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Learning about wave optics: the effects of combining external visualizations with extreme case reasoning

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In this study, we investigated whether combining external visualizations with extreme case reasoning may the development of a conceptual understanding of wave optics. For purposes of answering our research question, we conducted a pretest-posttest quasi-experiment, which included 179 students from a first-year introductory physics course at the University of Zagreb, Croatia. Students who were guided through extreme case reasoning in their wave optics seminars significantly outperformed their peers who received conventional teaching treatment. Findings from our study suggest that combining external visualizations with extreme case reasoning facilitates the development of visually rich internal representations, which are a good basis for performing mental simulations about wave optics phenomena. Besides, it has been also found that many students use the "closer to the source implicates greater effect" p-prim when reasoning about certain relationships, such as the relationship between fringes' dimension and slits-screen separation.

Keywords: Wave optics; extreme case reasoning; p-prims; misconceptions.

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1. Introduction

Wave optics has many applications in the field of lasers, microcomputers and electronic detectors. We can say that its applications extend to all areas of modern science, engineering, and technology [1]. In everyday life, wave optics is useful to understand some phenomena, such as interference of light on peacock feathers and colored appearance of a soap bubble [2]. Generally, wave optics significantly contributes to learning one of the most important physics concepts, which is the wave concept. Consequently, learning wave optics is very important for conceptual understanding of other areas of physics, *e.g.,* solid-state physics and quantum mechanics [3, 4]. However, many students struggle with developing a basic understanding of wave optics [5–7]. Earlier studies have shown that students often do not understand whether they should use geometric or wave optics to solve standard textbook problems related to light phenomena [4,8]. Understanding of wave optics requires simultaneous thinking about spatial and temporal aspects of wave motion. However, research showed that human working memory is highly limited [9]. That is why thinking about wave phenomena induces high cognitive load [10]. Furthermore, reasoning about wave optics is additionally obstructed by the fact that students lack intuitive mental models about wave optics [11].

For developing a deep understanding of wave optics, students have to go far beyond intuition. Actually, examples from the history of physics show that deeper truth is often hidden under the surface of everyday experience. In many cases throughout history, scientists discovered this truth by using analogies and extreme reasoning [12,13]. Stephens and Clement stress that extreme case reasoning is at work when, "in order to facilitate reasoning about a situation A (the target), a situation E (extreme case) is suggested, in which some aspect of situation A has been maximized or minimized" [14]. For example, it seems that Galileo Galilei used extreme case reasoning for mentally simulating what would happen to the motion of a sphere moving between two smooth inclined planes facing each other. He concluded that as the angle of the second plane tends to zero, the distance covered by the sphere tends to infinity [15].

Any teaching approach, the extreme case reasoning approach included, should take into account basic principles of cognitive psychology. According to cognitive load theory, it is useful to distinguish between intrinsic, extraneous, and germane cognitive load [16]. When preparing lessons, the goal should be to maximize the germane load, minimize extraneous load, and adjust intrinsic load [17]. To increase the level of germane load, it is advisable to use visual representations. According to Nersessian, the use of visual representations and mental simulations are compatible activities with the goal of developing mental models. Mental simulations that are based on extreme case reasoning can help us to optimize the cognitive load [18]. However, poorly designed lessons can create cognitive overload [19, 20]. If we want to include extreme case reasoning in our lessons, it is desirable to optimize the cognitive load using external visualizations, step-by-step guidance, and highlighting the most important information. In our study, we combined external visualizations and extreme case reasoning with the purpose to make more comprehensive the abstract mechanisms that are at the core of the superposition of waves. In our opinion, one of the most useful visualizations in wave optics instruction is the phasor diagrams. Concretely, the most important aspects of waves and wave superposition can be effectively represented by using phasor diagrams [21, 22]. Phasors are rotating vectors that represent light waves. Thereby, phasor magnitudes correspond to amplitudes of waves. Furthermore, in phasor diagrams, the phase differences between waves are represented as angles between the corresponding phasors. Finding a resultant wave at a certain point of space boils down to the addition of phasors, *i.e.*, vectors. Consequently, phasors help us to explain the occurrence of interference patterns.

2. Research question and research design

In this study, we conducted a pretest-posttest quasi-experiment to determine whether combining external visualizations and extreme case reasoning can help university students to become more successful in understanding wave optics phenomena. In our opinion, the significance of this research is related to the fact that there was no earlier research on the pedagogic potentials of using extreme case reasoning in wave optics instruction. For example, in this paper, it is shown how extreme case reasoning may be applied for purposes of explaining some relatively complex relationships such as the relationship between the number of slits and width of fringes. Finally, the significance of this study also stems from the fact that it offers some new conceptual questions and describes misconceptions and p-prims that were not reported in earlier research.

