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The entropy production rate a bridge between thermodynamics and chemical kinetics

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It is shown how through the entropy production rate a natural unification between the formalism of classical thermodynamics and chemical kinetics is achieved. It is also shown how the entropy production rate represents an alternative way to the sensitivity analysis method in order to determine the fundamental steps in a reaction mechanism.

Keywords: Irreversible thermodynamics; entropy thermodynamics; evolution; entropy in education; science in school.

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1. Introduction

Traditionally, the idea that thermodynamics deals with equilibrium and the chemical kinetics of change has been widely accepted [1]. While to some extent this classification is correct within the framework of classical thermodynamics, where time appears implicitly, it leads undoubtedly to an apparent divorce between thermodynamics and kinetics.

Consequently, in undergraduate courses, students face a dilemma in the study of chemical reactions.

This apparent divorce between thermodynamics and chemical kinetics can be resolved naturally through the thermodynamic formalism of irreversible processes, in particular through the production of entropy per unit time [2].

This communication aims to show how through the entropy production rate, on the one hand, the formalism of classical thermodynamics and chemical kinetics is unified. On the other hand, it can be used in chemical kinetics as an alternative method to that of the sensitivity analysis for the selection of the fundamental steps in a reaction mechanism.

The paper is organized as follows: we first briefly review the basic definitions in classical thermodynamics in Thermodynamics framework section. Section The method of dominating steps, focuses on the development of a thermodynamic outline, based on the rate of entropy production in chemical kinetics as an alternative method to sensitivity analysis for the selection of the fundamental steps in a reaction mechanism. Finally, some comments and remarks are presented in the Conclusion.

2. Thermodynamics framework

Unlike other quantities widely used in thermodynamics, such as work, heat, etc., entropy has always been a "dark" and controversial subject within thermodynamics [3-7].

According to De Donder [8] the entropy change of a system dS_s is given by

$$dS_s = \delta S_e + \delta S_i, \tag{1}$$

where δS_e is entropy flow, that is, the entropy exchanged with the surroundings, while δS_i is the entropy production due to irreversible processes. Thus, the fundamental postulate of the Second Law establishes that every system evolves in the direction of $\delta S_i > 0$, and reaches equilibrium when $\delta S_i = 0$ [8].

For an evolving system subjected to constraints, for example temperature T and the pressure P constants, the entropy production δS_i can be evaluated through the variation of Gibbs's free energy [8] dG_{T_p} as:

$$\delta S_i = -\frac{1}{T} dG_{T_p}. \tag{2}$$

The temporal variation of the expression of Eq. (1) represents the entropy production rate as:

$$\frac{\delta S_i}{dt} = -\frac{1}{T}\frac{dG_{T_p}}{dt}, \tag{3}$$

where $\delta S_i/dt \equiv \dot{S}_i$ represents the entropy production rate [2]. The term dG_{T_p}/dt can be developed by means of the chain rule as a function of the degree of advance of the reaction ξ as:

$$\frac{dG_{T_p}}{dt} = \left(\frac{\partial G}{\partial \xi}\right)_{T_P} \frac{d\xi}{dt}, \tag{4}$$

where $(\partial G/\partial \xi)_{T_P}$, according to De Donder and Van Rysselberghe [9] represents the affinity $A \equiv -(\partial G/\partial \xi)_{T_P}$, and the term $d\xi/dt$ is the reaction rate ξ. The rate of entropy production Eq. (3) can be written as:

$$\dot{S}_i = \frac{1}{T} A\dot{\xi} = -\frac{1}{T}\Delta G \dot{\xi}, \tag{5}$$

where $A = -\Delta G$ [8].

The formula, Eq. (5) establishes a bridge between classical thermodynamics, "thermodynamics force" $\frac{1}{T}A$ and chemical kinetics, the "flow" $\dot{\xi}$ the reaction rate. The affinity A can be evaluated from the isotherm of the reaction [10] by the equation:

$$A{=}RT\ln K_C{-}RT\sum_{i=1}^{k}v_k\ln C_k{=}RT\ln\left(\frac{K_C}{\Pi C_k^{v_k}}\right), \quad (6)$$

where $K_C = k_f/k_b$ is the Guldberg-Waage constant, R is the gas constant, k_f, k_b are the specific rate constants of the forward and backward reaction steps respectively, C_k is the concentration of the k-th specie, and the v_k are the stoichiometric coefficients that are taken, by agreement, as positive for the products and negative for the reactants.

Therefore, the Eq. (6) can be written as:

$$A = RT\ln\left(\frac{k_f\Pi C_{k(f)}^{v_k(f)}}{k_b\Pi C_{k(b)}^{v_k(b)}}\right). \quad (7)$$

The rate of reaction $\dot{\xi}$ for an elementary step of reaction can be written as:

$$\dot{\xi} = \left(\dot{\xi}_f - \dot{\xi}_b\right) = k_f\Pi C_{k(f)}^{v_k(f)} - k_b\Pi C_{k(b)}^{v_k(b)}, \quad (8)$$

where $\dot{\xi}_f, \dot{\xi}_b$ are the forward and reverse reaction rate respectively. Substituting formulas, Eq. (8) and Eq. (7) in to Eq. (5) is obtained:

$$\dot{S}_i = R\left(\dot{\xi}_f - \dot{\xi}_b\right)\ln\frac{\dot{\xi}_f}{\dot{\xi}_b} \geq 0. \quad (9)$$

The Eq. (9) analogously to Eq. (5) ends the controversy related to the apparent "divorce" between classic thermodynamics and chemical kinetics.

