The quantum beam splitter revisited without a vacuum state

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In this article we explain in a new light two fundamental concepts of quantum optics, the quantum beam splitter and the quantum interferometer, in terms of two state quantum wave functions. This method is consistent with the concept of entanglement, and hence the algebra needed to describe them is reduced to additions and products of the components of the quantum states. Furthermore, under the premises of this method it is possible to study quantum states of greater complexity, like those arising from the addition and products of single photon states.

Keywords: Quantum beam splitter; quantum interference; entanglement.

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1. Introduction

The beam splitter is an important element in quantum optics. They play an important role in linear optics quantum computing, where the transmission of wave packets through beam splitters is usually described by unitary transformation matrices. Through the experience of teaching introductory quantum optics to undergraduates we have noticed that students usually have problems understanding the role of vacuum states which are necessary in several quantum concepts and experiments such as the Cassimir effect, Lamb's displacement, orthonormality of quantum coherent states, and vacuum noise and its distortion. At the most fundamental level two examples stand out due to the numerous questions raised by the students, namely *the quantum beam splitter* and *the quantum interferometer*. Most introductory courses on quantum optics [1, 2] acquaint the students with these subjects by invoking the zero photon state, *i.e.* the vacuum state $|0\rangle$. The reasoning behind it is that the algebra of the ladder operators does not preserve the usual properties. In other words, the mathematical transformation imposed by the operator, for example the beam splitter, is not unitary. And hence, it is necessary to introduce the extra state that allows to preserve the transformation.This state is necessary to have an unitary transformation, ensure the conservation of energy and of the number of photons.

For this reason in this article, we endeavor to describe and discuss the interaction between single photons and linear mediums, like the beam splitter, based only on simple algebraic operations of the quantum components of single photon states without the need to invoke the creation and annihilation operators, although following a rigorous mathematical analysis [1–3]. In Sec. 2 we will describe the quantum beam splitter in terms of two quantum components. In Sec. 3 we will employ our mathematical description to study the single photon interference in a Mach-Zehnder inteferometer (MZI). In Secs. 4 and 5 we will study how to use this tools to outline the concept of entanglement and polarization entangled states, respectively. Finally in Sec. 6 we present our conclusions.

2. The quantum beam splitter

In order to develop some intuition of our method we begin by studying the case of two quantum states in a beam splitter. When a photon goes through a beam splitter its output can be described as follows, see Fig. 1,

$$|\Psi\rangle_e = \frac{1}{\sqrt{2}}\left(|\Psi\rangle^T + i\,|\Psi\rangle^R\right), \tag{1}$$

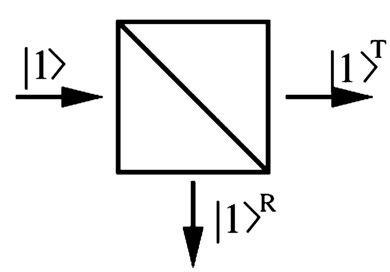

FIGURE 1. Possible outputs resulting from the interaction between a single photon and a quantum beam splitter. When the beam splitter is 50:50, in other words the probability that the photon is transmitted or reflected is 50%.

where the superindexes T and R indicate the transmitted and reflected outputs respectively, which are sufficient to denote the direction of each possible outcome and $i = \sqrt{-1}$ indicates the phase change $e^{i(\pi/2)}$ due to reflection.

Equation (1) can be rewritten in terms of the single incident photon that enters the beam splitter on port 1, $|\Psi\rangle^1 = |1\rangle$ as:

$$|\Psi\rangle_e = \frac{1}{\sqrt{2}}\left(|1\rangle^T + i\,|1\rangle^R\right), \tag{2}$$

Equation (2) indicates that the probability of the photon being reflected or transmitted is the same *i.e.* 50%. This is the case with two paths where each one has associated a probability amplitude, whose implications are well discussed by Feynman [10]. We will denote this state as *wave state*, and we will use it to identify the linear combination of states. It is important to remark that if we put a detector at each output we would observe anticorrelation, [4–6], meaning that the incident photon will be either reflected or transmitted, but not both.