3. Methods and materials

3.1. Participants and curriculum

This study included 179 first-year students (mostly 19-year-olds) from the Faculty of Chemical Engineering and Technology in Zagreb. Students were enrolled in a typical two-semester introductory physics course for scientists and engineers. This course consists of two hours of lectures and two hours of seminars per week. In general, teaching approaches in this course can be referred to as traditional, *i.e.*, the lectures emphasize the transfer of information and providing proof for the most important equations, while the emphasis in seminars is on solving quantitative physics problems.

We divided the total sample of students into 4 subgroups. Two subgroups received a traditional treatment while the remaining two received the experimental treatment. The number of students in each subgroup was not greater than 47. The gender distribution in all subgroups was approximately equal, and every subgroup had a higher proportion of female students (71%: 29%).

Before their university education, students from our sample already had the opportunity to learn about basic concepts of wave optics in their high-school education.

3.2. Treatment

Our research was conducted within the regular introductory physics seminars. Before receiving the treatment in seminars, the students from all four subgroups had the same traditional lectures on wave optics. At the seminars, two subgroups received traditional treatment that is characterized by discussion and solving of quantitative physics problems. On the other hand, in the two remaining subgroups, the traditional approach was enriched by the use of extreme case reasoning and visualizations which were designed to help the students to comprehend the abstract mechanisms that are at the core of wave optics phenomena. The same concepts were covered in all subgroups. Also, the seminars in all subgroups were led by the same teaching assistant, and the teaching treatment lasted for 90 minutes.

Traditional treatment was based on summarizing and applying the most important principles that had been covered in lectures. The teaching assistant solved the quantitative problems on the blackboard, whereby the solving process was accompanied by classroom discussion. In the experimental subgroups, students solved two numerical problems less (Item 1 and Item 6) than in the control subgroups because considerable time was devoted to visualizations and extreme case reasoning. Problems were selected to include the following important phenomena: interference on the double-slit and optical grid (Table I). In the experimental subgroups, before solving quantitative problems, the teaching assistant presented light interference and diffraction through external visualizations.

Explanation of the phase concept was influenced by didactic approaches in German grammar school books such as Metzler Physik and Dorn-Bader Physik [23, 24]. In electromagnetic waves, the electric field vector oscillates over time and space. The different oscillatory states of the electric field represent different phases. These phases can be effectively visualized by a rotating vector called phasor (Fig. 1a). As

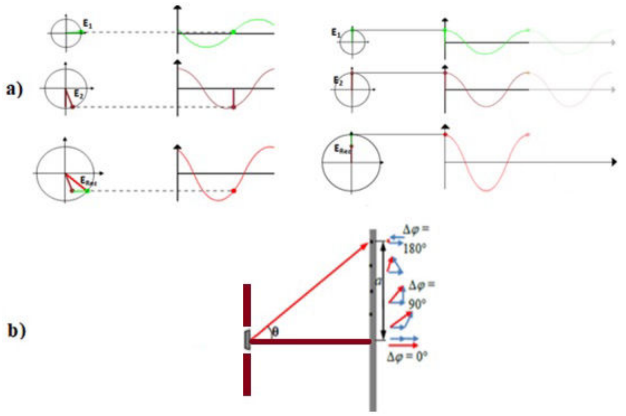

FIGURE 1. Absolute constructive and absolute destructive interference are only extreme cases. The superposition of waves results in a continuous distribution of light along with the screen; as the diffraction angle increases, the phase angle also increases, and the resultant electric field decreases.

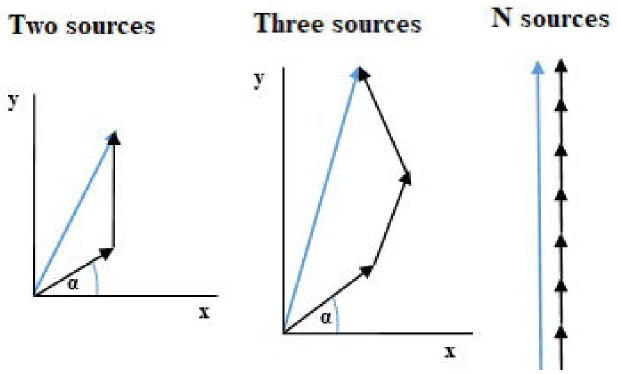

FIGURE 2. The amplitude of the resultant electric field vector increases with the number of sources.

