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The concept of entropy, from its origins to teachers

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Research on alternative conceptions shows that the entropy concept is among the most difficult for students to understand. Entropy is a concept with a complex history and has been the subject of diverse reconstructions and interpretations. This paper presents a brief review of the origin of the concept (Clausius and Boltzmann), a description of some influential textbooks (Planck, Fermi and Gibbs) and a comparative review on textbooks and how the concept of entropy is conceived and taught by university teachers. The results show that there is a great variety in the descriptions and meanings of entropy provided in textbooks, confusing students and teachers. A brief discussion focuses on new conceptual approaches to entropy. We conclude by suggesting how history can contribute to teachers and students' awareness about changes in the meaning of entropy and their acceptance of a more relevant commitment in their understanding of physical concepts.

Keywords: Entropy in education; science in school.

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1. Introduction

At virtually all education levels, research on thermodynamics concepts indicates that students have difficulties understanding them. Research on previous ideas or alternative conceptions shows that to students, fundamental topics such as the difference between heat and temperature and the second law are among the most complex in structure and representation [1-4]. One of these difficult concepts is undoubtedly entropy, for which problems are found not only among students but also among teachers.

The answers that students provide when asked about entropy in an expanding gas in an adiabatic system make reference to increases in entropy that result from an increase in disorder [5,6]. Also, understanding problems in university students are common when considering the entropy changes in systems and their surroundings [7]. Those cases are typical of the confusion caused by deficiencies in teaching of the concept rather than in students' understanding difficulties or in their existing ideas. Another issue that fosters this problem is the confused treatment of the macroscopic and microscopic conceptions of entropy in textbooks, as well as the idea of "disorder", which appears in textbooks without convincing or sufficient explanation [8] that results in a diversity of meanings [9]. In the case of teachers, there are no precedents regarding how to understand entropy and considerations to make when teaching it; this is the topic of this study.

Entropy itself, as we will observe below, is a concept with a complex history [10,11] and has been the subject of diverse reconstructions and interpretations, as well as more formal or axiomatic formulations [12,13] which have made it more difficult to teach. In addition, entropy has influenced other fields such as information theory, which has produced greater dispersion in its conceptualization. Considering all of these factors, we can formulate some questions: What remains of the original interpretations of entropy and its historical development? How have original ideas of entropy been transformed into those in textbooks that are used by teachers? How do teachers conceive entropy and how they teach it to students?

In the next four sections, we attempt to answer these questions at least partially. In the first section, we provide a brief review of the origin of the concept of entropy and we describe the work of Clausius and Boltzmann. In the second section, we briefly describe the work of those that offer the base for the actual textbooks; we will focus on Planck, Fermi and Gibbs. The third section is on teachers and the textbooks they use; a comparative review is performed on the treatment of the concept of entropy in textbooks and how it is conceived and taught by interviewed university teachers.

2. Origins of entropy

2.1. Entropy as a transformational content

The concept of entropy is part of the reasoning used by Rudolf Clausius to tackle the problem of finding a mathematical expression to describe all transformations of a body through heat exchange between that body and another one or the environment. In 1865 he published an integrated volume with several previous and new works (In this work we use the 1867 English version). The aim of this paper is not a historical analysis, for that purpose we consider the Clausius book is appropriate, in it he describes his ideas about entropy, in particular his Memoirs VI and Memoirs IX [14] Clausius begins with the Carnot studies related to heat engines cycles and attempts to clarify heat-energy relations with regard to what he calls the *Equivalence principle in energy transformations*, to which he assigns at least two meanings: work can be transformed in heat and reciprocally and heat cannot pass for itself from a cold body to a hot one.

In his Memoir VI, Clausius regards as equivalent transformations those that can be mutually replaced and for which the expression Q/T is equivalent to the amount of heat at temperature T produced by (external) work. Then he finds the general expression:

$$\int \frac{dQ}{T} = 0 \tag{1}$$

Clausius thus relates the second principle (second law) to a non-compensated relation (irreversible processes), which is described by Eq. (2) and to which he assigns the meaning of *transformation content or equivalence-value*.

$$\int \frac{dQ}{T} \leq 0 \tag{2}$$

Clausius also attempts to answer the question of how heat H contained [15] in a body is able to produce work, so he introduces a microscopic view and establishes that work is produced on account of a modification of the arrangement of the particles that compose the body, which requires the overpowering of internal and external resistances. In this way, Clausius combines macroscopic and microscopic processes, for which he was criticized by Maxwell and Tait [16,17].

Clausius finds that the heat provided by a body, dQ [18], is equivalent to the variation in heat contained by the body, dH, plus the work produced by the body during the modification of its particle arrangement, $dL = ATdZ$ (A for an equivalence factor, T for absolute temperature and Z for disgregation). Using these factors, he finds the expression for a reversible and irreversible (non-compensated, as he calls them) process (the inequality is for the irreversible process):

$$\int \frac{dQ + dH}{T} + \int dZ \geq 0 \tag{3}$$

"He describes the disgregation of a body as the measure of the degree in which the molecules of the body are dispersed" [19].