3. Quantum interference

We consider now a Mach-Zender interferometer to analyze the case of interference with single photons, see Fig. 2. Here a phase control, θ, has been added to modify the phase of the photon traveling along the arm T. In this particular setup there are two beam splitters, each of them can be described in terms of Eq. (1). A single photon $|\Psi\rangle^1 = |1\rangle$ enters to the first beam splitter. Notice that this implies that the inputs at the second beam splitter are of the same form as Eq. (2), we only add the change in phase. Therefore the output of the second beam splitter, $|\Psi\rangle_s$, can be written recalling Eq. (1) as:

$$|\Psi\rangle_s = \frac{1}{\sqrt{2}}\Bigg(\frac{1}{\sqrt{2}}\left[e^{i\theta}\,|1\rangle_t^T + i\,|1\rangle_t^R\right] + \frac{i}{\sqrt{2}}\left[e^{i\theta}\,|1\rangle_r^T + i\,|1\rangle_r^R\right]\Bigg), \tag{3}$$

where the index t and r denote the transmitted and reflected paths of the second beam splitter.

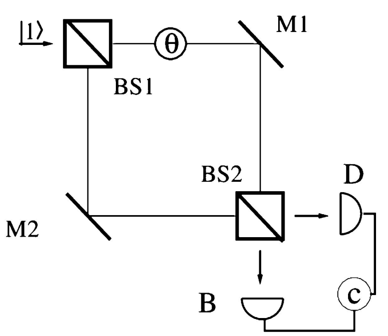

FIGURE 2. Sketch of the Mach-Zehnder interferometer. A single photon state, $|1\rangle$ is incident at the first beam splitter, BS1. The output state $|\Psi\rangle_e$ is then taken to a second beam splitter, BS2, where its output is later analyzed and correlated by detectors D, B, and correlator C. M1 and M2 are the mirrors.

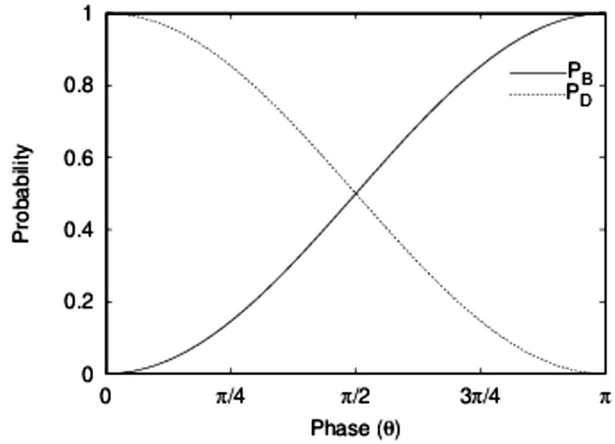

FIGURE 3. Probability vs phase angle of the two exits from the Mach-Zender (B and D), after a single photon enters through the input 1.

Rearranging the terms of Eq. (3), we can define two wave equations, the reflected (right output),

$$|\Psi\rangle_B = \frac{1}{2}\left(e^{i\theta}|1\rangle_t^T - |1\rangle_r^R\right) = \frac{1}{2}\left(e^{i\theta} - 1\right)|1\rangle_B, \tag{4}$$

$$|\Psi\rangle_D = \frac{i}{2}\left(e^{i\theta}|1\rangle_r^T + |1\rangle_t^R\right) = \frac{i}{2}\left(e^{i\theta} + 1\right)|1\rangle_D. \tag{5}$$

Thus, the probability that the photon takes either the B or D output are given by:

$$P_B = \left|\frac{1}{2}\left(e^{i\theta} - 1\right)\right|^2 = \frac{1}{2}\left(1 - \cos(\theta)\right), \tag{6}$$

$$P_D = \left|\frac{i}{2}\left(e^{i\theta} + 1\right)\right|^2 = \frac{1}{2}\left(1 + \cos(\theta)\right). \tag{7}$$

In Fig. 3 the probability as function of the phase is presented for both exits B and D.We observe that the phase can determine the exit of the photon. When $\theta = 0$ the photon will exit though D port, when $\theta = \pi/2$ the photon has equal probability to exit through either port, and when $\theta = \pi$ the photon will exit through B port.