earlier stated, the phasor length corresponds to the amplitude of the electric field vector. Furthermore, the angle that the phasor closes with the x-axis at some instant t is called a phase angle and determines the state of oscillation of the electric field vector at instant t. The teaching assistant visually presented to students an example showing the motion of the rotating vector over time. He pointed out how the y-component of the rotating vector changes over time according to the sinusoidal law. In the other part of the seminar, the teaching assistant described the Young's experiment. Students observed visualization of the interference fringes on the screen, where constructive and destructive interference has been explained by the difference in optical path lengths of the waves originating from the slits. This was illustrated with phasors. Two coherent light waves were presented by phasors, and students observed in visualizations how the sum of these two phasors affected the irradiance at various points of the screen (Fig. 1b). The two blue-colored phasors from Fig. 1b correspond to waves that originate from two sources (*i.e.*, from the two slits), while the resultant phasor is represented by the red-colored vector. If the phase difference between vectors is zero, then their superposition results in maximal amplitude of the resultant wave, *i.e.* a maximum is observed. On the other hand, if the phase difference is 180 degrees, then the amplitude of the resultant wave is zero, and a minimum is observed. Of course, when the phase difference for the two waves is between 0 and 180 degrees, the amplitude of the resultant wave is between zero and maximum value. The students were guided to mentally simulate how the continuous change from one extreme case (zero phase difference) to the other extreme case (phase difference equal to π), results in a continuous change of light intensity across the screen. This could potentially help the student to change the misconception according to which the distribution of light intensity on the screen is discrete, *i.e.*, only places of maximal constructive and maximal destructive interference are distinguished [25].

The next topic was an optical grid where students could see the interference pattern on the screen, as well as the explanation of the obtained pattern in the phasors approach. Increasing the number of slits increases the number of waves/phasors that superimpose on the screen. This results in obtaining brighter maxima; that is, the intensity of maxima increases (Fig. 2). The minima are generated when the sum of phasors is equal to zero, that is, when the phasors form a closed polygon. Our visualization shows that in the case when we increase the number of phasors, (*i.e.*, slits), the angle between the successive phasors, for which the first minimum is obtained, becomes smaller. After students were guided to see that a smaller phase angle corresponds to a smaller diffraction angle. Consequently, they concluded that increasing the number of slits influences the phase angle (for which the first minimum is obtained) to become smaller which means that interference fringes become narrower (see Fig. 3). In the extreme case, when the number of slits is very

TABLE I. Brief description of items that were solved at the seminar. The asterisk stands for items that were solved only in control subgroups.

*Item 1	Item 2	Item 3	Item 4	Item 5	*Item 6
Calculating the wavelength of light used in Young's experiment.	Calculating the wavelength of light and diffraction angle in Young's experiment.	Calculating the separation between two light sources used for creation of intereference pattern.	Calculating the constant of optical grid.	Calculating the number of lines per mm for optical grid.	Calculating the diffraction angle for certain maxima $(m = 1, 2, 3)$ for optical grid.
Open-ended	Open-ended	Open-ended	Open-ended	Open-ended	Open-ended

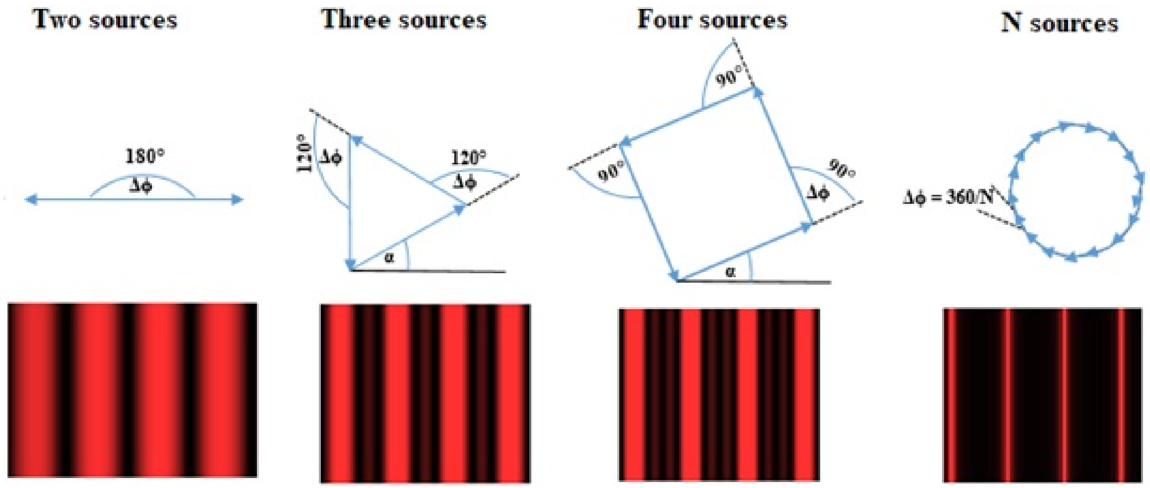

FIGURE 3. Angles between consecutive phasors that form the first minimum become smaller, making the lines narrower when we increase the number of sources.

large, as it is the case in optical grids, the fringes become very sharp and narrow, which means that interference fringes become narrower (see Fig. 3). In the extreme case, when the number of slits is very large, as it is the case in optical grids, the fringes become very sharp and narrow.

In the last part of our experimental seminars, students watched an additional educational video in which wavefronts were used for explaining what happens when waves encounter a single-slit or a double-slit. For the single-slit, it has been only shown why waves, unlike large particles, can reach the region of geometric shadow, while for the double-slit experiment, students could observe how the superposition of secondary waves gives rise to dark and bright interference fringes. They were also shown how changing the slit separation affects the superposition of the two waves visualized by two overlapping wavefront representations.