In this manner,

$$S - S_0 = \int \frac{dQ}{T} = \int \frac{dH}{T} + \int dZ \tag{4}$$

for an isolated system. Clausius chose for S the name entropy to give significance to a process that involves energy, as he describes:

> I propose to call the magnitude S the entropy of the body, form the Greek word ($\tau\rho\omega\pi\eta$) transformation. I have intentionally formed the word entropy so as to be as similar as possible to the word energy [20].

Regarding the elements he introduces in his formulation, Clausius notes that

> …these quantities, all having in common to be determined by the present state of the body without need to know how it was achieved, are six in number: 1) content of heat, 2) content of work, 3) the sum of these two quantities or content of heat, work or energy, 4) the transformation value of the heat content, 5) the disgregation which can be considered as the transformation value of the present arrangement of particles, 6) the sum of the two precedent quantities or the transformation value or entropy [14].

As observed, for Clausius, entropy has a meaning that is related to energy transformation, and this is placed in internal heat, H, as in the transformation of the arrangement of particles or disgregation. In this way, he attempts to give entropy a macroscopic and microscopic meaning. The relation to heat and temperature is the same as that used today, but the concept of disgregation, or Z, has been lost for several reasons. One of these reasons is doubtless the lack of clarity in the meaning of Z and how to operate using it, as noted by Moulines [21] and Uffink [11]. In the second edition of the Mechanical Theory of Heat in 1876, Clausius abandons the disgregation (Z), that is the microscopic interpretation, and focuses only on the relation for entropy (the equality holds for reversible processes) as known in textbooks.

$$S - S_0 \geq \int \frac{dQ}{T} \tag{5}$$

2.2. Entropy as a probability of particle arrangement

As in Clausius, we describe Boltzmann's ideas through an influential book: Lectures on Gas Theory first published in 1896 (part I) and 1898 (part II) that shows an integration of previous Boltzmann works. We use the Dover edition of 1964. In contrast to Clausius, Boltzmann attempts not to combine macroscopic and microscopic points of view but to determine macroscopic behavior using the internal constitution of the substance, that is, the particles. For instance, one of his objectives was to prove that the basic facts of the second law of thermodynamics and the statistical laws of particle or molecule movement in a gas are more than superficially similar [22]. Boltzmann proposes assumptions where remarks: 1) adopting mechanical analogies and differential equations to describe internal movements and 2) matter is not continuous but consists of discrete molecules that are unobservable because of their size.

Using these and other assumptions, he establishes the order and disorder possibilities of a molecule arrangement:

> …from the point of view of mechanics any arrangement of molecules in a container is possible; in an arrangement, the variables that determine the movement of molecules may have different mean values in a region of space filled with gas than in another region…such distribution is called molar-ordered…if the molecular arrangement does not present regularities that vary from a finite region to another one -if it

> is molar-disordered- then however that…small groups of molecules may show definite regularities. A distribution that shows such regularities will be called molecular-ordered. …When these spatial groupings are not limited to particular places …but are, in average, equally distributed… the distribution may be called molar-disordered [22].

Boltzmann notes that the most probable state of a gas in a container is such that the molecules do not retain any specific configuration, but rather, are totally random. He calls this state "molecular-disorder" and regards it as a necessary property to proof the gas theory theorems or the Maxwell velocity distribution [22].

Using the behaviour of the product of the components of a logarithm as the sum of the logarithms, without regarding a physical meaning as mentions d'Abro [23], he establishes that for a volume, Ω, the logarithm of the probability, $\mathcal{B}$, of the molecular arrangement and state distribution in several gases is as follows:

$$\ln \mathcal{B} = -\sum \Omega H \tag{6}$$

"We saw that -H represents, apart from a constant, the logarithm of the probability of the state of a gas considerer" [22].

When multiplying this value by a factor, RM (M = the mass of the hydrogen molecule), as a gas constant and considering that β = 0 (Boltzmann previously made the consideration that β= 0 when the molecules are spherical), he obtains the following [24]

$$RM \ln \mathcal{B} = R \sum \frac{k}{\mu} \ln \left(\frac{T^{3/2}}{\rho} \right) \tag{7}$$

Boltzmann equates this element to the total entropy of the gases in question, or entropy as the natural logarithm of possible states of a gas. Finally, he notes that:

> The fact that in nature the entropy tends to a maximum shows for all interactions (diffusion, heat conduction, etc.) of actual gases the individual molecules behave according to the laws of probability in their interactions, or at least that the actual gas behaves like molecular-disordered gas which we have in mind [22].

Boltzmann establishes an expression for entropy that, from the microscopic point of view, accounts for the possible states of a thermodynamic system and that entropy tends to a maximum and shows that there is a tendency to the most probable state, that is, the state in which molecules do not present particular spatial configurations or arrangements. We can note the similitude among the expressions $S = RM \ln \mathcal{B}$ and $S = k \ln W$, the usual in textbooks.