Now, lets consider the case of one photon incident in each input of the beam splitter as shown in Fig. 4. Assuming both states arrive to the beam splitter simultaneously, it is possible to write the output states as the product of two states described by Eq. (8).

$$|\Phi\rangle_s = \frac{1}{2}\left(|1\rangle^{T_1} + i|1\rangle^{R_1}\right)\left(|1\rangle^{T_2} + i|1\rangle^{R_2}\right), \tag{8}$$

expanding the product above yields:

$$|\Phi\rangle_s = \frac{1}{2}\Bigg(|1\rangle^{T_1}|1\rangle^{T_2} + i|1\rangle^{T_1}|1\rangle^{R_2} + i|1\rangle^{R_1}|1\rangle^{T_2} - |1\rangle^{R_1}|1\rangle^{R_2}\Bigg). \tag{9}$$

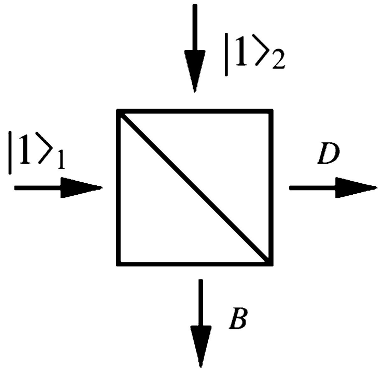

FIGURE 4. Two identical photons enter each at each of the inputs of a beam splitter. Observe that we distinguish them by labeling them with the numbers 1 and 2 respectively

Given that the first and fourth terms are indistinguishable and they have opposite signs, the equation above is reduced to:

$$|\Phi\rangle_s = \frac{i}{2}\left(|1\rangle^{T_1}|1\rangle^{R_2} + |1\rangle^{R_1}|1\rangle^{T_2}\right), \tag{10}$$

translating it to the propagation directions D and B, and recalling that single photon states are indistinguishable, the above equations is reduced to:

$$|\Phi\rangle_s = \frac{i}{2}\left(|2\rangle_B + |2\rangle_D\right), \tag{11}$$

which can be interpreted in a similar way as Eq. (1). This would imply that the state in Eq. (11) refers to the case where two photons exit through either path D or B with a 50% probability. However, if we calculate the probability of having the photon pair exiting through each port we get,

$$|\Phi\rangle_B = |\Psi\rangle_D = \left|\frac{i}{2}\right|^2 = \frac{1}{4}. \tag{12}$$

Hence, we conclude that the output state in Eq. (11) is not normalized. Therefore, we introduce a normalization factor to fix this problem defined as:

$$|n\rangle \to \sqrt{n!}|n\rangle, \tag{13}$$

that

$$|n\rangle|n\rangle \to \sqrt{n!}|n\rangle. \tag{14}$$

By using this factor into Eq. (11), this turns into:

$$|\Phi\rangle_s = \frac{i}{\sqrt{2}}\left(|2\rangle_B + |2\rangle_D\right), \tag{15}$$

and hence the probability that the photon pair exits any output in Eq. (15) is the expected 50%.

It is worth noting that any normalization factor introduced should be applied to both the initial and final states.

