Comment on "How to get mechanical work from a capacitor and a couple of batteries" by E.N. Miranda [*Rev. Mex. Fís. E* **52** *(2006) 215*]

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Received 30 November 2018; accepted 12 December 2018

We show that the diagram describing the process to produce mechanical work from the capacitor-batteries system proposed in the reference *Rev. Mex. Fís. E* **52** (2006) 215, is inconsistent with the analytical results obtained in that paper. Thus, the correct diagram and the process to produce mechanical work are presented.

Keywords: Electrical engine; capacitor.

Demostramos que el diagrama que describe el ciclo para generar trabajo mecánico a partir del sistema capacitor-baterias propuesto en la referencia *Rev. Mex. Fís. E* **52** (2006) 215, es inconsitente con los resultados analíticos reportados en dicho trabajo. Así, el ciclo para generar trabajo y el diagrama correcto son presentados.

Descriptores: Máquina eléctrica; capacitor.

PACS: 41.20; 84.32; 84.50 DOI: https://doi.org/10.31349/RevMexFisE.65.101

In Ref. 1, Miranda has analyzed the mechanical work done by a parallel plate capacitor with variable separation. The net work is calculated assuming that the device follows a closed cycle in the charge-voltage space. Thus, an expression for the net work is obtained by the author (Eq. (10) in Ref. 1), concluding that if we want to gain mechanical work the conditions $V_2/x_3 > V_1/x_1$ and $V_2 > V_1$ needs to be fullfilled. These conditions are deduced from the fact that we want a negative work, which represents work done by the system.

Using the same notation than in Ref. 1, the closed cycle considered by the author is the following (see Fig. 1 in [1]):

i) Step I (A→B): The capacitor is expanded at constant charge, from V_1 until it reaches V_2. The gap increases from x_1 to x_2.

ii) Step II (B→C): The capacitor is expanded at constant voltage, from q_1 until it reaches q_3. The gap increases from x_2 to x_3.

iii) Step III (C→D): The capacitor is compressed at constant charge, from V_2 until it reaches V_1. The gap decreases from x_3 to x_4.

iv) Step IV (D→A): The capacitor is compressed at constant voltage, from q_3 until it reaches q_1. The gap decreases from x_4 to x_1.

Thus, the obtained results for the work in each step are the following:

$$W_{A,B} = \frac{\epsilon_0 A V_1^2}{2x_1^2}(x_2 - x_1), \tag{1}$$

$$W_{B,C} = \frac{\epsilon_0 A V_2^2}{2}\left(\frac{1}{x_2} - \frac{1}{x_3}\right), \tag{2}$$

$$W_{C,D} = \frac{\epsilon_0 A V_2^2}{2x_3^2}(x_4 - x_3), \tag{3}$$

$$W_{D,A} = \frac{\epsilon_0 A V_1^2}{2}\left(\frac{1}{x_4} - \frac{1}{x_1}\right). \tag{4}$$

It is important to mention that in the process to reproduce the results reported in Ref. 1 we found that some of the equations where incorrect. Next, we are going to list those errors:

1. In the equation for the energy as a function of q at paragraph four second line, it should be: $E = q^2/2C$.
2. In Eq. (8) the subscript for V is incorrect, the correct one is the Eq. (2) presented above.
3. In Eq. (11) the subscript for q in $q_1 = C_3V_2$ is incorrect, the correct equation should be: $q_3 = C_3V_2$.

In the next, we are going to analyze the relation between the gaps in the different steps, in order to show that the proposed $V - x$ diagram is incorrect:

a) From the condition to get mechanical work from the system, we get that $x_3 < x_1(V_2/V_1)$, but according to Eq. (3) in Ref. 1 $x_2 = x_1(V_2/V_1)$, so we obtain that $x_3 < x_2$.

b) From $x_2 = x_1(V_2/V_1)$ and knowing the implicit condition $V_2 > V_1$, we obtain that $x_2 > x_1$.

c) From the condition to get mechanical work from the system, we get that $x_1 > x_3(V_1/V_2)$, but according to Eq. (3) in Ref. 1 $x_4 = x_3(V_1/V_2)$, so we obtain that $x_1 > x_4$.

d) From $x_4 = x_3(V_1/V_2)$ and knowing the implicit condition $V_2 > V_1$, we obtain that $x_4 < x_3$.