It is clear that Clausius and Boltzmann present entropy using two different conceptions. In the case of Clausius, entropy is linked to energy transformations, and although he intends to consider molecular behavior, he does not succeed in establishing the relation between macroscopic and microscopic views. Boltzmann describes entropy in terms of the states of arrangements and particle movement but uses a different meaning of entropy and does not succeed in accounting for a macroscopic behavior in terms of the microscopic view.

3. Influential textbooks: Entropy as a fundamental thermodynamic entity

The formal development of entropy as a systematic theory, was performed by Duhem, Jaunann and Lohr [25]. However, as our interest here is in textbooks and how entropy is taught today, we will follow the path that takes us through the writings in influential textbooks on thermodynamics. Thus, we will use the thermodynamics writings of Planck and Fermi with regard to classical thermodynamics and those of Gibbs with regard to statistical mechanics. The emphasis is on how the entropy concept is developed, but there is no pretention to do a history of thermodynamics; there are many excellent history works [10,16,26,27].

3.1. Entropy: Preference for a final state

Directly related to Clausius is Planck, who was profusely interested in entropy and irreversible processes, or as he called it, "my favorite topic" [28].

Planck considers in his textbook Treatise on Thermodynamics [28] two basic situations. The first uses the first law of thermodynamics and arrives at the expression for entropy Φ:

$$\Phi = M \left(C_v \ln T + \frac{R}{m} \ln v + \text{const.} \right) \tag{8}$$

R is the universal gas constant and v is the specific volume. In the second situation, he considers heat absorption and arrives at this expression for entropy change in a reversible process [29]:

$$d\Phi = \frac{Q}{T} \tag{9}$$

Planck develops the entropy expressions for certain irreversible processes, or natural processes, as he calls them. In addition, he establishes an additive property for entropy for the case of several systems or bodies in a thermodynamic system. Planck also provides a new interpretation of the second law, and with it, the directionality of thermodynamic processes. The directionality is expressed as follows:

> As the problem of a process being reversible or irreversible depends only of the nature of its initial and final states and not in the manner it is developed, then in the case of an irreversible process, the final state is, in a way, more important than the initial state as if nature, we could say, 'prefers' the final state over the initial one. I discovered a measure of this preference in Clausius' entropy and found the meaning of the second law

> of thermodynamics in the principle that in every natural process the sum of all entropies of all involved bodies increases [28].

Planck's developments and contributions were fundamental to further developments and textbooks, for example, Sommerfeld wrote: "From my view, and that of Planck, the essence of the second law consists in the existence of entropy and the impossibility of its decrease under definite conditions" [31] but as noted by Tarsitani and Vicentini, [8] modern texts tend to forget or overlook subtle details of a suitable phenomenological approach and the abstract processes pertaining to phenomenological experience.

3.2. Entropy: A state function

In the same manner as Planck, Fermi introduces the second law, noting the limitations of the possibility of transforming heat into work and stating that if there were no such limitations,

> It would be possible to construct a machine which could, by cooling the surrounding bodies, transform heat, taken from its environment, into work. Since the supply of thermal energy contained in the soil, the water, and the atmosphere is practically unlimited, such a machine would, to all practical purposes, be equivalent to a *perpetuum mobile*, and is therefore called a *perpetuum mobile* of the second kind [32].

Fermi, describes the properties of entropy in a manner that was followed by teachers and textbooks. He develops entropy following the properties of cycles; starts with a system that is subject to a cyclic transformation, during which the system receives several amounts of heat from a set of sources at several temperatures. Using the amount of heat received during a cycle between two states (O, i). Considering these transformations to be cyclic and deduces that for two states A and B, for an irreversible processes in an isolated system,

$$S_A - S_B \geq \int \frac{dQ}{T} \tag{10}$$

For Fermi, entropy is only a property of cycles and a state function. The property expressed in the theorem stating that the value of the integral for a reversible transformation depends only on the extreme states in the transformation and not on the transformation itself allows for the definition of a new state function for a system. Entropy is defined in the following way:

> A certain equilibrium state O of our system is arbitrarily chosen and is called the standard state. Being A some other equilibrium state, and consider the integral $S(A) = \int_0^A dQ/T$ taken along a reversible transformation. Given the preceding theorem, and given that the state O is unchanging, we can say that $S(A)$ is a function of the state A only. We call this function the entropy of state A [32].

As observed, Fermi's aim to explain entropy (not only its mathematical expression) is reduced to the definition of a state function without concern for its phenomenological meaning. Fermi, in the following pages of his text, relates Boltzmann expression with entropy in the sense that the former is an interpretation stating that entropy of an isolated system can never decrease during any transformation and develops a mathematical description in terms of properties of a sum of probabilities.

3.3. Entropy: Essential concept in thermodynamics

In his first works on thermodynamics, published in 1873, Gibbs highlights the fundamental role of entropy, and following Clausius, establishes an equation with only state variables, including entropy:

$$d\epsilon = td\eta = pdv$$
$$(\text{in today terms}: \ dU = Tds - PdV) \tag{11}$$

Using entropy as a state variable, Gibbs [33] also develops geometric processes for representing thermodynamic states and constructs thermodynamic surfaces using the variables entropy, energy and volume, which allow him to analyze phase changes and the coexistence of various phases. This work's geometrical treatment won him, above all, the recognition of Maxwell [17].