After visually representing wave phenomena and introducing students with analogies and extreme cases (which lasted 20 minutes), the assistant began the problem-solving session. In this process, the assistant encouraged the students to use the previously introduced visual models for purposes of reasoning about phenomena described in the given quantitative problems.

3.3. Assessment instrument

To reduce the risk of compromising internal validity due to potential interaction between pre-test and teaching treatments, we decided to use different instruments for pretest and posttest [26].

However, in both, the pretest and posttest, we measured a conceptual understanding of wave optics phenomena with a focus on interference and diffraction of light. In most items, students needed to use knowledge from the wave optics domain to interpret, explain and predict certain phenomena. According to Michael and Modell, providing accurate predictions of scientific phenomena and processes is a very good indicator for understanding a certain scientific content [27].

TABLE II. Brief description of pretest items.

Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
What is the difference between phases of the wave at two different points in space? (wavefront representation)	What is the difference between phases of the wave at two different points in space? (sinusoidal representation)	How number of slits influences the diffraction pattern?	Phase difference between two coherent waves based on given path-length difference	Nature of interference of two coherent waves at a certain position, based on given path-length difference	How change of light color influences appearance of the pattern in a double-slit experiment?
Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice

TABLE III. Brief description of pretest items.

Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
What is the phase difference between the two points? (sinusoidal representation)	How covering of upper half of the slits with opaque material influences the interference pattern?	What happens to the diffraction pattern if we replace a single slit with a circular aperture?	How adding a third slit influences interference pattern?	How rotation of slits by 90° influences the interference pattern?	How screen-slits distance influences the diffraction pattern?	How number of slits influences the diffraction pattern?	How distance between the slits affects the interference pattern in a double-slit experiment?
Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice	Multiple-choice

Consequently, our test items were designed to require a transfer of knowledge, *i.e.*, they included situations that were not explicitly covered in instruction. Besides, we wanted our items also to be effective when it comes to uncovering typical student misconceptions.

The pretest and posttest versions of the Basic Understanding of Wave Optics Survey (BUWOS) consist of six and eight items, respectively. Each correctly solved item from the pretest and posttest was awarded one point. In Table II and Table III, we provide a brief description of our test items.

Based on posttest data, we calculated the point-biserial coefficients for our items. Taking into account that the point-biserial coefficient for Item 4 was negative, this item has been excluded from the BUWOS scale but retained for purposes of individual item analyses. After Item 4 has been excluded, Cronbach's alpha coefficient for our instrument was calculated. It amounted to 0.48, which is relatively low but higher, then value that is considered to be acceptable in the research by McKagan, Perkins, and Wieman [28]. This could be because questions from BUWOS were primarily designed to activate misconceptions, which are often mutually inconsistent, leading to a relatively low degree of internal consistency of the instrument [30]. Because of the relatively low reliability of our instrument, we decided to strengthen our evidence about between-treatment differences by also providing item-level analyzes. The average difficulty index for our posttest items amounted to 0.49, which is near the optimal value [29]. On the posttest, Items 7 and 8 proved to be very demanding with difficulty indices of 0.27, and 0.29, respectively. When it comes to the pretest, its average difficulty index was 0.27 and the most difficult items were Item 5 and Item 6 with difficulty indices of 0.02 and 0.09, respectively. In the posttest context, all the item difficulty indices were in the desired range from 0.2 to 0.8.

3.4. Research design

For investigating the effectiveness of our experimental teaching method, we used the pretest-posttest quasi-experimental design. The students from the two control subgroups received traditional treatment characterized by discussing and solving quantitative problems. On the other hand, students from the two experimental subgroups revisited the wave optics concepts through extreme case reasoning and visualizations, before proceeding with solving quantitative problems. Students from all subgroups wrote the pretest one week before the treatment, and they wrote a posttest immediately after the treatment. For conducting the pretest and posttest, we allocated 20 minutes of time.

4. Results and discussion

In this chapter, we will first present the scores of the control and experimental subgroups on the pretest and the posttest. Then we will merge the individual subgroups into one control group and one experimental group for purposes of testing for the significance of between-treatment differences. To that end, the analysis of covariance (ANCOVA) will be used. Finally, we will attempt to identify and discuss the most difficult items and most common students' errors at the pretest and posttest.

4.1. Pretest and posttest scores across subgroups

From Table IV, it is evident that at the pretest, the CG2 control subgroup was the most successful, and the CG1 control subgroup was least successful. On average, the control subgroups were slightly more successful than experimental subgroups. When it comes to the posttest, the results clearly

TABLE IV. Average pretest and posttest scores for students from experimental subgroups (EG) and control subgroups (CG). Theoretically, the pretest scale ranges from 0 to 6, while the posttest scale ranges from 0 to 7. Standard deviations are shown in brackets.