As we can see, from this analysis we conclude that x_4 is the smaller and x_2 the biggest gap in the whole cycle. The ordering of the gaps is easy to understand analyzing the physical meaning of the process described by the cycle depicted in the (q, V) plane as shown in Fig. 2 of Ref. 1. The $V - q$ diagram in Fig. 2 of [1] shows the following process: starting from point A, an increment of voltage at constant charge move the system at point B, then an increment of charge at constant voltage move the system to the point C, subsequently a decrease of the voltage at constant charge move the system at point D, and finally a decrease of charge at constant voltage return the system to the point A. From the formulas for the charge $q = CV$ with $C = \epsilon_0 A/x$, we obtain that $q = \epsilon_0 AV/x$. Thus, from that result we can see that for the increment of charge from the point B to point C, a reduction of the gap (x) is needed. However, in the Fig. 1 of Ref. 1 an increment of the gap (x) is shown, in clear contradiction with the analytical results.

From the stated conditions to get mechanical work from the system and the Eq. (3) in [1], we obtain that $q_3 > q_1$. This result is implicit in Fig. 2 of Ref. 1. Also it is important to mention that the slope of the Steps I and III in the $V - x$ diagram are proportional to q_1 and q_3, respectively. Thus, after the analysis presented above, we can construct the correct $V - x$ diagram in order to get mechanical work from the system, as shown in Fig. 1 of this comment.

In conclusion, the correct process to produce work from the capacitor-batteries system is as follow:

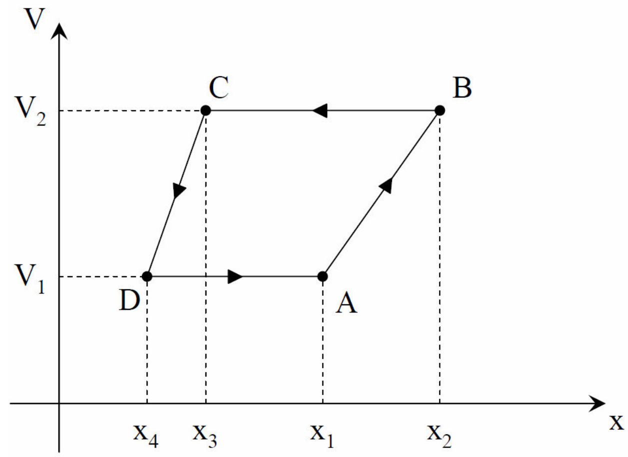

FIGURE 1. Voltage-gap $(V - x)$ diagram for the closed cycle (see text).

i) Step I (A→B): The capacitor is expanded at constant charge, from V_1 until it reaches V_2. The gap increases from x_1 to x_2.

ii) Step II (B→C): The capacitor is compressed at constant voltage, from q_1 until it reaches q_3. The gap decreases from x_2 to x_3.

iii) Step III (C→D): The capacitor is compressed at constant charge, from V_2 until it reaches V_1. The gap decreases from x_3 to x_4.

iv) Step IV (D→A): The capacitor is expanded at constant voltage, from q_3 until it reaches q_1. The gap increases from x_4 to x_1.

Acknowledgments

One of the authors R. de Coss-Martínez gratefully acknowledges a student fellowship from the “Programa de Becas de Excelencia” of the Secretaría de Educación del Gobierno del Estado de Yucatán and Telmex Foundation. One of the authors C. Valez-Pinzon gratefully acknowledges a student fellowship from the Consejo Nacional de Ciencia y Tecnología (CONACyT, México).

1. E.N. Miranda, *Rev. Mex. Fís. E* **52** (2006) 215.