Gibbs devotes special attention to the equilibrium states in thermodynamic systems and notes that:

> . . . it is an inference naturally suggested by the general increase of entropy which accompanies the changes occurring in any isolated material system that when the entropy of the system has reached a maximum, the system will be in a state of equilibrium [17].

After this work, he wrote his Elementary Principles in Statistical Mechanics in 1902. Gibbs arrives at a formulation of statistical mechanics with greater mathematical power and using a generalization of Boltzmann's ideas on the probabilistic considerations for entities such as particles or molecules, he extend this to any system with the idea of a system ensemble:

> . . . for some purposes, however, it is desirable to take a broader view . . . We imagine a great number of systems of the same nature, but differing in the configurations and velocities which they have at a given instant, and differing not merely infinitesimally, but it may be so to embrace every conceivable combination of configurations and velocities [10].

Gibbs establishes several types of ensembles; those that satisfy equilibrium conditions are called canonical (systems with the same temperature and volume). Using these canonical systems, he produces a generalization of entropy to diverse and mixed systems. Gibbs wrote: ". . . the η in the statistical equation has been completely defined as the average value in a canonical ensemble of systems of the logarithm of the coefficient of probability of phase" [34] that correspond to an equation as:

$$\eta = \log P \tag{12}$$

P is a coefficient or probability related to energy of an ensemble in a phase space. This generalization is an important one for all thermodynamic systems and their interactions.

Entropy is a concept that as observed in the abovementioned, has become essential in the description of physical systems. From the beginnings of the twentieth century until today, several developments have been made, including those related to quantum mechanics, [13,35] as well as various philosophical studies [11,12,21,36]. However, given the purpose of this work, we will not consider those that occurred at a later time and do not appear in basic university textbooks (an exception is Swendsen, [37] or in corresponding teaching.

4. Teachers and textbooks. There use: Entropy without meaning

In the following we show teachers interpretations and how they introduce entropy to their students and the characteristics of the main textbooks they use.

4.1. Teachers

To know how teachers explain and work with entropy, interviews were conducted with 12 thermodynamics teachers: 7 from a Chemistry Faculty (30% of all teachers that teach thermodynamics in that Faculty) and 5 from a Science Faculty - Physics (70% of all teachers that teach thermodynamics in that Faculty). Table I provides some characteristics of this sample.

In assembling the interview guide, the following aspects were considered: A) difficulties that teachers perceive in students when learning the concept of entropy, B) point of view and conception of entropy, C) the teachers' historical knowledge of the development of entropy and their use in classroom. Aspect A has the purpose of determining whether the teachers in the sample have observed signs of special difficulty among their students in attempting to understand the concept. Aspect B is used to determine which view the teacher's use (macroscopic or microscopic) and their entropy interpretations. Aspect C reveals whether teachers consider or know the origins of the concept and whether they use history in their course. The interview guide is provided in the annex.

Interviews were 30 minutes long on average. They were audio recorded and transcribed in full.

4.2. Results

Below are the results, ordered in accordance with the three aspects described and classified as being provided by the chemistry teachers or the physics teachers. The collected data are provided in Tables II and III.

Aspect A. Students difficulties in understanding entropy. Regarding the difficulties observed by the interviewed teachers among their students, it is noteworthy that 41% (5/12)

relate the problem to the students' lack of mathematical background for understanding entropy, specially their problems with calculus, 16% to student lack of interest, another 16% to students' existing ideas, as entropy as heat or entropy as a substance, and one teacher (8%) to word confusion, with another teacher (8%) stating that his students do not have any difficulty in understanding entropy. It is worth mentioning that 75% of teachers say that their students, in reference to previous courses, express entropy as "disorder" but are unable to explain how it is composed or what meaning it has in a thermodynamic system. Others only say that their students have existing ideas regarding entropy but were not able to elicit someone (25%).

As can be recognized, a high percentage of teachers focus on mathematics as the problem with student understanding more than on their own approach to the teaching of entropy. Regarding questioning on their student assessment, there is a correlation with what was stated above: teachers are satisfied if their students solve application problems, and they have a preference (50%) for the Abbot and Vanness book, [38] which presents topics using a problem resolution approach.

Aspect B. Teachers interpretation of entropy. Most of the teachers interviewed (75%) address the concept by beginning with Carnot cycles and advancing to the Clausius expression, as described in textbooks and following the Fermi way of treatment, without referencing the Clausius interpretation of energy transformability. One teacher says that he introduces entropy following Boltzmann (as a measure of disorder). Two

TABLE I. Teachers in the sample.

Science Faculty (UNAM)			Chemistry Faculty (UNAM)		
Sample	Teaching level	Years of experience	Sample	Teaching level	Years of expe rience
5 (70% of the total of College teachers)	College (5)	More than 15 years	7 (30% of the total of College teachers)	College (7) MsC (1)	More than 15 years

teachers use the axiomatic development in the Callen book, [39] one of them distinguishing among energetic processes and the other discussing "absurd" situations and the impossibility of some thermal processes.

