# Understanding the arithmetic of color

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In this work we propose to introduce Arithmetic for the Mixture of Colors, which can be defined from dividing the electromagnetic spectrum of white light in three thirds, and by associating each of them to an algebraic expression of the three primary additive and subtractive colors. Then, we will give the rules for adding or subtracting to predict the new mixture of resultant colors.

*Keywords:* Mixture of colors; chromatic arithmetic.

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## 1. Introduction

If we ask randomly to any person on the street how much is three plus two, probably, we will get the answer without any problem. If, instead of asking them to sum these two numbers, we ask them to subtract them, practically any person would answer that the result is one. But, if now the questions are: which color do we obtain when we superpose a red light with a blue one? Or, which color results by mixing cyan paint with yellow one? Then the answer is no longer easy, and it will be hard to find someone to give a proper answer. Maybe, the first obstacle in giving an answer to these questions is that just a few people can identify a cyan pigment, or to name a color magenta (it is most common to call it Mexican pink).

In order to explain how light and pigments can be combined to obtain different colors, the summation of color as light (additive mixture) and the superposition of coloring materials (subtractive mixture) are used [1,2].

Therefore, in this work the following considerations will be assumed:

- White light is formed by the whole visible spectrum, *i.e.*, the one that contains all the rainbow bands.
- This white light or visible spectrum will be divided in three thirds, see Fig. 1. The first one corresponds to the short wavelengths (blue), the second one to the medium wavelengths (green), and the third one to the long wavelengths (red), in the same way as in a computer projector or a TV.
- Each of these bands is defined as an Additive Primary. In our proposal, we will represent it as a light source that emits in blue (B), another one that emits in green (G), and another one that emits in red (R).
- An additive mixture means to project at the same time and on the same place two or more light sources, so there is an overlap of these emitted lights. The representation of this mixture will be represented with the plus sign (+).
- In this sense, the additive mixture of the three thirds of the rainbow forms again the original white light, which we will represent algebraically as:

$$\text{B+G+R} = \text{white light} \tag{1}$$

- When, instead of using a light source as a primary color, we use a material to obtain pigmentation, we referred to this material as subtractive primary.
- A subtractive primary is a material designed to absorb one third of the white light spectrum. For this reason there are three subtractive primaries: one which absorbs the red band, one which absorbs the green band, and one which absorbs the blue band of the spectrum. The minus sign (-) indicates the absorption characteristic of each to these primary colors.
- To make a subtractive mixture, we need to mix two or more pigments. The representation of this mixture is through a sum, in which each term of the sum is a subtractive primary color.
- When in a subtractive mixture we obtain a result of 2B, it will be substituted by B (which means $2\text{B}\approx\text{B}$), and the same will happen with 2R by R, and 2G by G; because the number 2 only indicates that we obtained two times blue (or red or green), and it is perceived as the same color.

## 2. Arithmetic of the additive mixture

Now we will use the thirds of Fig. 1 as blue light, green light, and red light in order to obtain new colors [3].

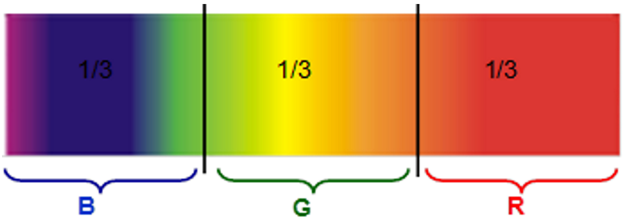

FIGURE 1. The three thirds, R, G, B of the white light spectrum.

If we superpose blue with green light, the obtained mixture will be defined as cyan (C) light. Then, the cyan light will be algebraically represented as follows:

$$C= B+G. \tag{2}$$

Notice that the red band of the visible spectrum is missing from the cyan light for it to be white light.

Now, if the superposed lights are blue and red, the new obtained color is called magenta (M), and it is algebraically represented as:

$$M = B + R\,. \tag{3}$$

Now the missing band of the visible spectrum for the magenta light to obtain white light is the green one.

By mixing green and red light we obtain yellow (Y) light, which can be represented by:

$$Y= G+R\,. \tag{4}$$

Another way to visualize it is that we observe a yellow color due to the fact that we are missing the blue rainbow band.

Figure 2 shows the experimental results of the three binary mixtures with the three primary colors R, G, B, to obtain cyan, magenta, and yellow, as was previously discussed in Eqs. 2-4.

Now, what should we obtain if, instead of using red, green, and blue lights, we use projectors that give us cyan, magenta, and yellow lights? Let's solve these cases:

What should we obtain from the mixture of yellow and magenta lights? Let's write the proposed algebraically expression for the additive mixture:

$$Y + M \;= ¿?$$

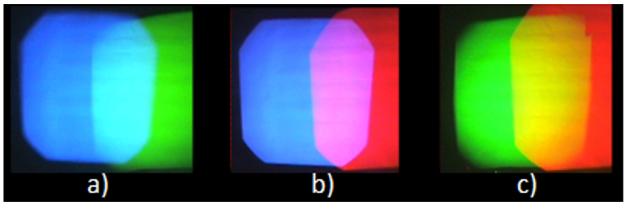

FIGURE 2. Experimental results of the binary mixtures from the Primary Additives: Red, Green, and Blue to obtain a) Cyan, b) Magenta, and c) Yellow.

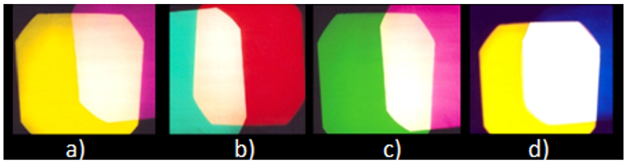

FIGURE 3. a) Yellow and Magenta; b) to d) Experimental results of mixing complementary colors to obtain white light.

