A new rolling friction coefficient measurement method based on the work-energy theorem

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The rolling friction coefficient represents a characteristic parameter for the rolling motion. To determine the rolling friction coefficient, we propose a simple method based on the conservation of energy. The measuring setup includes the tools and materials that are simple and easy to obtain, such as the spring, ruler, and a small laboratory wheeled cart. In this paper, we have determined the rolling friction coefficient for several masses of the test carts and different lengths of spring compression. When the spring is compressed, its energy is directly proportional to the square of the length of the spring compression. The initial speed of the cart was determined using the law of energy conservation, and the length that the cart goes before stopping is measured with a ruler. The conclusion is that the value of the rolling friction coefficient is usually very small and that the mass of an object will affect the friction force but will not affect the rolling friction coefficient. Besides determining the rolling friction coefficient, this technique should help students comprehend the concept of friction easily.

Keywords: Rolling friction; method of measuring the rolling friction coefficient; elastic spring; wheeled laboratory cart.

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1. Introduction

With rare exceptions [1-3] the rolling friction is not frequently included in general introductory physics textbooks. It should be mentioned that an even more challenging situation exists when introducing rolling friction in introductory physics textbooks. Detailed analyses of this mechanism and measurements of the coefficient of rolling friction can be found in Refs. [4-6]. Every introductory physics textbook discusses kinetic and static friction, and many papers are devoted to explaining these forces [7], but it is well known that measuring static and kinetic friction in educational labs can be troublesome [8,9]. The force that resists the motion of a body rolling on a surface is called the rolling resistance or the rolling friction. If we let a body of mass m, with initial velocity $\vec{v}_0$, roll on a surface, down a long horizontal table, it will eventually come to rest. This means that while the body is in motion, it experiences an average acceleration $\vec{a}_{avg}$ oriented in a direction opposite to its motion. In this case, we can declare that the table exerts a force of friction, whose average value is $m\vec{a}_{avg}$, on the rolling body. The frictional force on each body is directed opposite to the motion relative to the other body. The ratio of the magnitude of the maximum of the rolling friction force to the magnitude of the normal force is called the coefficient of rolling friction: $\mu_r = F_r/N$, where F_r and N are the rolling friction and the normal force, respectively. Since rolling resistance can be very small, determining the coefficient of rolling friction is not an easy task. Many papers are devoted to teaching force of friction on a rolling object. In Ref. [10] the introduction to rolling friction is studied in relation to the height that a ball attains up an inclined track after rolling down a given height along another inclined track opposite from the first. A precise calculation of rolling friction in student labs are complicated as rolling friction effects are small to observe (approximately two orders of magnitude less than kinetic friction) while the measurements of the time of experiment (usually about 1s) and object distance traveled (usually about one meter) are small, tending to make the measurements unreliable. A novel method of measuring the resistive forces of kinetic and rolling frictions of a sliding block and a rolling cart was proposed in Ref. [11]. The method is based on studying the decay of spring/mass oscillations caused by kinetic or rolling resistive force. In the note [12], a ball oscillating on a concave trackway is used to find the coefficient of rolling friction using typical laboratory equipment. The simple experiment that can be carried out by students and the very reliable results can be obtained. In Ref. [13] an alternative and simple technique to measure the coefficient of rolling friction for a hard ball rolling on a hard, horizontal surface is described. The method involves attaching a small mass to the side of the ball so that the ball processes slowly along a spiral path. Since the radius of curvature decreases slowly with time, the decrease in ball speed can be measured over a relatively long path length on a small area surface. To make matters worse, the coefficient of rolling friction is much smaller than the coefficient of kinetic friction [1-5] so student measurements of rolling friction are even more troublesome. A numbered experiments have been performed to understand rolling friction. The coefficient of rolling friction turns out to be small compared to static or kinetic friction. For example, in Ref. [14] an experimental investigation carried out where a Maxwell's wheel to acquire some initial energy and then roll along a pair of horizontal tracks until it comes to a stop. The coefficient of rolling friction determined from the offset distance, shows a power law dependence on the dimensionless energy ratio. Different authors have used different models to study theoretically and experimentally rolling friction in undergraduate level physics courses. For example, in Ref. [15] a model of rolling resistance based on the idea of bulge formed by a soft underlay in front of the rolling body is presented.

