Geodesic time travel in Gödel's universe

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This work is an introduction at a beginning graduate or advanced undergraduate level to Kurt Gödel's foray into cosmology. After an elementary introduction to the basics of Einstein's theory of gravitation, we simply present the Gödel's solution and the geodesic equations associated with it. This equations are then explicitly solved obtaining its full set of temporal geodesics. Armed with such explicit expressions, the geodesic time-travelling possibilities of Gödel's universe are discussed. We search for their time-like closed geodesics that, following Gödel's analysis, other people has imagined as possible routes for time-travel. We next exhibit that such time-travelling possibility do not exist in his model universe. This is done in the most straightforward way possible, framing the discussion as to serve as a simple example for students of General Relativity.

Keywords: Time travel; Gödel universe geodesics.

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1. Introduction

Kurt Gödel, one of the greatest logicians of all time [1], discovered in 1949 a solution to Einstein's field equations, describing the Gödel universe (GU), which some people claimed that there exist geodesics which run smoothly back into themselves; *i.e.* the GU was imagined endowed with closed time-like and geodesic world lines. Furthermore, in this spacetime matter is rigidly rotating respect to, as Gödel put it, a *compass of inertia*. You may imagine such inertia compass as a set of gyrocompasses fixed to every galaxy in GU and such that all such galaxies rotate in unison about its prescribed parallel-transported normals, so indicating that the entire GU rotates rigidly in the opposite sense. Therefore, GU is homogeneous but cannot be isotropic, a feature that prevents the definition of a unique time valid for the whole universe [2]. The notion of causality itself, implying that a cause happens earlier than its effect, was moreover challenged by the existence of such closed time-like geodesics, since a time-machine for travelling into one's own past had become a possibility sanctioned by general relativity (GR). Time travel lead to questions as, 'how can we understand that someone could kill her own ancestors destroying in this way the very conditions for her existence?' As we recognize that these are the sort of discussions that attract students into GR, it is important to offer physics students an introduction to GR and to cosmological features like of GU from a not too-complicated standpoint, as we intend to offer in this contribution. Incidentally, it is apt to mention that Gödel himself began his scientific career in 1924 as a physics student at Vienna University and that he was keenly interested in physics for the rest of his life [1, 3]. Gödel managed to obtain a cosmological solution in GR with the above-mentioned temporal structure and some other features that, as he was well aware, do not represent the universe we live in. For example, a universe in which matter rigidly rotates [4] means that GU is homogeneous but not isotropic. Moreover, it is an example of a cosmology exhibiting properties associated with the rotation of the universe as a whole. The conflict with observations notwithstanding, he maintained that if GR permits such behavior then it should be studied in detail. The Gödel metric (3) solves the Einstein field equations with a homogeneous perfect fluid source [2, 3], see Eq. (6). Moreover, it has been regarded as an important pedagogical example [6] as it may illustrate some of the remarkable behavior GR predicts. GU is known to allow closed timelike and closed null *curves*, but as we are going to exhibit here, it contains **no** closed temporal *geodesics*, that is, that no observer can travel in such a way in Gödel's model universe. GU is also known to be geodesically complete, the domain of definition of every geodesic is the whole real line [5]— containing neither a singularity nor a horizon —of the sort occurring in black-hole solutions [6]. Gödel's cosmological solution was one of the first to admit the possibility of time travel, but such non-causal possibility occurs in other metrics like Kerr's and Stockum's [6, 8, 9]. This GU feature seems to have been considered as the most important point of the solution by Gödel himself, who was supposedly trying to show that Einstein's equations were not consistent with our basic concept of time. For, in GU, the usual distinction of *later than* and *earlier than* is no longer permitted, simply because a time machine enabling one to travel into ones own past became a GR-sanctioned possibility [1, 6].