	EG1 2	EG2	CG1	CG2
Pretest	1.72	1.46	1.23	2.02
	(1.00)	(1.08)	(0.88)	(1.13)
Posttest	3.79	4.44	2.76	2.64
	(1.89)	(1.24)	(1.25)	(1.32)

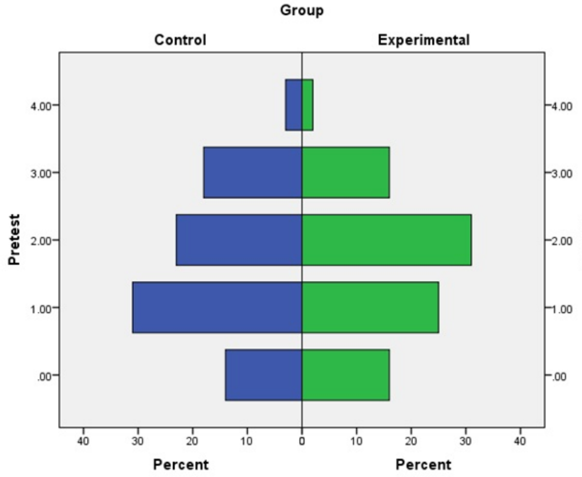

FIGURE 4. Distribution of pretest scores for the experimental and control group. The pretest scale ranges from 0 to 6.

show that students from experimental subgroups EG1 and EG2 scored higher than students from control subgroups CG1 and CG2.

Considering the consistency of between-treatment differences across all our subgroups, we decided to merge the individual subgroups into one control and one experimental group.

4.2. Between-group differences in score distributions

Figure 4 shows the distribution of pretest scores in the control and experimental group. The most common result in both groups is two points out of six points for the experimental group and one point out of six points for the control group. Although the distribution shape is relatively similar in both groups, it is possible to see that the control group has a lower share of students with zero points and a greater share of the students with three or four points. Furthermore, the percentage of students who scored two points or lower in the control group was 76.3%, while for the experimental group, there were 80% such students.

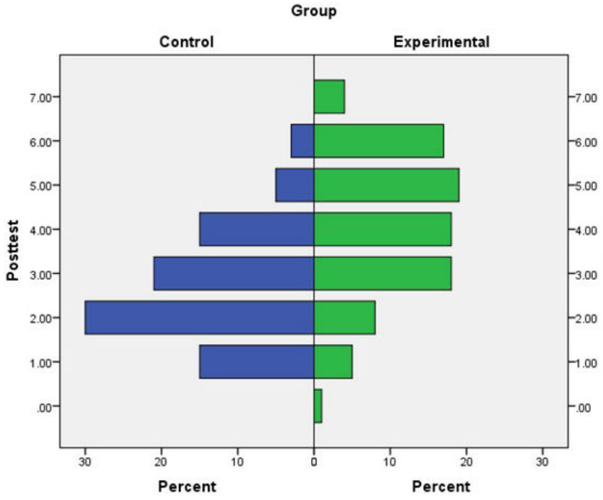

FIGURE 5. Distribution of posttest scores for the experimental and control group. The posttest scale ranges from 0 to 7.

TABLE V. Average pretest and posttest scores for the experimental and control group. Theoretically, the pretest scale ranges from 0 to 6, and the posttest scale ranges from 0 to 7. Standard deviations are given in brackets.

	Pretest	Posttest
Control group	1.60	2.70
	(1.08)	(1.28)
Experimental group	1.58	4.13
	(1.04)	(1.61)

Figure 5 shows the distribution of posttest scores for the experimental and control group. We can notice that the distribution for students from the experimental group is shifted towards higher scores. Furthermore, the percentage of students who scored 2 points or less at the posttest is 50.6% for the control group compared to 15.6% in the experimental group.

Table V summarizes between-group differences at the pretest and posttest.

From Table V, we can conclude that, at the pretest, the control group and experimental group scored 26.6% and 26.3%, respectively. At the posttest, the control group scored 38.5%, while the experimental group scored 59%.

First of all, it is important to note that pretest scores were very low in both groups although the pretest questions were aligned with the high-school curriculum in Croatia. This indicates low effectiveness of the achieved high school curriculum in Croatia, at least when it comes to the development of conceptual understanding about wave optics. This could be explained by the fact that teaching in Croatian schools predominantly follows traditional approaches characterized by a passive student role [31]. Another explanation for relatively low achievement at the pretest as well as at the pretest is related to the fact that wave optics content is intrinsically complex and demanding even for top-performing students [4, 5].

Pretest results also indicate that, before participating in our treatment, students from our sample had a very low level of conceptual understanding of wave optics, which means that the post-treatment level of knowledge largely relates to the effects of the seminar.