Therefore, in the case of an arbitrary number of indistinguishable photons, n_1 and n_2, respectively, being incident at each port of the beam splitter, the initial normalization should be $\sqrt{n1!}\sqrt{n2!}$. Lets test this normalization factor for the case where two states of two photons each are incident at the input ports of the beam splitter. Without normalization we have that:

$$|2\rangle_1|2\rangle_2 \to \frac{1}{4}\Big(\left[\left\{|1\rangle^{T1} + i|1\rangle^{R1}\right\}\right]^2 \times \left[\left\{|1\rangle^{T2} + i|1\rangle^{R2}\right\}\right]^2\Big), \tag{16}$$

and hence the output state is:

$$|\Phi'\rangle_s = -\frac{1}{4}\left(|4\rangle_B + |4\rangle_D - 2|2\rangle_B|2\rangle_D\right). \tag{17}$$

If instead we use the normalization mentioned earlier, we obtain:

$$|\Phi'\rangle_s = -\frac{1}{2}\times\frac{1}{4}\left(\sqrt{24}|4\rangle_B + \sqrt{24}|4\rangle_D + 4|2\rangle_B|2\rangle_D\right), \tag{18}$$

where the first factor is the inverse of the initial normalization, thus:

$$|\Phi'\rangle_s = -\sqrt{\frac{3}{8}}\left(|4\rangle_B + |4\rangle_D\right) - \frac{1}{2}|2\rangle_B|2\rangle_D, \tag{19}$$

which is the correctly normalized state [11]. A careful reading of Eq. (19) shows that the quantum interference destroys the output combinations $|1\rangle_B|3\rangle_D$ and $|3\rangle_B|1\rangle_D$.

Let us summarize the algebraic rules concerning the quantum beam splitter:

a) Every photon that enters the beam splitter has one output state that is the the lineal combination of the transmitted and reflected quantum components.

b) If the initial state has more than one incident photon, that enter simultaneously and with the same phase to the beam splitter, the output state is the product of the *wave states* of every photon. In the event of photons entering the beam splitter with some random delay, then the output state will be a lineal combination of each output *wave state*.

c) Once the products and additions of the output states are calculated, then they can be normalized by means of Eq. (13), if, and only if, the states are indistinguishable; otherwise, each distinguishable group should be treated separately.

3.1. Interference with an input *wave state*

By virtue of symmetry it is easy to foresee that if one was to change the initial entrance port in the single photon interference experiment, then the above discussion will remain valid. Moreover, it would have the same mathematical description. However, when instead of a single photon we introduce a *wave state*, such as:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle^1 + |1\rangle^2\right), \tag{20}$$

where the superindex denotes the incidence port in the first beam splitter. After the first beam splitter the intermediate state, $|\psi_e\rangle$, is:

$$|\psi_e\rangle_D = \frac{1}{2}\left(|1\rangle^{1T} + i|1\rangle^{2R}\right), \tag{21}$$

$$|\psi_e\rangle_B = \frac{1}{2}\left(i|1\rangle^{1R} + |1\rangle^{2T}\right). \tag{22}$$

In the output of the second beam splitter $|\psi_s\rangle$, we obtain:

$$|\psi_s\rangle = \frac{1}{2\sqrt{2}}\Big(e^{i\theta}\left(|1\rangle_t^{1T} + i|1\rangle_t^{2R} + i|1\rangle_r^{1T} - |1\rangle_r^{2R}\right) + i|1\rangle_t^{1R} + |1\rangle_t^{2T} - |1\rangle_r^{1R} + i|1\rangle_r^{2T})\Big). \tag{23}$$

Similar to the previous cases, the labels tells us which is the output port:

$$|\psi_s\rangle_D = \frac{1}{2\sqrt{2}}(ie^{i\theta} - e^{i\theta} + i + 1)|1\rangle_D, \tag{24}$$

$$|\psi_s\rangle_B = \frac{1}{2\sqrt{2}}(e^{i\theta} + ie^{i\theta} - 1 + i)|1\rangle_B. \tag{25}$$

And the square of the probability amplitudes are:

$$P_D = \frac{1}{2}\left(1 - \sin\theta\right), \tag{26}$$

$$P_B = \frac{1}{2}\left(1 + \sin\theta\right). \tag{27}$$

When comparing against the earlier case, we observe that the interference remains. The only difference is the change of phase, which arises from the fact that the output probability for both ports has to comply with $P_{B,D} = 1/2$.