Most non-vacuum cosmological solutions allow defining a universal time coordinate, τ, the so-called cosmological time, thanks to the existence to a system of 3-spaces everywhere orthogonal to the world lines of matter. The non-existence of such a system, hence of τ, is equivalent to a rotation of matter relative to a system of free-falling observers, each carrying giroscopes with parallel(-transported) angular momenta, *i.e.* rotating respect the aforementioned compass of inertia [2]. As Gödel proposed a homogeneous rotating universe, the use of such cosmological time is not allowed as closed time-like curves (that are not geodesics!) are possible. Let us clearly state that we do not pretend to derive the Gödel solution in this paper, we only discuss its geodesic temporal properties, for a more or less simple derivation see Ref. 3. This paper is organized as follows, after giving a very brief cursory introduction to Einstein equations in Sec. 2, we obtain the complete set of solutions to GU geodesic equations. We analyse such curves and, after writing them in what we call hypercylindrical coordinates, we exhibit that closed, future-pointing, time-like geodesics do not exist in GU. We expect our solution and subsequent discussion be of help to students trying to get a grip on the basic techniques needed in the study of GR.

2. General relativity basics

Einstein general relativity is a geometric theory of gravitation in which Newton's force of gravity is replaced by the curvature of 4-dimensional spacetime. But, what do we mean by *curvature* of a space(time)? as described in Ref. 14, the term is used as "an analogy, a visual way of extending ideas about three–dimensional space to the four dimensions of spacetime." GR also serves to explain certain facts that are not even mentioned in Newton's theory, as the cancelation of gravitational fields by accelerated motions. That is, as the acceleration in a gravitational field is independent of the mass of the body then gravitational interactions behave as pseudo-forces, or inertial forces, that can always be cancelled (at least in a small spacetime region) by the proper selection of a reference frame. Such cancelation is known by the name of *weak principle of equivalence.* The so-called *strong principle of equivalence*, on the other hand, states that the results of any experiment (gravitational or not) in a free-falling laboratory is independent both of the velocity of the laboratory and of its location. These two principles suggests that gravity behaves as an inertial force or, as stablished by Einstein's general relativity, that is geometric in nature. Accepting such basic tenet, it then follows that the metric alone suffices to determine the effect of gravity. We pinpoint also that GR is the only theory of gravity that satisfies both previously stated equivalence principles. Those changes in the basic framework of gravitation obliges masses free from non-gravitational interactions to move not in "old-fashioned" straight lines but along certain natural trajectories in spacetime (remember, there are no forces just the "bumps and hollows" of spacetime), such important trajectories, the *geodesics*, may be described as those minimizing the proper-time between any events A and B (or minimizing the interval between them). Such interval, or rather, square of the interval, $d\tau^2$, is in differential form and for a region completely *free* from gravity

$$\begin{aligned} d\tau^2 &= dt^2 - dx^2 - dy^2 - dz^2 = dt^2 - dr^2 \\ &= dt^2\,(1 - \mathbf{v}\cdot\mathbf{v}) \end{aligned} \tag{1}$$

where we are using units such that the speed of light is unity, $i.e.\ c = 1$, $\mathbf{v} = d\mathbf{r}/dt$ is the particle's standard 3-velocity, the $\cdot$ represents the usual dot product, and $x_A = (\mathbf{r}_A, t_A)$, $x_B = x_A + dx$ are the coordinates of the spacetime events A and B, separated by the infinitesimal interval $dx = (d\mathbf{r}, dt)$. Notice that the interval, ds, can be real, zero, or purely imaginary, corresponding to spacetime points separated in such a way that A and B may be connected by an inertial observer, a ray of light, or free from any possibility of causal connection. Notice also that the interval along a worldline connecting A and B can be measured by a clock riding on such world line. That is, ds, may be called either the interval or the proper time, $d\tau = ds$, along the worldline. To describe the motion of matter, we introduce the 4-velocity, $v^a = dx^a/d\tau$, and the 4-acceleration, $a^c = dv^c/d\tau$, both defined along a worldline. The 4-velocity is a 4-vector tangent to the particle's worldline and complying with $v^a v^b g_{ab} = 1$, and $v^c\, a^d g_{cd} = 0$. Here, as in the rest of the paper, we use the summation convention where repeated indices are regarded as summed over from 0 to 3. The $g_{\mu\nu}$ stands for the metric tensor whose coefficients can be arranged in matrix form as

$$\mathrm{g}_{\text{flat}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{2}$$

this metric is the one corresponding to the *flat spacetime* of special relativity which is called the Minkowsky metric. In the more general, non-flat spacetime of general relativity the components of $\mathrm{g}(x)$ are in general not constants but functions of the spacetime point x^μ, such dependence accounts for the curvature of spacetime which is the manifestation of the action of gravity.