It is well accepted that the loss of energy in rolling motion occurs because of the deformation and subsequent reformation of the surface resulting in hysteresis which is responsible for the loss of energy. As a result of the deformation, reaction force of the surface on the rolling object acts at a point which is slightly offset from the centre of the object in the direction of motion.

Rolling friction is a tricky topic that needs to be carefully addressed both in classroom lessons and in laboratory activities. A more realistic description of rolling motion requires inclusion of rolling friction. A simple experiment that is described in this paper can be used as a didactic method within the context of "guided discovery." That encourages the experimental investigation of unfamiliar phenomena, and the students will be motivated to enquire further into the understanding of their results. The procedure proposed here includes more students' activities: (a) elaboration of the formal model of rolling friction, (b) application to the different situations, (c) measurement and treatment of data, (d) calculation of the friction coefficient of rolling friction and comparison of the obtained value with the values from the textbooks (e) analyses of the factors influencing this coefficient. Moreover, this lab integrates different topics in introductory physics including: kinematic relations (differential and integral) between velocity, displacement, and distance traveled; the work-kinetic energy theorem.

Here we have described a simple method for the determination of the rolling friction coefficient, based only on the conservation of energy. Small rolling carts on metal tracks have become common in introductory physics laboratories at universities and secondary schools. Instead of rolling friction force, the rolling friction coefficient is calculated by measuring the distance the cart, pushed by elastic spring, takes to come to rest. The method is very simple and accurate. Some errors can occur due to the measurements of the spring lengths, but that can be minimized by taking careful readings of the measured values.

2. Theoretical background and design of the approach

A small rolling cart with mass m is attached to the end of the spring in the horizontal position. The length of the uncompressed spring is x_0. In the equilibrium position, the weight of the rolling cart is equal to the normal force (Fig. 1). When the spring is compressed by a certain length x, it is found that the applied force is directly proportional to the length of the spring compression.

While the spring compression displacement is x, the potential energy of the spring is $E_p = (1/2)kx^2$, where k is the spring constant (Nm^{-1}). In this work a standard elastic spring that can be compressed was used. The spring constant

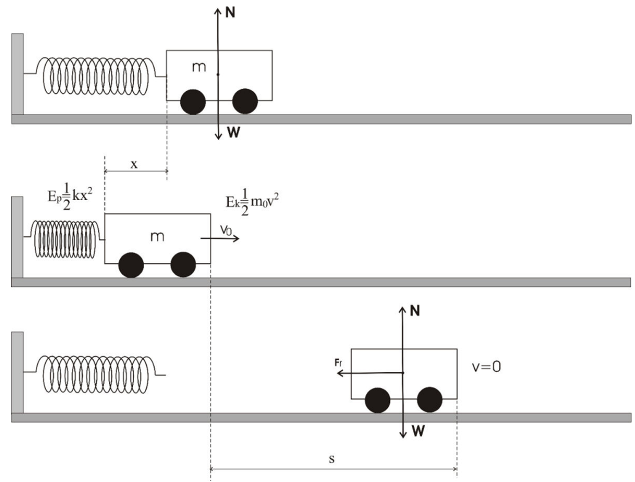

FIGURE 1. The design of the rolling friction coefficient measurement.