3. The method of dominating steps

It is known that the sensitivity analysis of differential equations has been used successfully in the study of reaction mechanisms in chemical kinetics [11]. Edelson's pioneering works [12-14] they allowed not only the determination of the fundamental reaction steps in a mechanism, but also its reduction. Later, Turanyi [15,16] implemented the method, achieving a drastic reduction in the mechanism of the famous Belousov-Zhabotinsky reaction (BZ), (GTF) model [17] from 80-reaction steps to 42-reation steps.

As an alternative method to the sensitivity analysis, we proposed the use of the entropy production rate based on Eq. (9), a so-called by us *Method of the Dominating Steps* [18]. For this; we postulate that those reaction steps that exhibit a higher entropy production value would be the fundamental ones in a reaction mechanism.

Let be a mechanism composed of n-reaction steps and m-species, represented by (10) such that

$$\begin{gathered} x_{1/i} = x_{2/i} \\ \vdots \\ x_{m-1/n} = x_{m/n}. \end{gathered} \quad (10)$$

Thus, we have that the entropy production rate of the n-reaction step is given by

$$\dot{S}_{i/n} = R\left(\dot{\xi}_{f/n} - \dot{\xi}_{b/n}\right)\ln\frac{\dot{\xi}_{f/n}}{\dot{\xi}_{b/n}} \geq 0. \quad (11)$$

TABLE I. The entropy production rate of the fundamentals reaction steps for reduced GTF model (adapted from [18]).

Reaction step*	$\dot{S}_i\left(\frac{\text{J}}{\text{s}\cdot\text{K}}\right)$	Reaction step*	$\dot{S}_i\left(\frac{\text{J}}{\text{s}\cdot\text{K}}\right)$
1	4.7×10^{-4}	17	1.8×10^{-4}
2	2.9×10^{-4}	21	1.2×10^{-6}
3	4.5×10^{-6}	22	1.3×10^{-3}
5	2.0×10^{-6}	24	5.3×10^{-6}
7	2.0×10^{-6}	26	6.3×10^{-6}
9	8.6×10^{-5}	29	1.1×10^{-6}
10	7.8×10^{-5}	38	1.6×10^{-6}
11	2.6×10^{-3}	39	3.7×10^{-6}
12	2.6×10^{-3}	46	2.6×10^{-6}
13	3.1×10^{-5}	56	2.6×10^{-6}
14	2.1×10^{-5}	61	2.4×10^{-6}
15	7.0×10^{-4}	71	1.2×10^{-6}
16	5.1×10^{-4}	77	2.6×10^{-6}

The n-reaction step will be dominant compared to $(n-1)$-reaction step if it is true that: $\dot{S}_{i/n} > \dot{S}_{i/n-1}$. To illustrate the use and efficacy of the Method of the Dominating Steps, as an example, we will show its use in the reduction of the original GTF17 model with 80-reaction steps to 26-reaction steps [18,19] as shown in Table I. An essential condition is that the selected reaction steps reproduce the observed experimental behavior [19].

On the one hand, it should be noted that the 26-reaction steps selected are among the 42-reaction steps set by Turanyi *et al.* [17] through the use of the sensitivity analysis to the GTF model. On the other hand, this drastic reduction not only incorporates the full richness of the pioneering work of Field *et al.* [20] (FKN model), periodic oscillations, and it is enough to account, in particular, for the experimental results reported by Ruoff [21], chaotic behavior.

Finally, we have successfully extended the method to the study of the glycolysis mechanism of cancer [22-25]. In the last year, glycolysis of cancer has been a target in oncology [26] Most tumor cells show a higher glycolytic rate than normal cells; this phenomenon is known in the literature as the Warburg's effect. The main objective was to identify reaction steps that can be potential targets in cancer treatment.

From the model proposed by Marín *et al.* [27] for HeLa and AS-30D tumor cells, 9-reaction steps were identified out of 20-reaction steps. In this sense, 3 of them coincide with the so-called "metabolic control points"; the other six have been identified as potential targets in cancer treatment [28-30].

4. Conclusions

In summary, in this paper we arrive at the following conclusions:

1. The rate of entropy production, as a generalization of the Second Law of thermodynamics, represents per se an extreme criterion that marks the directionality of natural processes on a macroscopic scale and constitutes a bridge between the formalism of classical thermodynamics and the chemical kinetics.

2. In biophysical-chemical systems, the rate of entropy production, as a non-extreme criterion, generalizes the so-called "maximum entropy criterion" [31] and constitutes a complementary method to the sensitivity analysis of differential equations, which allows determining the fundamental reaction steps in a reaction mechanism.

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