So, how can we describe the gravitational interaction using no forces? We have to accept first the idea that *physics is simple only when analyzed locally*, gravitation should be, at difference with Newton's action-at-a-distance approach, a completely local phenomena. We should accept that the gravitational interaction is mediated by the local structure of the spacetime, which is a 4-dimensional space or a 4-dimensional *manifold*[i]. We have to accept that spacetime tells matter how to behave and, in turn and because we want a closed theory, that matter determines the local curvature of spacetime, the curvature acts back on matter determining its motion. We should forget everything about the old-fashioned notion of force of gravity and start thinking that the particles of matter or the quanta of fields when not acted by non-gravitational interactions follow the straightest possible paths in spacetime: they should travel along *geodesics* [12]. So, the metric tensor, $g_{\mu\nu}$, plays the role of the gravitational field in general relativity. In our Gödel case, the free particles should follow the GU geodesics; our first task then is to derive the possible geodesic paths in GU and then to prove impossible that travelling on such paths we could manage to travel back in time.

3. The geodesic equations for GU

The Gödel spacetime is a stationary solution of the Einstein equations with nonvanishing cosmological constant Λ whose matter content for comoving observers consists of dust with constant density ρ[ii], as shown in Eq. (6).

The Gödel's metric solves the Einstein field equations with a homogeneous perfect fluid source given in (6). For comoving observers such matter can be thought of as consisting of dust with constant density ρ, the associated spacetime admits closed timelike and closed null curves but contains no closed timelike nor closed null geodesics. This is one of the things we want to exhibit here. Moreover, all possible geodesics in GU never encounter a singularity or meet a horizon, as do happen with the geodesics of the Schwartzchild solution [5]. Assuming $c = 1$ Gödel's metric may be written as

$$ds^2 = a^2\big(dx_0{}^2 - dx_1{}^2 + (\exp(2x_1)/2)dx_2{}^2 - dx_3{}^2 + 2\exp(x_1)dx_0dx_2\big), \tag{3}$$

or, in matrix form,

$$\mathsf{g}_{GU} = a^2 \begin{pmatrix} 1 & 0 & \exp(x_1) & 0 \\ 0 & -1 & 0 & 0 \\ \exp(x_1) & 0 & \exp(2x_1)/2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{4}$$

where the x_μ, $\mu = 0, 1, 2, 3$, are the space-time coordinates and a is Gödel's constant parameter, related to the angular velocity Ω of matter respect the compass of inertia by $a = 1/(\sqrt{2}\Omega)$ [2, 3]. The metric (3) satisfies Einstein equations,

$$R_{ab} + (\Lambda - \frac{1}{2}R)g_{ab} = -\sqrt{8\pi G}\, T_{ab}, \tag{5}$$

where $\Lambda = -1/(2a^2)$ is the value of cosmological constant used by Gödel, ρ is the matter density, G the gravitational constant and the energy-momentum tensor is

$$T^{ab} = \rho\, u^a u^b, \tag{6}$$

with u^a the 4-velocity of matter. From (3), we may also get the non-zero Christoffel symbols,

$$\begin{aligned} &\Gamma_{012} = \Gamma_{120} = \Gamma_{210} = a^2 \exp(x_1)/2, \\ &\Gamma_{122} = -\Gamma_{221} = \Gamma_{212} = a^2 \exp(2x_1)/2, \\ &\Gamma_{01}{}^0 = 1, \quad \Gamma_{22}{}^1 = \exp(2x_1)/2, \\ &\Gamma_{10}{}^2 = -\exp(-x_1), \quad \Gamma_{12}{}^0 = \Gamma_{02}{}^1 = \exp(x_1)/2, \end{aligned} \tag{7}$$