4. Entanglement

Taking advantage of the methods derived before in this article, we can derive the mathematical description of entanglement with a pair of photons in a beam splitter.

In Fig. 5, a state containing the product of two polarized and correlated photons is incident on one of the ports of a beam splitter. Without loss of generality we will choose for our treatment port 1, and describe the initial state $|\phi\rangle_i$ as:

$$|\phi\rangle_i = |H\rangle_1|V\rangle_1. \tag{28}$$

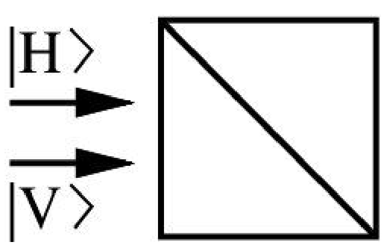

FIGURE 5. Two photons with orthogonal polarization are incident on the same port of a beam splitter.

And hence the output $|\phi\rangle_s$ is given by means of Eq. (1) as:

$$|\phi\rangle_s = \frac{1}{2}\left(|H\rangle_D + i|H\rangle_B\right)\left(|V\rangle_D + i|V\rangle_B\right), \tag{29}$$

which can be rewritten as:

$$|\phi\rangle_s = \frac{1}{\sqrt{2}}\left(i|\psi^+\rangle + |\psi'^-\rangle\right), \tag{30}$$

where

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_D|V\rangle_B + |H\rangle_B|V\rangle_D\right), \tag{31}$$

$$|\psi'^-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_D|V\rangle_D - |H\rangle_B|V\rangle_B\right), \tag{32}$$

which are entangled states. Therefore, if one measures the coincidences between the two outputs of the beam splitter, in other words performs a correlation measurement, the result would be an unique entangled state $|\psi^+\rangle$ [12, 13]. Up to this point it was not necessary to set labels for the beam splitter outputs, since the symmetry of the experiment guarantees that either reflected or transmitted outputs would be equivalent. Concerning the correlation measurement, this is a post-selected entanglement measurement, and hence in it we cannot distinguish the output state in Eq. (31) from the Bell state in Eq. (32).

The story is entirely different if we select the initial state to be somewhat different. Assume that a horizontally polarized photon enters the beam splitter through port 1, and a vertically polarized photon is incident on the port 2, see Fig. 6. We can write the initial state as:

In this case the initial state $|\Phi\rangle_i$ is:

$$|\Phi\rangle_i = |H\rangle_1|V\rangle_2. \tag{33}$$

By performing a mathematical development close to that outlined in Eq. (3), we can describe the output of the beam splitter when he incident state is Eq. (33) as:

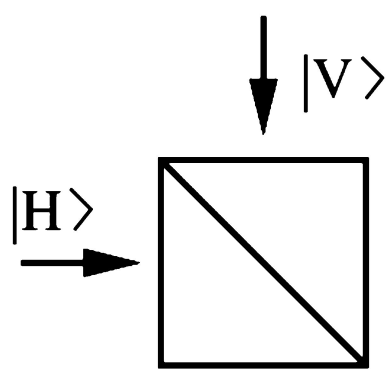

FIGURE 6. Two photons with orthogonal polarization are incident each at every of a beam splitter.

$$|\Psi\rangle_i = \frac{1}{\sqrt{2}}\left(i|\psi'^{+}\rangle + |\psi^{-}\rangle\right), \tag{34}$$

where

$$|\psi^{-}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_D|V\rangle_B - |H\rangle_B|V\rangle_D\right), \tag{35}$$

$$|\psi'^{+}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_D|V\rangle_D + |H\rangle_B|V\rangle_B). \tag{36}$$

Notice that the state described by Eq. (34) is a Bell state that can only be discerned by a coincidence -correlation experiment.