The interview had the purpose of allowing the teachers to express the physical meaning they give to entropy and how they attempt to aid their students in understanding. There is dispersion with regard to the way in which they describe entropy (some teachers provide more than one description).

Entropy determines spontaneous process direction (4/12); Entropy is a quality of available energy (1/12); Entropy has no definition, nor interpretation (4/12); Entropy is the expression $\Delta S = \Delta Q/T$ (3/12); Entropy is the heat available for the production of work (1/12); Entropy is a thermodynamic property (2/12); Entropy is a concept arising from the necessity of explaining spontaneous changes (1/12); Entropy is invented to account for irreversible processes (1/12); Entropy is volume in phase space (1/12); Entropy represents the degradation of energy (1/12); Entropy means the loss of restrictions of the system (1/12); Entropy means disorder (2/12).

The problem with these expressions is that most teachers think that each of them implies a sufficient description of entropy and that it is enough for students to understand entropy. These descriptions include the declaration that entropy has no definition and not physical meaning. Teachers said that is a concept that emerges by necessity or that it is invented with a particular purpose (teleological vision), as if it was not developed and did not have foundations in the conceptual and phenomenological structure of thermodynamic processes. Teachers, usually do not connect the different descriptors they use for entropy and do not discuss with their students that neither of these sentences are enough to understand this complex concept.

It is interesting that the idea of disorder (or order) does not appear in the entropy interpretation made by some teachers (16%). Some of them refer to energy (16%), and it is remarkable that some (32%) do not assign to entropy any interpretation at all, although it is a physics concept. Many of the teacher's descriptions are evidently wrong and others are properties of entropy, but isolated do not help students to understand entropy.

TABLE II. Results for the conception of entropy and physics teachers' methods of teaching.

How teachers teach entropy (Science Faculty: Physics)				
Teacher	Student difficulties	Treatment	Entropy interpretation	Role assigned to history
1	Believe that thermodynamics is not important	Carnot cycle; entropy equations; second law	Natural directionality of processes. Quality of useful energy	Introduces history with Carnot cycles
2	They possess "distorted" concepts	Follows Callen textbook	Entropy does not have a physical interpretation	Believes that it has a heuristic role but does not use history in class
3	They have no problems	Carnot cycle; entropy equations; second law	Loss of systems constraints.	Remarks on importance of Gibbs's work
4	There is confusion between heat and temperature and between reversible and irreversible processes	Carnot cycle; entropy equations; second law	Quantity transformed in processes.	Introduces history using Carnot cycles
5	The high school teacher does not teach well	Carnot cycle; entropy equations; second law	Entropy as volume in phase space. Order and disorder in a phase space	Is not relevant

TABLE III. Results for the conception of entropy and chemistry teachers' methods of teaching

How teachers teach entropy (Science Faculty)				
Teacher	Student difficulties	Treatment	Entropy interpretation	Role assigned to history
1	Do not understand the concepts	Follows Callen textbook	Heat is useful for performing work	Does not consider it
2	Mathematics are poor	Carnot cycle; entropy equations; second law	Degradation of energy, time arrow	Does not consider it
3	Understand variables that are trajectory dependent	Carnot cycle; entropy equations; second law	Level of organisation or disorganisation of a system	Does not consider it
4	Have deficiencies in mathematics	Carnot cycle; entropy equations; second law	A thermodynamic property. Quantity of heat transferred per time unit	Does not consider it
5	Confusion between trajectory and state functions	Carnot cycle; entropy equations; second law	Property of systems; directionality criteria	Does not consider it
6	Do not think logically	Boltzmann equation and random concept	Spontaneity criteria of processes	Does not consider it
7	Deficiencies in mathematics. Do not understand PV diagrams	Carnot cycle; entropy equations; second law	Is the time arrow, the directionality of processes	Asks students to read popular science texts on history of thermodynamics

Aspect C. Knowing and use of history. Considering the results for aspects A and B, it is not surprising that teachers do not know or consider the historic development of the concept of entropy. It is worth mentioning that teachers from the Chemistry Faculty (58%) do not consider any historical view, whereas teachers on the Science Faculty (33%) say that they subscribe more importance to history and that they understand that beginning their curriculum development with the Carnot cycles constitutes consideration of the historic view. Only one teacher asks students to read a popular science book with a partially historic approach [40].

None of the interviewed teachers uses Boltzmann's developments, although they mention them or address entropy as disorder. It is noteworthy that one teacher on the Chemistry Faculty states that history is not necessary and that what must be accomplished is the development of mathematical ability in students.

During the interviews, the Clausius Eq. (4) was shown to the teachers to determine whether they had any knowledge of it. Only one teacher in the sample said that he had an unclear remembrance of this equation.