the non-vanishing components of the Ricci tensor

$$\begin{aligned} &R_{00} = 1, \quad R_{22} = \exp(2x_1), \\ &R_{02} = R_{20} = \exp(x_1), \end{aligned} \tag{8}$$

the curvature scalar

$$R = \frac{1}{a^2}, \tag{9}$$

and the geodesic equations

$$\frac{d^2x_\alpha}{d\tau^2} + \Gamma_\alpha^{\beta\sigma}\frac{dx_\beta}{d\tau}\frac{dx_\sigma}{d\tau} = 0. \tag{10}$$

Note that the previous equation clealy exhibits that the "acceleration" (the $d^2x_\alpha/d\tau^2$ term) along the geodesic has no components outside the tangent plane of the spacetime manifold, so the motion is completely determined by the bending and the deforming of it.

For the specific GU case, the geodesic equations that follow from (10) are

$$\ddot{x}_0 + 2\dot{x}_0\dot{x}_1 + \exp(x_1)\dot{x}_1\dot{x}_2 = 0, \tag{11}$$

$$\ddot{x}_1 + \exp(x_1)\dot{x}_0\dot{x}_2 + \exp(2x_1)(\dot{x}_2)^2/2 = 0, \tag{12}$$

$$\ddot{x}_2 - 2\exp(x_1)\dot{x}_0\dot{x}_1 = 0, \tag{13}$$

and

$$\ddot{x}_3 = 0, \tag{14}$$

were the overdots stand for derivatives respect to the proper time τ.

Equation (14) can be inmediately solved to get

$$\dot{x}_3 = C \qquad \text{and} \qquad x_3 = C\tau + c_3 \tag{15}$$

where C is the starting value of the third component of the $4-$velocity, $\dot{x}_3$, and c_3 is the starting value of x_3. Next, we realize that we can complete a square in metric (3), rewrite such equation, and, dividing twice by $d\tau$, we get what we may call a first integral of the geodesic equations

$$\begin{aligned} &(\dot{x}_0 + \exp(x_1)\dot{x}_2)^2 - (\dot{x}_1)^2 \\ &- \exp(2x_1)(\dot{x}_2)^2/2 - (\dot{x}_3)^2 = 1. \end{aligned} \tag{16}$$

We then proceed to solve for the $\dot{x}_\mu$ with $\mu = 0, 1, 2$, we first take the product of (12) times $\dot{x}_1$ and the product of (13) times $\exp(2x_1)\dot{x}_2/2$, finally adding the results, we get

$$\dot{x}_1\ddot{x}_1 + \frac{1}{2}\exp(2x_1)\dot{x}_1(\dot{x}_2)^2 + \frac{1}{2}\exp(2x_1)\dot{x}_2\ddot{x}_2 = 0, \tag{17}$$

integrating this last equation, we obtain

$$B^2 = (\dot{x}_1)^2 + \frac{1}{2}\exp(2x_1)(\dot{x}_2)^2, \tag{18}$$

where B is a constant. From (18), we obtain $\dot{x}_2$ in terms of $\dot{x}_1$

$$\dot{x}_2 = \sqrt{2}\exp(-x_1)[B^2 - (\dot{x}_1)^2]^{1/2}. \tag{19}$$

Now, using (15) and (18), in (16) we obtain

$$D \equiv \sqrt{2(1 + B^2 + C^2)} = \sqrt{2}\,(\dot{x}_0 + \exp(x_1)\dot{x}_2). \tag{20}$$

Now, on analysing (27), we may obtain

$$\begin{aligned} \dot{x}_0 &= (D/\sqrt{2}) - \exp(x_1)\dot{x}_2 \\ &= (1/\sqrt{2})\left(D - 2[B^2 - (\dot{x}_1)^2]^{1/2}\right); \end{aligned} \tag{21}$$

then, we found it convenient to introduce the auxiliary variable θ through

$$\dot{x}_1 \equiv B\sin\theta, \tag{22}$$

using this last definition, Eq. (19) and Eq. (21), in (12) we obtain

$$\frac{d\theta}{ds} = B\cos\theta - D. \tag{23}$$

which expresses a useful relationship between τ and θ, as we exhibit in what follows.

