# Comment to the article "On thermal waves velocity: some open questions in thermal waves physics", Rev. Mex. Fis. E 62 (2016) 1-4

O. Delgado Vasallo

*Universidad Politécnica Metropolitana de Hidalgo,*
*Boulevard Acceso a Tolcayuca # 1009 Ex Hacienda de San Javier, 43860, Tolcayuca, Hidalgo, México.*

E. Marín

*Instituto Nacional Politécnico, Centro de Investigación en Ciencias Aplicadas y Tecnología Avanzada,*
*Unidad Legaria, Legaria 694, Col. Irrigación, México, 11500, D.F., México.*
*e-mail: emarin@ipn.mx*

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It is showed here that an upper limit must exist for the speed of thermal waves because Fourier's laws of heat conduction cannot be used at very short length scales, but only over a length larger than the mean free path of the heat carriers.

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In a recent article published in this journal [1] some considerations about thermal waves are presented, in particular about their velocity. It is showed that for thermal waves resulting from the solution of the parabolic heat diffusion equation (also known as second Fourier's law, as it is a consequence of first Fourier's law for heat conduction) for a semi-infinite one dimensional body with thermal diffusivity $\alpha$ and a sinusoidal heat source at its surface modulated at a frequency $f$, the group velocity,

$$V_g = 2(2\alpha \times 2\pi f)^{1/2} = 2(2\alpha\omega)^{1/2} = \omega\mu \qquad (1)$$

can exceed the value of the speed of light in vacuum, $c$, for $f > fc$, where $f_c = c^2/(16\pi\alpha)$, *i.e.*, for very high modulation frequencies. In the above equation

$$\mu = (\alpha/\pi f)^{1/2} \qquad (2)$$

is known as the thermal diffusion length, a characteristic propagation distance of the thermal wave.

The above result is physically impossible because it violates relativity theory. One solution for this paradox was given in the refereed manuscript in terms of the hyperbolic heat diffusion equation (deduced by substituting First Fourier's Law by Cattaneo's Equation [2]), which predicts non-attenuated thermal waves propagating at the so-called second sound velocity

$$u = (\alpha/\tau)^{1/2}, \qquad (3)$$

where $\tau$ is the relaxation time or build-up time that must exists for the onset of the thermal flux after a temperature gradient is imposed on the sample. Since $\tau$ must be of the order of magnitude of $\Lambda/u$, where $\Lambda$ is the mean free path of the heat carriers, then we can conclude from Eq. (3) that

$$u \sim \alpha/\Lambda \qquad (4)$$

The second sound velocity takes values ranging from hundreds to thousands of m/s for typical solids.

An alternative solution to the above mentioned paradox should be given in the following way: Fourier's laws can only be defined for lengths larger than the mean free path of the heat carriers, since they involve the concept of temperature, the basic parameter of Thermodynamics, which is related to the average energy of a system of particles [3]. Therefore, the maximal frequency, $\omega_{\max}$, that can be achieved to generate thermal waves is that for which the thermal diffusion length becomes approximately equal to the mean free path, *i.e.* $\mu \sim \Lambda$. Using Eq. (2) one can see that this condition is achieved for

$$\omega_{\max} \sim 2\alpha/\Lambda^2 \qquad (5)$$

Substituting this value into Eq. (1), the maximum possible group velocity becomes

$$V_{g\,\max} \sim 4\alpha/\Lambda \qquad (6)$$

a value of the same order of magnitude as that given by Eq. (4).

In conclusion, thermal waves cannot propagate at velocities greater than $V_{g\,\max} \sim u$ and therefore they cannot travel at the speed of light in vacuum, as expected.

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