4.3. Investigating the significance of the observed between-group differences

We investigated the between-group differences on the posttest by running an analysis of covariance (ANCOVA) that allowed us to control for between-group differences on the pretest [32]. Before conducting ANCOVA, we first checked whether the assumption of independence between covariate (result at the pretest) and treatment variable (group) was met. Thereby, we could show that the between-group differences at the pretest were not significant (t (177) = 0.112, p = 0.91).

Furthermore, the interaction between covariate and treatment variable was also not statistically significant (F (1, 175) = 0.002, p = 0.96). A visual examination of the Q-Q graphs for the control and experimental group led us to the conclusion that our data approximately satisfy the normality assumption [33]. Finally, an inspection of Leven's statistics (F (1, 177) = 0.47, p = 0.49) showed that the assumption of homogeneity of variance was met, too.

The results of ANCOVA indicate that between-group differences in posttest are statistically significant (F (1, 176) = 41.01, $p < 0.001$, partial $\eta_2 = 0.19$), after controlling for between-group differences at pretest. From the given results, we can conclude that students from the experimental group significantly outperformed their peers from the control group.

The main difference between the two teaching approaches was that the students from the experimental groups not only solved and discussed quantitative problems in their seminars but also developed visual mental models about the superposition of light waves, which is at the heart of generating interference and diffraction patterns [5]. According to Greca and Moreira, to develop a deep conceptual understanding of physical phenomena and processes, it is important to develop appropriate internal visualizations [34]. Students from experimental subgroups had the opportunity to observe and discuss external visualizations of wave optics phenomena, thereby using a language that goes beyond the language of mathematics. On the other hand, the discussions in control subgroups were mostly anchored in formal, mathematical contexts which once again proved to be relatively ineffective when it comes to developing deep conceptual understanding of physics [35].

Our results are consistent with the idea that visualizations and reasoning about extreme cases help students to create vivid intuitive mental models about physical phenomena [14,36,37-41].

4.4. Between-group differences on individual items

Taking into account the relatively low reliability of our assessment instrument it is very useful to enrich our discussion of between-group differences by analyzing student achievement at the level of individual items. Table VIII shows a summary of between-group differences in individual posttest items.

From Table VI, we can see that students from the experimental group outperformed their colleagues from the control group on items 1, 2, 3, 5, 6, 7, 8. The highest differences in favor of the experimental group were observed on items 3, 5, and 8. In these items, between-group differences in percentages of correct answers amounted to 30%, 22%, and 23%, respectively. In Item 3 students were expected to predict how replacing a single slit with a small circular aperture would affect the diffraction pattern. The circular aperture could be "simulated" by rotating the slit through all angles from 0 to 2π, which would result in rotating the single slit pattern through all the different angles from 0 to 2π. If we mentally merged all the individual single slit patterns, the result would be a two-dimensional symmetric pattern consisting of concentric rings. In the experimental subgroups, the students trained to perform mental simulations and extreme case reasoning, which could have helped them in arriving at the correct answer. In Item 5, students were expected to predict how rotating the double-slit by 90° would influence the interference pattern. Having developed a visual model about how secondary waves originate at the slit and superimpose on the screen to generate the interference pattern, students from the experimental subgroups could transfer that kind of thinking to the described situation and arrive at the correct answer (*i.e.*, now the maxima and minima would be places along the vertical direction). Finally, in Item 8, students were shown two interference images formed by passing monochromatic light through the original and modified experimental setup of Young's double-slit experiment, and students were required to recognize what had happened to the original setting in two experimental setups. The students were expected to recognize that, for the second setup, the fringes were wider. In the experimental subgroups, students trained to mentally simulate how changing slit separation affects the overlapping of wavefronts that correspond to the two waves that originate from the double slit.

Large differences in favor of the experimental group were also detected in many other posttest items. One such item was Item 7 that required the students to use knowledge of the relationship between the number of slits and width of fringes.

4.5. Students' misconceptions and conceptual change

The most frequently chosen distractors on the pretest and posttest are shown in Table VII and Table VIII, respectively. At pretest, the structure of students' answers was very similar across all subgroups, which is the reason why we decided to merge all pretest data and discuss pre-treatment misconceptions at the level of the whole student sample (Table VII). Post-treatment misconceptions have been reported separately

TABLE VI. Proportion of correct answers on individual posttest items. Standard deviations are given in brackets.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
Control group	0.36	0.39	0.57	0.39	0.61	0.41	0.16	0.18
	(0.48)	(0.48)	(0.49)	(0.49)	(0.48)	(0.49)	(0.37)	(0.38)
Experimental group	0.55	0.62	0.87	0.28	0.83	0.46	0.36	0.41
	(0.49)	(0.47)	(0.32)	(0.45)	(0.37)	(0.50)	(0.48)	(0.49)

TABLE VII. Most frequently chosen distractors at the pretest.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
Pretest (overall)	B (60%)	A (33%)	C (23%9)	B (58%)	C (36%)	B (22%)

for the experimental and control group. However, from Table VIII, it is evident that even after the treatment, students from both groups share similar misconceptions on most items.