4.3. Textbooks

Several analyses have been performed of thermodynamic textbooks that account for diverse problems and construction

TABLE IVA. Treatment and definition of entropy in textbooks

Textbooks (Thermodynamics)	Treatment	Interpretation
Abbott M. & Van Ness, H.	Defines internal energy; introduces entropy in terms of measurable coordinates of the system; describes the second law based on entropy; mentions that it is not useful for students to learn through history.	Consider entropy as a primitive concept. Not provide any argument.
Callen, H.	Introduces entropy as a variational function that defines the states of equilibrium; mentions Gibbs but does not provide any explanation of his ideas. Describes macroscopic systems in terms of transitions of quantum states.	Entropy as the logarithm of the number of microstates. Describes entropy as disorder and uses analogies from outside thermodynamic systems.
García-Colín, L.	Describes the Carnot cycles and the relationships of reversible processes and describes entropy as an equivalence relation between heat and temperature.	Entropy as a thermodynamic variable that is a point function and provides its properties.
Piña, E.	Describes the Carnot cycle, makes a generalisation of heat - temperature relations and defines the integral between two thermodynamics states.	Describes entropy as an integral that increases when there are no restrictions on the system.
Pippard, A. B.	Uses the reversible Carnot cycle; makes a generalisation of heat-temperature relations and defines entropy as the integral between two thermodynamics states.	Describes Clausius inequality of reversible processes and define entropy as a state function and a property of systems and their restrictions.
Reif, F.	Describes states of equilibrium and non-equilibrium, uses the logarithm to describe the variations of energy.	Describes entropy as the Boltzmann logarithm function and the probability of states of particle arrangement.

characteristics [8]. In this instance, we will make a brief comparison of three types of textbooks: general physics, thermodynamics and physical chemistry and physics textbooks at the high school level. The books analyzed are provided in Tables IVA, B. This sample contains only books that teachers declare they use with or recommend to their students. The elements that each author considers in developing entropy in his text are indicated. These elements are view (macroscopic or microscopic), essential features in the argument and the definition of entropy.

As can be noted, most books have a postulate-based viewpoint in which entropy is introduced as a consequence of thermodynamic cycles and the second law. However, differences exist in language. For instance, Pippard [41], Piña [42] and Chue [43] consider entropy a system property linked to certain restrictions. Others, such as García-Colín [44], introduce entropy as a criterion to determine whether a process, among various states, can be realized. Abbott and Vanness [38] state that there is no explicit definition, so they consider entropy a primitive concept.

TABLE IVB. Treatment and definition of entropy in textbooks.

Textbooks (Physical-Chemistry)	Treatment	Interpretation
Morris, J.	Introduces Entropy as a mathematical function; describes Entropy as a measure of the random distribution of energy.	Introduces three definitions: S - measure of the stability of the system. S - measure of the random distribution of energy. S - measure of system disorder.
Canales, M.	Describes heat transfer as unidirectional and notes that systems tend to equilibrium spontaneously.	Entropy as the measure that increases when the distribution is random. Uses the Boltzmann logarithm function and the Clausius differential equations without explanation of how they are related.

Thermodynamics textbooks reproduce the classical treatments of Fermi with varying degrees of mathematical refinement and do not observe a need to provide an explanation of a concept that they remark "is obtained from the second law, from which entropy will be defined as a thermodynamic variable" [44].

The review reveals polysemic references to entropy (Thermodynamic variable, System-intrinsic property, Primitive concept, etc.) that are undoubtedly related to the difficulties in understanding entropy [45].

Two books in the sample provide a broad treatment of the microscopic view [39,46]. In these books, the probabilistic view is centered on the term disorder. For instance, in Díaz Peña, [46] "…disorder is related to the probability or the number of ways of placing a set of objects. The greater that number is, the greater is the probability that these objects are ordered at random; that is to say, that they are disordered" [46].

It is worth noting that instead of establishing a representation using thermodynamic systems, further development is centered on card shuffling or object arrangement, from which the analogy using entropy is inferred, whereas in the work of Boltzmann and Gibbs, the approach is centered on physical processes.

In Callen, [39] thermodynamics is a formalism sustained in a few simple hypotheses, and entropy is a central concept. He defines entropy as "…one of the extensive parameters, as energy, volume, the number of moles and magnetic moment" and links it to probability, arguing that if external restrictions are removed, the number of permissible states increases" [39]. Thus concludes that entropy can be identified using the number of microstates, following in part the Gibbs formulation. To make sense of the expression $S = k \ln W$, this author refers to Shannon's [47] information theory, introducing an explanation taken from a field other than physics in an attempt to give entropy a physical meaning and not vice versa.

Patterns can be observed when reviewing the teachers and textbooks results:

Teachers and texts emphasize the mathematical aspects and ignore the efforts at intelligibility displayed by Clausius and Boltzmann. The interpretations formulated by Clausius in terms of energy transformability and its relation to particles (term Z) are not even mentioned as a historical referent, and regarding Boltzmann, only in a few books is noted the deduction of his equation in terms of possible states.

Entropy; a polysemic concept. It is clear with the first influential textbooks that entropy has a number of different connotations (Planck, Fermi, etc.), which is increased by teachers and textbooks. Thus, entropy is altered from a non-compensated transformation to a thermodynamic property or a criterion for distinguishing a reversible transformation from a non-reversible one, a postulate and a measure (disorder).