Now, by substituting the values of Y and M from Eqs. (2) and (4), the mixture is as:

$$\begin{aligned}(G + R) + (B + R) &= G + R + B + R\\ &= (B + G + R) + R\\ &= \text{white light} + R\\ &= \text{white light with reddish tendency.}\end{aligned}$$

The experimental results are shown in Fig. 3(a), where we have two ansatz: each of the superposed primaries has the same energy, and it is irrelevant in which order we sum them. That is (G + R + B) = (B + R + G).

An important contribution of these kinds of sums is that we can make a simple arithmetic to predict complementary colors, for which the following definition will be used: if additive sum of two sources light produces white light, then these source lights will be called complementary. The complementary source can be found by using the algebraic expressions derived before for the cyan, magenta, and yellow mixtures.

Cyan has been defined as the primary that does not have red in its spectrum, so, by superposing red to cyan, *i.e.*, its complementary, we obtain:

$$C + R = (B + G) + R = \; B + G + R = \text{white light!}\,.$$

Figure 3(b) shows the experimental result of this mixing.

In the same way we can illustrate that green is the complementary color of magenta:

$$\begin{aligned}M + G &= (B + R) + G\\ &= B + R + G\\ &= \; B + G + R\\ &= \text{white light!}\end{aligned}$$

The experimental result is shown in Fig. 3 (c).

And, that the complementary of yellow is blue, obtaining:

$$\begin{aligned}Y + B &= (G + R) + B\\ &= G + R + B\\ &= \; B + G + R\\ &= \text{white light!}\end{aligned}$$

The experimental result is shown in Fig. 3(d).

In the following section, we show the proposed arithmetic for the color pigment mixing known as subtractive mixture.

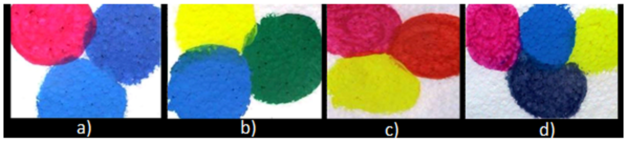

FIGURE 4. Subtractive mixtures: a) Cyan + Magenta = Blue, b) Cyan + Yellow = Green, c) Magenta + Yellow = Red, and d) Magenta + Cyan + Yellow = Black.

## 3. Arithmetic of a subtractive mixture

In this section we will mix pigments that are used to the coloration process. The primary colors that are proposed are called Subtractive Primary: cyan ($C_{\text{pigment}}$), magenta ($M_{\text{pigment}}$), and yellow ($Y_{\text{pigment}}$). Based on our initial considerations we have the following algebraic representations for them.

$$C_{\text{pigment}} = B + G - R \tag{5}$$

$$M_{\text{pigment}} = B + R - G \tag{6}$$

$$Y_{\text{pigment}} = G + R - B\,. \tag{7}$$

Now, we can find the resultant color if we mix a cyan with a magenta pigment (in the same proportion):

$$C + M = (B + G - R) + (B + R - G) = 2B\,. \tag{8}$$

The sum gives us twice blue. This does not mean that the obtained blue will be more saturated, but that we have obtained twice the amount of material, which is observed as just blue. Figure 4 (a) shows the experimental result of this mixing.

When the cyan and the yellow pigments are mixed in the same proportion, the new obtained pigment is known as green:

$$C + Y = (B + G - R) + (G + R - B) = 2G \approx G. \tag{9}$$

Indicating that the green part of the cyan and the yellow pigments has been reflected, and the observed color is green. Figure 4 (b) shows the resultant of the experiment.

By mixing the magenta and the yellow pigments in the same proportion, we obtain the red pigment:

$$M + Y = (B + R - G) + (G + R - B) = 2R\ R. \tag{10}$$

This indicates that only the red part of the magenta and the yellow pigments is reflected. Figure 4 (c) shows this mixture of colors.

And, by mixing cyan, yellow, and magenta pigments in the same proportion, we obtain:

$$\begin{aligned} C + M + Y &= (B + G - R) + (B + R - G) + (G + R - B) \\ &= (2B - B) + (2R - R) + (2G - G) \end{aligned} \tag{11}$$

But, as we mentioned above, 2R simply means that red (R) is observed; so, by subtracting the same color (R - R), we obtain zero. The same thing happens with each of the terms in parenthesis in Eq. (11). That is:

$$C + M + Y = (B - B) + (R - R) + (G - G) = 0 \tag{12}$$

Which means *absence of reflected light, i.e.*, black. This happens due to the fact that when the cyan and the magenta pigments are mixed, the cyan absorbs the red part of the magenta, leaving only the blue part. And, when the latter is mixed with the yellow pigment, the yellow absorbs the blue part of the spectrum, meaning that; each pigment absorbs a third of the visible spectrum of Fig. 1, which results in the absence of reflected light, see Fig. 4d).

## 4. Conclusions

We have shown a very simple and easy way to interact with primary colors; as light sources and as pigments, and how they behave in nature when are mixed. Also, we have shown a simple arithmetic procedure to predict the new color by adding or subtracting light sources or pigments. Finally, chromatically, adding and subtracting is just to play with the three thirds of the visible electromagnetic spectrum.

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1. L.R. Daniel, *El color y su medición*, 1era. Edición (América Lee, Buenos Aires, 1978).
2. Billmeyer and Saltzman's, *Principles of color Technology*, Third Edition (Roy S. Berns, John Wiley and Sons Inc., New York, 2000).
3. H.M. Leo, *Color Vision*, (Sinauer Associates Inc. Publishers Sunderland, Massachusetts, 1981).