Solving Eq. (23), we get

$$\begin{aligned} \tau &= -\sqrt{\frac{4}{(D^2 - B^2)}}\arctan\left[\left(\frac{D+B}{D-B}\right)^{1/2}\tan\left(\frac{\theta}{2}\right)\right] \\ &\equiv \frac{2}{(D^2 - B^2)^{1/2}}\,\sigma, \end{aligned} \tag{24}$$

were we have introduced σ as an scaled measure of the original interval τ. From now on, we employ σ as our proper time parameter using it instead of τ and, as follows from (18) and (20), we assume $D \geq B$. Furthermore, using (24), the relationship between θ and σ is

$$\tan\sigma = -\sqrt{\frac{D+B}{D-B}}\tan\left(\frac{\theta}{2}\right). \tag{25}$$

From the previous transformations, we get

$$\cos\theta = \frac{1 - \alpha\tan^2\sigma}{1 + \alpha\tan^2\sigma}, \quad \sin\theta = -2\sqrt{\alpha}\frac{\tan\sigma}{1 + \alpha\tan^2\sigma},$$
$$\text{where} \quad \alpha \equiv \frac{D-B}{D+B}, \tag{26}$$

and, after obtaining $\dot{x}_1$ from Eq. (22), we can use (24), (25), and (26), to write

$$\frac{dx_1}{d\sigma} = \dot{x}_1\frac{ds}{d\sigma} = -\left(\frac{4B}{D+B}\right)\frac{\tan\sigma}{1 + \alpha\tan^2\sigma}. \tag{27}$$

The first-order geodesics Eqs. (19) and (21), now become

$$\frac{dx_0}{d\sigma} = \left(\frac{2}{D^2 - B^2}\right)^{1/2}\left[D - 2B\left(\frac{1 - \alpha\tan^2\sigma}{1 + \alpha\tan^2\sigma}\right)\right], \tag{28}$$

$$\frac{dx_2}{d\sigma} = 2e^{-c_1}\left(\frac{2B^2}{D^2 - B^2}\right)^{1/2}\frac{(1 - \alpha\tan^2\sigma)\sec^2\sigma}{(1 + \alpha\tan^2\sigma)^2}, \tag{29}$$

and, in spite of the fact that we already know its solution (15), we add

$$\frac{dx_3}{d\sigma} = \frac{2C}{(D^2 - B^2)^{1/2}}. \tag{30}$$

We have gotten the four first-order geodesic equations in terms of σ, which we can then proceed to write for the x_μ $\mu = 0, 1, 2, 3$, as

$$\begin{aligned} x_0 = &-\left(\frac{2D^2}{D^2 - B^2}\right)^{1/2}\sigma \\ &+ 2\sqrt{2}\arctan(\sqrt{\alpha}\tan\sigma) + c_0, \end{aligned} \tag{31}$$

$$x_1 = \log\left(\frac{1 + \alpha\tan^2\sigma}{1 + \tan^2\sigma}\right) + c_1, \tag{32}$$

$$x_2 = 2e^{-c_1}\left(\frac{2B^2}{D^2 - B^2}\right)^{1/2}\frac{\tan\sigma}{1 + \alpha\tan^2\sigma} + c_2, \tag{33}$$

$$x_3 = \frac{2C}{(D^2 - B^2)^{1/2}}\,\sigma + c_3. \tag{34}$$

These solutions, with $c_0 = c_1 = c_2 = c_3 = 0$, $\alpha = 1/4$, $C = 1/3$, are plotted in Fig. 1, we want to pinpoint that they