The interpretation of entropy as a "measure of disorder" has been justified outside the field of thermodynamics. To make entropy understandable, various teachers and textbooks have promoted access to other fields such as information theory, biological processes and theories about systems, which, although they have enhanced entropy as a concept, do not clearly indicate the various functions of the concept in those fields, promoting ambiguity in the physical interpretation of entropy.

The measure of disorder as a definition of entropy appears to appeal to intuition for easier understanding, but it actually reduces it. This expression of general character leads only to the view that entropy only measures whether a spatial system arrangement has a certain order or tends towards an equal probability configuration. In this way, the concept loses meaning in terms of thermodynamic systems themselves and their relation to the second law and energy [6,48].

5. Discussion

Entropy is one of the most important concepts in physics but also one of the most abstract and difficult to visualize. This difficulty might have originated from the lack of an enduring, substantial image underlying collective imagination [49]. This lack is apparent in Boltzmann's development, which establishes entropy using the similarity in the mathematical-functional behavior of its expressions regarding the probable states of particle arrangements or in the developments made after Clausius [32,33] that relate it to a state function or a system property, also on the basis of its mathematical behavior. Its own origin in Clausius gives way to ambiguity in meaning when he looks for an association with energy and the possibilities of change, with regard to a somewhat speculative microscopic interpretation.

In addition, Shannon's acceptance of von Neumann's suggestion of using the word entropy given the resemblance of his information equation to that of probability in Boltzmann constitutes an additional source of confusion that adds to the idea of disorder [16].

The identification of entropy with disorder, although it is referenced in Boltzmann in his description of molecularly ordered or disordered configurations, has been more of a hindrance than an explanation for the understanding of entropy, as is stated in various physics, philosophical, historic analyses [11,13,35,49,50,51] and even metaphorical analysis [48].

The role we have called *influential textbooks* has been relevant in the introduction of a more structured and axiomatic mathematical derivation of the development of entropy (in addition to widening its range of applications and new thermodynamic developments) and has influenced later textbooks, and as a consequence, teachers.

Teachers present entropy using a great diversity of denominations, as has been shown in the results. This presentation is indicative, in some cases, of the intention to make entropy intelligible (criterion of process spontaneity, loss of system restrictions, system intrinsic property, and so forth). However, any student who pays attention to these attempts finds that they are not very comprehensible, [8,52] and in addition, if the student reads a couple of textbooks, his confusion will surely be greater. It is not surprising that the term "disorder" is assumed by students; it is a referent that is correlative with their daily life context, and it is an expression that offers something to grasp [9], although it cannot be understood how the disorder of a thermodynamic system can be interpreted using the expression $\Delta S = \Delta Q/T$ or how a probabilistic process can be introduced using a mechanic conception (Daub 1969).

5.1. What meaning of entropy must be taught?

Several paths have recently been followed to elucidate the concept of entropy. For instance, Falk [49] proposes the reintroduction of historic substantialist conceptions (Black's heat or Carnot's caloric), providing an intuitive idea of entropy in terms of its increase and the impossibility of its decrease, which would set it apart from the connotation of entropy that is linked to energy.

From an axiomatic perspective Lieb and Yngvason [50,51] propose a development excluding the historic concepts of heat, thermodynamic cycle, temperature, etc. They establish the relation of adiabatic accessibility between states X and Y of a system and several rules (reflexivity, additivity, transitivity, scaling) and conditions (the system can be cut adiabatically into two parts, adiabatic accessibility is stable with respect to small perturbations) and a function $S(X)$ that represent all the possible states of X, Y of that system that can be reached adiabatically, and a comparison hypothesis between states. They establish that:

> $S(X)$ is the maximal fraction of substance in the state X_1 that can be transformed adiabatically into the state X with the aid of a complementary fraction of substance in the state X_0. This way of measuring S in terms of substance is reminiscent of an old idea, suggested by Pierre Laplace and Antoine Lavoisier, that heat be measured in terms of the amount of ice melted in a process [51].

From a different perspective, Leff [13] suggests a new interpretation in terms of *spreading* (in which he finds certain similitude to Clausius' *disgregation*) that allows him to describe entropy in macroscopic and quantum systems more clearly.

> The spreading metaphor is powerful, and offers a physically motivated, transparent alternative to the metaphors discussed above. . . The spreading metaphor addresses this deficiency by focusing on temporal shifts between configurations. In 1996, the spreading metaphor was used to introduce and motivate the development of thermodynamics pedagogically [13].

With the spreading metaphor the dynamics in terms of energy, as well as spatial and time configurations even in equilibrium states is recognized and describe more precise the continual shifts from one microstate to other. "In 2002, Lambert argued in favor of energy-based language to replace the inadequate metaphor of disorder in chemistry textbooks" [13].

Recently Swenden [35] proposes a definition of entropy derived on probabilities following Boltzmann and shows how such definition is better than others to consider the properties of entropy in a classical ideal gas of distinguishable particles and useful for teaching.