5. Incoming entangled state

In this section we study what happens when the initial state at one of the inputs of the beam splitter is entangled, *e.g.* a Bell state $|\psi^{+}\rangle$. Introducing an entangled state to the beam splitter implies that sometimes a portion enters the beam splitter by port 1, and in others it goes through port 2. An schematic representation of this process is shown in Fig. 7.

If the input state has the form

$$|\psi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2\right),$$

then, the output state can be written as:

$$|\Psi\rangle_s = \frac{1}{2\sqrt{2}}\big([|H\rangle_D + i|H\rangle_B][|V\rangle_B + i|V\rangle_D] + [|V\rangle_D + i|V\rangle_B][|H\rangle_B + i|H\rangle_D]\big). \tag{37}$$

Expanding the products we get:

$$\begin{aligned}|\Psi\rangle_s = \frac{1}{2\sqrt{2}}(&|H\rangle_D|V\rangle_B + i|H\rangle_D|V\rangle_D \\ &+ i|H\rangle_B|V\rangle_B - |H\rangle_B|V\rangle_D \\ &+ |V\rangle_D|H\rangle_B + i|V\rangle_D|H\rangle_D \\ &+ i|V\rangle_B|H\rangle_B - |V\rangle_B|H\rangle_D).\end{aligned} \tag{38}$$

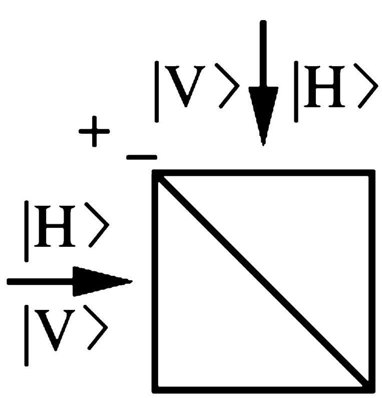

FIGURE 7. Bell states $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$ as inputs into a beam splitter.

Reducing the above equations by eliminating opposite terms yields:

$$|\Psi\rangle_s = \frac{i}{2\sqrt{2}}(|H\rangle_D|V\rangle_D + |H\rangle_B|V\rangle_B + |V\rangle_D|H\rangle_D + |V\rangle_B|H\rangle_B). \tag{39}$$

Which results in:

$$|\Psi\rangle_s = \frac{i}{\sqrt{2}}(|H\rangle_D|V\rangle_D + |H\rangle_B|V\rangle_B). \tag{40}$$

Observe that the equation above is almost identical to the initial state, except for a $\pi/2$ change in phase, or:

$$|\Psi\rangle_s = i|\psi'^{+}\rangle. \tag{41}$$

It is important to point out that this process is reversible except for a phase change. If we introduce $|\psi'^{+}\rangle$ as the incident state instead, we will obtain at the output the state $|\psi^{+}\rangle$.

Up to now, we have seen that when two identical photons enter, simultaneously, through both ports of a beam splitter, the outcome can be understood as destructive interference of the photons when they exit both outputs at the same time. Therefore, any coincidence measurement, with zero time delay will be null. This fact is the basis of the Hong-Ou-Mandel interferometer [14]. However, when photons are distinguishable, for example when they have different polarizations, the interference will not happen if the state is not entangled, recall to this matter Eqs. (31) and (35). As remarked, this is not exclusive to polarized states, and can be expanded to those where photons are differentiated for having different energies or color. For example, Aspect *et al*. used entangled states made of photons with different wavelengts, 551 nm and 422.7 nm, with opposite polarization produced by a ^{48}Ca jet [15–17] as the initial state in the so called the proof of the existence of the photon experiment. In this work Aspect showed the anti-correlation between the outputs of a beam splitter. In other words, what their experiment, and many others since, shown is that if one of these entangled states is incident at one of the ports of a beam splitter, then, interference is observed. Moreover, it takes place even with photons of different energies, *i.e.* color, and the coincidences of such experiment are null correlations, when the time delay is zero. Incidentally, this also the result when the an EPR state, $|\Psi\rangle_e = |\psi^{-}\rangle$, enters the beam splitter, since the output will be the same $|\Psi\rangle_s = |\psi'^{-}\rangle$, and the coincidences between the output ports is once again null, at zero time delay.