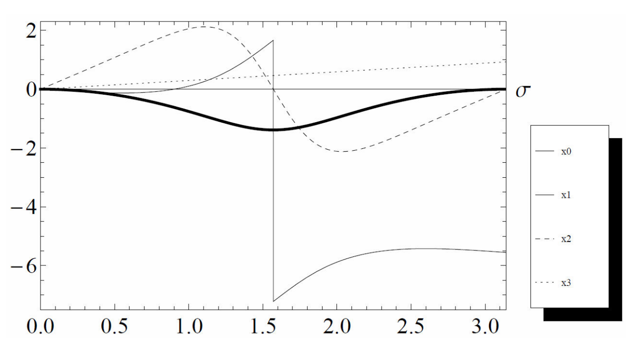

FIGURE 1. Solution of the geodesics equations. The x_0-coordinate is plotted as the thin line showing a discontinuity, x_1 is plotted as a thick continuous line, x_2 is plotted as a dashed line, and x_3 is the dotted line. The obvious discontinuity in the plot is completely harmless as it is just a coordinate discontinuity, as explained in the text.

they are of the same form that Chandrasekhar and Wright found [7]. However, there is a difference between the graph of the x_0-coordinate in [7] and the one shown here. In [7] x_0 has an apparent maximum at $\sigma = 3\pi/4$ that we can not found here and in our result it shows a discontinuity at $\sigma = \pi/2$ that is nowhere in sight in [7]. This difference is rather extraordinary because we are plotting our solution, that is the same as Chandrasekhar's, and also because the discontinuity was to be expected because of the *ArcTan* function. However, as we show in the next section, this is a discontinuity that disappears after a simple change of coordinates and so there is nothing to worry about since it is not part of the spacetime structure but just a consequence of the chosen coordinates.

It is convenient to change variables to the coordinates (r, φ, t, z) through the transformations,

$$e^{x_1} = \cosh 2r + cos\varphi \sinh 2r, \tag{35}$$

$$x_2\, e^{x_1} = \sqrt{2} \sin\varphi \sinh 2r, \tag{36}$$

$$\tan\left(\frac{\varphi}{2} + \frac{x-2t}{2\sqrt{2}}\right) = e^{-2r} \tan\frac{\varphi}{2}, \tag{37}$$

$$x_3 = 2z. \tag{38}$$

In the new coordinates, we have

$$ds^2 = 4a^2(dt^2 - dr^2 - dz^2 + (\sinh^4 r - \sinh^2 r)d\varphi^2 + 2^{3/2} \sinh^2 r d\varphi dt); \tag{39}$$

which explicitly exhibit the cylindrical symmetry of the metric—they can be regarded as a sort of "hyper-cylindrical" coordinates. The coordinate r is a generic function of σ unless c_1 takes certain values, for example, if

$$e^{2c_1} = 1/\alpha, \tag{40}$$

r becomes independet of σ, accordingly $1 \leq \cosh 2r \leq \sqrt{2}$ and

$$\cosh 2r = \frac{1}{2}\left(\frac{1}{\sqrt{\alpha}} + \sqrt{\alpha}\right). \tag{41}$$

On account of the previous equations, if we take

$$\varphi = 2\sigma, \tag{42}$$

and use (31) and (42), equation (37) becomes

$$t = \sqrt{2}(1 - 1/2\cosh 2r)\sigma = \beta\sigma. \tag{43}$$

The solution of the geodesic equations simplifies due to the constancy of $x_3 = z$ and r. Note that no trace of the singularity is found in hypercylindrical coordinates ergo the singularity found is merely an artifact of the description —as is the case of the singularity near the poles of a sphere in 3D-spherical-coordinates that disappear in cartesian coordinates.