Any of these interpretations implicitly carries a historic-epistemological point of view that in any case, must be clarified and justified in a textbook or with students. However, although all of these interpretations introduce alternate ways of making sense of entropy, the discussion is open, and much time is required for new interpretations such as those described to reach the classroom in a coherent and broad manner.

Physical concepts have a history in which they are subject to diverse reinterpretations and formalizations until they acquire standard use or become definitively out of use through processes such as coalescence and transformation. Entropy not only follows a representational transformation process but also presents special difficulties, as with other relevant physical concepts that share a status of structural concepts in physical theories, which have had a historical development rich in diverse interpretations and polemics and have been subject to intense physics and philosophical debates.

We suggest in this paper that teachers should take into account the complexity of the entropy concept and that students require more elements to understand entropy in some introductory manner. Teachers must know and take into account a brief historic background to the development of entropy concept, take a position about it if they teach a macroscopic or microscopic approach first, [53] but also how the meaning of entropy has changed, [9] and when possible (desirable), be informed that clarifying efforts are in progress. Taking into account these elements helps students not only to understand entropy better but also to know about the development of scientific concepts. These considerations would have these advantages:

- Students would understand that entropy is a concept that has been in transformation and that debates are occurring even now.
- They would be provided with more elements for understanding that scientific knowledge is not static.
- They would be provided with elements for assessing, and thus, determining whether to give credit to texts that state that entropy has no physical meaning or is a criterion for defining processes.
- They would be provided with new elements with which to appreciate the importance of science history in concept understanding and value historic experiments and developments [54].
- They would enrich new teaching proposals as the multiple instructional metaphor strategy [48].
- They would generate different expectations regarding implications in physical knowledge learning and their role as students in understanding the knowledge they gain.

Appendix

A. Interview guide

What student difficulties do you perceive when teaching entropy?

Do your students have a previous idea of entropy?

Please explain in detail how you develop the entropy concept in your class.

What physical meaning of entropy do you introduce to students?

How do you use the history of physics in your class?

Do you find it relevant to introduce the history of the physical concepts to your students?

In one study, researchers find that Master of Science students respond that in a reversible expansion of a thermally isolated gas, entropy increases. Has this result revealed problems in understanding entropy in these students?

In 1865, Clausius described entropy in the following terms:

$$\int \frac{dQ + dH}{T} + \int dZ, \qquad (13)$$

H is heat in the body and Z is “disgregation”.

Do you know this equation?

If yes, how do you interpret the equation? Do you inform to students about this equation?

Are there some relations between how do you teach entropy and the Clausius development? Please explain.

What evidence students were required to show to make you think that they understand entropy?

Which books you use in your class? Why?

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11. J. Uffink, *STUD HIST PHILOS M P*, **32** (2001) 305-394.
12. A. Ibarra, and T. Morman, *THEORIA*, **55** 21-38 (2006).
13. H. Leff, *FOUND PHYS* **37** (2007) 1744-1766.
14. R. Clausius, *The Mechanical Theory of Heat, with its Applications to the Steam-Engine and to the Physical Properties of Bodies*. (London. John van Voorst. 1867) (Google Digital Copy)
15. The language used by Clausius was usual at that time, but does not imply a caloric theory.
16. C. Cercignani and R. Penrose, *Ludwig Boltzmann*. (London. Oxford University Press, 2006). doi: 10.1093/acprof:oso/9780198570646.001.0001
17. M. J. Klein, "*Gibbs, Josiah Willard*", In C. C. Gillispie (Ed.), *Dictionary of Scientific Biography*, Vol. 5,. (New York, NY. Charles Scribner's Sons, 1981) p. 386-393.
18. dQ must be considered as an inexact differential, but there is no special symbol in the Clausius papers.
19. M.J. Klein, *HIST STUD PHYS BIOL* **1** (1969) 127 - 149.
20. R. Clausius, *Théorie mécanique de la chaleur*, (Sceux: Jacques Gabay, 1991), p. 357.
21. U. Moulines, "*The classical spirit in J. Willard Gibbs's thermodynamics*". In K. Martinás, L. Ropolyi, and P. Szegedi (Eds.), *Thermodynamics: history and philosophy facts, trends, debates*, (Singapore: World Scientific. 1991) p. 1-12.
22. L. Boltzmann, *Lectures on gas theory*. (Berkeley: University of California Press, 1964).
23. d'Abro, *A. The rise of new physics. Its mathematical and physical theories*, Vol. 1 (New York, NY: Dover Publications, 1951).
24. A detailed description is provided in Boltzmann [22], pages 73-74.
25. As cited in Müller [10].
26. C. Truesdell, *The tragicomical history of thermodynamics* (New York, Springer, 1980). pp. 1822-1854.
27. , S. G. Brush *The Kinetic Theory of Gases. An Anthology of Classic Papers with Historical Commentary.* (London: Imperial College Press, 2003).
28. M. Planck, *Scientific Autobiography and Others Papers*, (London: William & Norgate, 1950).
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54. C. Sichau, *SCI EDUC-NETHERLANDS* **9** (2000) 389-398.