k is determined experimentally. When the spring is released from compression, it gives the initial velocity $v_0 = \sqrt{(k/m)}x$ and the total kinetic energy $E_k = (1/2)mv_0^2 + (1/2)I\omega^2$ to the wheeled cart, where I and ω are the moment of inertia and the angular velocity of the wheels, respectively. The rotational kinetic energy of the wheel is $(1/2)I\omega^2 = (1/2)\cdot(1/2)m_w r_w^2\,(v_0/r_w)^2$ where m_w and r_w are mass and radius of the wheel, respectively. In our experiment we used the first approximation of the total kinetic energy of the object moving that contains only the translation kinetic energy[i] *i.e.*, $E_k \approx (1/2)mv_0^2$ because the rotational component is small compared to the translation part. The work of the friction force on the cart is

$$A = -F_r s = -\mu_r N s, \tag{1}$$

where s is the distance that the cart takes to come to rest, and $F_r = \mu_r N$ is the rolling friction force with the rolling friction coefficient μ_r. The work done on the rolling body (cart) by the resultant force acting on it is equal to the change of the kinetic energy of the cart

$$A = \Delta E_k = E_{k2} - E_{k1} = -\frac{1}{2}mv_0^2. \tag{2}$$

This equation is known as the work-energy theorem. Using these equations, we obtain the formula that can be used to estimate the rolling friction coefficient:

$$\mu_r = \frac{kx^2}{2Ns}. \tag{3}$$

From the Eq. (3), the unknown rolling friction coefficient μ_r, can be easily calculated by measuring the displacement x and distance s. The standard procedure described in Ref. [16] was applied to determine the relative uncertainty $\Delta\mu_r$:

$$\frac{\Delta\mu_r}{\mu_r} = \frac{\Delta k}{k} + 2\frac{\Delta x}{x} + \frac{\Delta N}{N} + \frac{\Delta s}{s}, \tag{4}$$

where $\Delta N/N = \Delta m/m$ (here Δm is the uncertainty of the mass of the used cart).

In the case of the indirectly measured value $Q(X, Y, Z, \ldots)$, the relative uncertainty could be estimated by adding the individual independent uncertainties in quadrature by

$$\delta Q \simeq \sqrt{\left(\frac{\delta Q}{\delta X}\right)_0^2 (\delta X)^2 + \left(\frac{\delta Q}{\delta Y}\right)_0^2 (\delta Y)^2 + \left(\frac{\delta Q}{\delta Z}\right)_0^2 (\delta Z)^2 + \ldots} \,. \tag{5}$$

If the function $Q(X, Y, Z, \ldots)$ is not linear, then the equality in Eq. (5) is only approximate; the smaller are the uncertainties $\delta X, \delta Y, \ldots$ with respect to the central values $X_0, Y_0, \ldots$, the better is the approximation. In our calculation for the function $\mu_r(k, x, N, s)$, the direct measured values are statistically independent, then instead of Eq. (5) the use of Eq. (4) is appropriate. By this way the maximum value of the uncertainly of μ_r is calculated.

In order to estimate the validity of the proposed method, the variation of friction force F with normal force N should be presented. This relationship is linear, with a slope that is equal to the coefficient of rolling friction μ_r. The procedure consists in finding the acceleration, assumed constant, of a cart rolling along a horizontal plane. The acceleration is calculated by $a = v_0^2/2s$, where s represents the distance the cart rolls until it comes to rest.

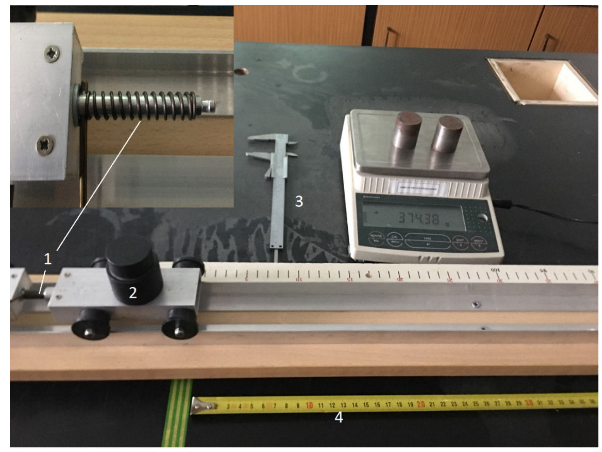

FIGURE 2. Experimental setup: (1) spring, (2) cart (container), (3) calliper ruler (4) meter.