With the explicit solution to the geodesic equations given above to show the non-existence of closed futue pointing world lines becomes an almost trivial task. Let us calculate first the 4-velocity of the geodesics, as

$$u^a = \frac{dx^a}{d\sigma} \tag{44}$$

which is future pointing. Additionally, as Gödel showed, a positive direction of time can be introduced in any temporal or null geodesic in such a way that we can be determine which of any two neighboring points is earlier and which is later. Furthermore, we need to check if there exist a number T such that $W^a(\tau) = W^a(\tau+T)$ for any value of the proper-time τ, where W stands for any one of r, ϕ or z. But the last one, z, clearly complies with the condition for any T because it is a constant. For the remaining coordinates such conditions should be clear on taking the derivatives of (31), (32), (33) and (34) and we may safely grant that we have proved that the supposed closed time-like geodesics in GU do not exist.

4. Conclusion

We have derived analytical solutions to the geodesic equations of Gödel's metric for general initial conditions. The general solution was also used to determine whether or not causality violations exist when traveling on geodesics. Chandrasekhar and Wright [7] presented an independent derivation of the solution. They concluded that there are no closed timelike geodesics and noted that this fact seems to be contrary to Gödel's statement that the "circular orbits" allow one to travel into the past or otherwise influence the past. We have informally proved that in Gödel universe there are no closed time-like geodesics and so that time travelling using one of the GU geodesics is not possible. But, of course, this not prevent the possibility of time-travel using any other kind of curves. There are more issues with GU that one should be aware of such as that it provides an example of an anti-Machian[iii] distribution of matter, *i. e.* one where the rotation of the compass of inertia bears no relation to the mass distribution in the universe.

The proof that in GU time travel cannot be performed along a geodesic has been given originally by Kundt [11] and independently by Chandrasekhar and Wright [7], but, as Chandrasekhar and Wright misinterpreted Gödel's paper, they claimed that their results were in contradiction to some of the statements in it. Because, in Ref. 7, Chandrasekhar and Wright did not find any closed time-like geodesics, they announced that Gödel's claim about the possibility of time-travel in his universe was incorrect. However, a careful reading of Gödel's papers would suffice to understand that he had never claimed that the closed time-like world lines in his model were geodesic, in GU geodesic time-travel is not allowed as we have shown again in this paper. The only conclusion then is that the claimed result in Ref. 7 is mistaken. This may be taken as a good lesson for any student of general relativity, as we shall never acritically accept as correct the claims made in any scientific work—the prestige of the authors notwhitstanding.

But you should notice that the features of GU bring to the fore two basic issues of general relativity. The first is the possible existence of closed time-loops, which means trouble for causality. The possible causal connectability of each point in this space-time with each other involves relevant questions as to whether our concepts of causal connectibility and causality are compatible with the consequences of GU solution. A possible answer to this is that it should be expected that nature has some (as yet unknown) mechanism preventing the formation of such universes, analogous perhaps to the speed limit in special relativity, which also could serve to preserve causality—but at present this is really just a hope not a solution. A case in point is the chronology postulate proposed expressely to avoid solutions like the non-geodesic but closed world lines that do exists in GU. The second issue is the non-unicity of simultaneity in some relativistic cosmologies though not in others. But this is not really a problem, as special relativity has taught us to accept it. Another point raised by Gödel paper is his demonstration that the possibility of the non-existence of an universal time is some thing that should not take for granted when working in GR, he was completely convinced that expanding (as the actual universe) and rotating solu tion to Einstein field equations did exist and that in such universes absolute time might also fail to exist [1,10]. Anyhow, as Einstein himself said [13]. “Gödel's paper is the most important one on relativity theory since my own original paper appeared.” We must say however that Einstein point of view on the importance of Gödel result is not shared by most researchers nowadays.

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$i.$ An n-dimensional manifold can be though as any space that locally looks like R^n, just think of a sphere that in a very small patch may be regarded as isomorphic to R^2 as we use on drawing city or country maps.

$ii.$ Comoving observers are observers who perceive the universe as isotropic. They are called “comoving” observers because they move along with the motion of galaxies that follow the overall expansion of the universe.

$iii.$ Mach principle can be loosely stated as “There is a sort of physical relationship between the motion and distribution of distant stars and the behaviour of local inertial frames.”

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