantum information system to storage is a fermion then the principles of Quantum Mechanics demand that the wave function of the initial state to be transferred must be[i]

$$|\Psi(\mathbf{r})\rangle_A = \frac{1}{\sqrt{2}}\Big(|\psi(\mathbf{r})\rangle - |\psi(-\mathbf{r})\rangle\Big), \qquad (6)$$

which is antisymmetrical with respect to exchange of the label $\mathbf{r} \to -\mathbf{r}$. In Eq. (6), the state $|\psi(-\mathbf{r})\rangle$ is defined through Eqs. (4) and (5). It is crucial to observe that according to the above, the final transferred wave function that does not suffer overlap with the state to be transferred is

$$|\Psi(\mathbf{r})\rangle_B = e^{-iH_{QB}T_f}|\Psi(\mathbf{r})\rangle_A, \qquad (7)$$

where the state to be transferred $|\Psi(\mathbf{r})\rangle_A$ is given by Eq. (6) and H_{QB} the Hamiltonian of the quantum bus is such that the overlap between $|\Psi(\mathbf{r})\rangle_A$ and $|\Psi(\mathbf{r})\rangle_B$ is null. As it was mentioned lines above, by construction of a legitimate quantum bus, the unitary operation $U_B = e^{-iH_{QB}T_f}$ must be independent of the initial state $|\Psi(\mathbf{r})\rangle_A$ which is given by Eq. (6). Such a condition demands that the Hamiltonian H of Eq. (5) and H_{QB} are independent of each other. One particular striking situation where the overlapping between $|\Psi(\mathbf{r})\rangle_A$ and $|\Psi(\mathbf{r})\rangle_B$ vanishes is when the quantum gate $U_B = e^{-iH_{QB}T_f}$ changes the parity of the initial antisymmetrical state $|\Psi(\mathbf{r})\rangle_A$ of Eq. (6) into a symmetrical final state $|\Psi(\mathbf{r})\rangle_B$. The later can be though of as a sort of quantum NOT gate which converts the state $|0\rangle$ into the state $|1\rangle$ and viceversa remembering that $\langle 0|1\rangle = \langle 1|0\rangle = 0$. In the context of the present work one would require that ${}_B\langle\Psi(\mathbf{r})|\Psi(\mathbf{r})\rangle_A = 0$. It is worth emphasizing that according to the principles of Quantum Mechanics, it can be concluded that the transference of a state through a spin chain (quantum bus) does not necessarily preserve the parity of a state.

3. Conclusions

We have considered the storage and transference of a quantum state through a quantum bus composed by a spin chain. The transference of the quantum state is carried through the spin chain. It has been pointed out that if the spin chain is short enough there is the risk of an overlap between the initial state and the storage final state. On the other hand, an overlap is avoided if the spin chain is long enough, however in such a situation appears an unwelcome decoherence deteriorating the storage and transference of the quantum state. In presence of decoherence the spatial reflection symmetry as given by Eqs. (4) and (6) would not be possible. Thus, it is necessary a decoherence free short spin chain as a quantum bus for transferring the initial state appearing with this an exchange degeneracy due to the overlap between the initial and final (transferred) state. We point out that such a loss of identity of the states S_A and S_B is circumvented through the Symmetrization Postulate of Quantum Mechanics for making distinguishable the states. Notice that the quantum transference and storage does not require a measuring process consequently cannot correspond to nothing similar to a quantum teleportation. As a main result we have found that a solution imposed by the principles of Quantum Mechanics is the symmetrization of the state to be transferred. With the above it is possible to nullify the ambiguities between the state to be transferred and the transferred state. The price that one must pay is that the parity of the initial state is not necessarily conserved.

Acknowledgments

We thank A. Salas and Y. García for helping in the preparation of the manuscript.

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i. Let us observe that according to the approach of [2], the transferred state is $|\psi(-\mathbf{r})\rangle$ while the state to be transferred is $|\psi(\mathbf{r})\rangle$. Such an approach is feasible only if the overlap between the states vanishes *i.e.* $\langle\psi(\mathbf{r})|\psi(-\mathbf{r})\rangle = 0$ which in general does not happen as Eq. (4) shows it.

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