The results summarized above bring us to some significant conjectures about quantum interference and entanglement with single photons in beam splitters.

A beam splitter is a device which changes one entangled state into another entangled, state with a different spatial distribution, *i.e.* :

$$|\psi^{+}\rangle \ \overrightarrow{bs} \ |\psi'^{+}\rangle, \tag{42}$$

$$|\psi^{-}\rangle \ \overrightarrow{bs} \ |\psi'^{-}\rangle. \tag{43}$$

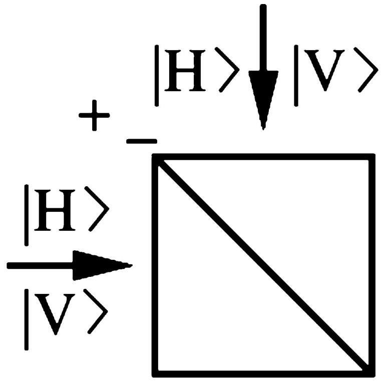

FIGURE 8. Bell states $|\phi^{+}\rangle$ y $|\phi^{-}\rangle$ incident at the input ports of a beam splitter.

A consequence of the previous results is that since that the beam splitter is a passive optical element, then it should obey *the conservation of correlation*. That is, if the initial state is not entangled the output state will not be entangled either, even when the output is a combination of entangled states, the total state will not be entangled. Then again, if the initial state is entangled, the components of the output and the total state will be entangled as well. Take for example an EPR state of the kind $|\phi^{+,-}\rangle$, this state is incident on a beam splitter as shown by Fig. 8; and it is described by:

$$|\Psi\rangle_e = |\phi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2\right), \tag{44}$$

and accordingly the output state will be:

$$|\Psi\rangle_s = \frac{1}{2\sqrt{2}}\big([|H\rangle_D + i|H\rangle_B]\,[|H\rangle_B + i|H\rangle_D] + [|V\rangle_D + i|V\rangle_B]\,[|V\rangle_B + i|V\rangle_D]\big). \tag{45}$$

Expanding,

$$\begin{aligned}|\Psi\rangle_s = \frac{\sqrt{2}}{2\sqrt{2}}(&|H\rangle_D|H\rangle_B + i|H\rangle_D|H\rangle_D \\ &+ i|H\rangle_B|H\rangle_B - |H\rangle_B|H\rangle_D \\ &+ |V\rangle_D|V\rangle_B + i|V\rangle_D|V\rangle_D \\ &+ i|V\rangle_B|V\rangle_B - |V\rangle_B|V\rangle_D),\end{aligned} \tag{46}$$

and simplifying by eliminating similar terms, we have:

$$\begin{aligned}|\Psi\rangle_s = \frac{i}{2}(&|H\rangle_D|H\rangle_D + |H\rangle_B|H\rangle_B \\ &+ |V\rangle_D|V\rangle_D + |V\rangle_B|V\rangle_B),\end{aligned} \tag{47}$$

which is also an entangled state.

6. Conclusions

By using a method entirely based on [*Schrödinger*] *wave states*, linear combination of single photon states through a beam splitter, we have been able to derive the results concerning interference with single a multiple photons.

We have also shown that using this method it is possible to obtain the initial states out of the mixed states, or *wave states*.

We believe that this intelligible analysis can help students in the first acquaintances with quantum optics to grasp the physics behind the interaction between photons and beam splitters. The mathematical methods is straightforward, and can be extended to study more intricate quantum states. We have proved this point by exploring *wave states* and we showed that when one introduces any EPR polarized entangled state through both inputs of a beam splitter, the latter becomes an entanglement idler.

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