Our discussion of most frequent misconceptions at pretest and posttest will be framed through three themes (1.phase concept; 2. two-source interference; 3. single slit diffraction, and optical grid).

1. Phase concept

Pretest items 1, 2, and 4, as well as posttest item 1, were designed for assessing students' understanding of the phase concept. In Item 1 of the pretest, students were expected to find the phase difference between two points in space based on the wavefront representation of a plane wave. The same task, only within the context of a sinusoidal representation, was described in pretest Item 2 and posttest Item 1. Finally, in pretest Item 4, students were expected to use their knowledge of the relationship between phase difference and path length difference to find the phase difference of two waves at a certain point of space.

A. Pretest

In Item 1, 60% of students chose distractor B which reflects the idea that the phase difference between two adjacent wavefronts amounts to π. Interestingly the same misconception was detected in students' answers to Item 2, only within the context of a sinusoidal representation of the light wave. Here 33% of students chose distractor A which reflects the idea that the phase difference between two adjacent "crests" of the wave amounts to π instead of 2π. Generally, it seems that, for students it makes more sense that the path length difference of one λ is related to a phase difference of one π rather than two π. Finally, pretest Item 4 referred to two point sources of monochromatic waves that were mutually coherent and separated by $\lambda/2$. Students were expected to reason about the phase difference of these two waves at position 1, which is at equal distance from both sources. The most frequently chosen distractor B (58%) says that the two waves arrive at position 1 in counter-phase (phase difference π). In traditional wave optics instruction, students are used to automatically relate halves of wavelengths with minima, and that is probably the reason why they associated the separation between sources with the occurrence of counter-phase in position 1.

B. Posttest

Item 1 from the posttest is very similar to Item 2 from the pretest; in both items, students were expected to determine the phase difference between points A and B of space, for a monochromatic wave that is represented by a sinusoid. The most common misconception in both groups was A (Control group - 58%, Experimental group - 42%,), which again reflects the idea that the phase difference between adjacent "crests" of the wave amounts to π.

2. Two source interference

In Item 6 of the pretest and items 2, 4, 5, and 8 of the posttest, students were expected to predict or explain how certain changes of the double-slit setup affect the appearance of the interference pattern. Item 5 from the pretest covers the phenomenon of two source interference, too.

A. Pretest

Item 5 refers to two point sources of monochromatic waves that are coherent with each other and separated by $\lambda/2$. In this item, students were required to answer how the given two waves interfere at a point that is a whole number of wavelengths away from the midpoint between the two sources. The observed point lies on the same line as the sources. The most frequently chosen distractor was C (36%). Distractor C says that neither constructive nor destructive interference occurs at the observed point. This could be related to the fact that now both, halves of wavelengths and the whole number of wavelengths are mentioned, and students cannot decide which association to activate. Earlier research shows that students often mistakenly believe that the mere path length defines wave interference at a particular point rather than the path length difference of the superimposing waves [4].

In Item 6, students were expected to predict how changing the color of laser light from red to purple would affect the appearance of the double-slit pattern. The most frequently chosen distractor for this item was B (22%) (the distance between adjacent lines will increase). This finding is in line

TABLE VIII. Most frequently chosen distractors at the posttest.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
Posttest (control)	A (58%)	C (26%)	A (38%)	C (29%)	C (18%)	B (28%)	A (31%)	C (40%)
Posttest (experimental)	A (42%)	C (18%)	A (12%)	A (40%)	C (11%)	D (25%)	D (32%)	A (31%)

with the results of previous research, and a possible explanation is that students try to establish an analogy with the dispersion of light through a prism in which the violet light deviates most [25].

B. Posttest

In Item 2, students were expected to predict how covering the upper halves of the slits with opaque material would affect the double-slit interference pattern. The most commonly chosen wrong answer in the control and experimental group was C (26% and 18%). Students believed that the interference fringes would elongate in the vertical direction which reflects the idea that changes of the height of the slit lead to similar effects as changes of the width of the slit, although for the horizontal dimension of the slit diffraction effects are prominent which cannot be said for the vertical dimension. This finding is in line with the results of the study by Mešić, Vidak, Hasović, and Đekić, who showed that students exhibit many difficulties in understanding the role of the vertical dimension of the slit [7].

In Item 4, students were expected to reason about the consequences of adding a third slit to the double-slit mask, whereby the separation between the slits is held constant. In the experimental group, the most frequent misconception was that adding a third slit would not affect the position of maxima and minima, whereas, in the control group, many students believed that at the original position of maxima now minima would appear and vice versa.

In Item 5, students were expected to predict how rotating the double-slit by 90° would affect the appearance of the interference pattern. The most common misconception in both groups was C (18% and 11%), which for the rotated-slits-setup shows a pattern consisting of only two fringes. It seems that many students believe that after rotation of the slits diffraction, effects are not prominent anymore. This could be related to the fact that university students often lack a basic understanding of the Huygens-Fresnel principle [7].

In Item 8, students were shown interference patterns for the original and modified double-slit setup. From the given figures, it could be directly observed that the fringes were wider for the modified setup. The most frequently chosen distractors were C (40%) in the control group and A (31%) in the experimental group. Distractor A reflects the misconception that increasing the width of the slits results in wider fringes, which could be related to students' misapplication of the ray model of light [4]. On the other hand, distractor C is related to the erroneous belief that decreasing the distance between screen and slits results in bigger fringes. This seems to be related to some kind of application of p-prims, such as "the closer to the source (*i.e.*, slits), the effect (*i.e.*, fringes) is bigger" [29].

3. Single slit diffraction and optical grid

In Item 3 of the pretest, as well as in items 3, 6, and 7 of the posttest, students were asked to reason about some characteristics of the diffraction pattern obtained by diffraction on a single slit or optical grid.

A. Pretest

In pretest Item 3, students were shown two optical grid patterns, whereby the pattern for the modified setup was characterized by narrower fringes. Narrower fringes can be obtained by using a grid with larger number of slits. However, many students claimed exactly the opposite by choosing distractor C (23%), which states that for the modified set up a grid with fewer slits was used.

B. Posttest

In Item 3, students were expected to predict how replacing a single slit with a circular aperture would affect the appearance of the diffraction pattern. The most frequently chosen distractor in control and experimental group was A with 38% and 12%, respectively. This distractor reflects the erroneous belief that the shape of the aperture does not affect the diffraction pattern. A possible explanation for such a result is that students from our sample (particularly the control group) are mostly focused on analytic representations of wave optics phenomena, and they did not cover equations for diffraction on circular aperture in their lectures. Students from the experimental subgroups were better prepared for visual reasoning about this situation.

In Item 6, students were required to predict how changing the grating-screen separation affects the appearance of the diffraction pattern. In the control group, students most often chose distractor B (28%), and in the experimental group, students most often chose distractor D (40%). Distractor B reflects the belief that increasing the grating-screen separation results in decreased separation of fringes, while distractor D reflects the erroneous idea that putting the screen farther away will make the fringes narrower. None of the treatments was successful in developing an understanding of the relationship between grating-screen separation and characteristics of the diffraction pattern.

In Item 7, two optical grid patterns were showed and in the pattern, for the modified setup the fringes were wider than in the pattern for the original setup. The most frequently chosen distractor in the control group was A (31%), and in the experimental group, it was D (32%). Similarly, as in the pretest, students from the control group erroneously believed that increasing the number of slits results in wider fringes. On the other hand, students from the experimental group believed that wider fringes are obtained by reducing the slits-screen separation, which is similar to their reasoning pattern for Item 6, as well as to the reasoning of control group students observed in solving Item 8. In all cases, it seems that students' thinking is guided by the p-prim "the closer to the source (*i.e.*, slits), the effect (*i.e.*, fringes) is bigger" [29].

F. Limitations of the study

The main limitation of this study is related to the relatively low reliability of the applied assessment instrument. A consequence of the low reliability is that we should be careful in summing raw scores and interpreting results on the test level. Therefore, besides providing results on the level of the whole test, we also provided evidence for between-group differences on the level of individual items. It has been shown that students from the experimental group outperformed their peers from the control group on a large majority of posttest items.

5. Conclusion

The context of wave optics instruction is very important for learning about one of the most important models of classical physics, which is the wave model. In this study, we investigated whether enriching traditional instruction with external visualizations and extreme case reasoning may facilitate the development of understanding about wave optics phenomena in university students.

We came to the following conclusions:

- External visualizations facilitate the development of visually rich internal representations of wave optics phenomena, which are a productive basis for performing mental simulations about interference and diffraction phenomena. Analytic representations are less functional in the context of qualitative conceptual problems.
- Extreme case reasoning is relatively effective when it comes to explaining the relationship between the number of slits and the width of the diffraction fringes. Traditional approaches that are focused on analytic representations fail to provide a “picture” about the mechanisms that relate the number of slits and width of fringes.
- In wave optics, students’ reasoning is often characterized by the use of p-prims. For example, many students use the “closer to the source (*i.e.*, slits) results in a bigger effect (*i.e.*, fringes)” p-prim.
- Students fail to correctly compare processes that happen along the x- and the y-axis of a slit. Consequently, they often fail to correctly predict what would happen if we would rotate the slits by 90°. This is probably related to a lack of basic understanding of the Huygens-Fresnel principle [7].
- Some misconceptions that had been identified in earlier studies were confirmed once again through this study (*e.g.*, “increasing width of slits results in wider fringes”) [4].

In our opinion, many of the observed student difficulties stem from the fact that in traditional wave optics, instruction students fail to develop a good understanding of the Huygens-Fresnel principle. For that reason, our future studies will be directed at designing conceptual approaches directed at developing the skill of using the Huygens-Fresnel principle for solving conceptual problems about light phenomena.

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