3. Results and discussions

The following theoretical considerations refer to the experimental device outlined in Fig. 2.

First, we measure the length of the unloaded spring $x_0 = 4.52$ cm. Then, using a standard calliper ruler, we measure the length of the spring compressed by the laboratory wheeled cart. We used the spring with constant $k = 1900$ N/m.

Based on the measured values of compressed spring lengths in the initial position ($x = 0.0195$ m), the distance that the laboratory wheeled cart goes before coming to rest (s) is measured for five different masses of the test cart. The contact surface of the table was made of the thin layer of special types of hard plastic known as bakelite. The work table was leveled horizontally so there was no problem keeping the cart moving in a straight track. The rolling friction coefficient is determined using Eq. (3). Table I gives the values of the measured results and the computed rolling friction coefficient.

Based on the measured values in Table I, we have obtained the mean value of the rolling friction coefficient $\overline{\mu_r} = 0.0571$. The other way to determine the rolling friction coefficient is to measure the acceleration of the laboratory cart. Figure 3 shows the linear regression of the rolling friction versus the normal forces for five different masses of the laboratory cart. It also shows the equation of the straight line and the value of the correlation coefficient that corresponds to the rolling friction coefficient of 0.058, which confirms that our method is accurate and reliable.

From the graphical interpretation of the results in Fig. 3 the equation of the linear regression of the rolling friction versus the normal forces is determined as

$$F_r = 0.0579N - 0.003.$$

Also, in Fig. 3 the linear and parabolic fit of the experimental results are presented together. It also shows that the deviation between these two fits is relatively small. The data in Fig. 3 suggesting some other effect is acting on the cart in addition to the one modeled here. Air drag acting on the cart

TABLE I. Rolling friction coefficient obtained by measurement of distance that the laboratory wheeled cart goes before coming to rest for five different masses of the cart.

m(kg)	s_1(m)	s_3(m)	s_3(m)	$s = \frac{s1+s2+s3}{3}$(m)	v_0(m/s)	μ_r	a(m/s^2)	F_r(N)	$N = mg$(N)
0.22904	2.67	2.68	2.65	2.667	1.776	0.060	0.591	0.135	2.247
0.040702	1.68	1.66	1.65	1.663	1.332	0.054	0.534	0.217	3.993
0.50566	1.34	1.32	1.36	1.340	1.195	0.054	0.533	0.270	4.961
0.55505	1.17	1.16	1.19	1.173	1.141	0.057	0.555	0.308	5.445
0.60876	1.01	1.00	1.01	1.007	1.089	0.060	0.589	0.359	5.972

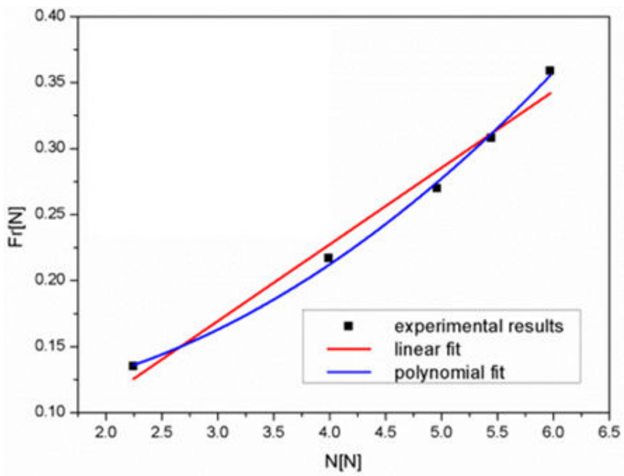

FIGURE 3. Graphic diagram of the experimental values of the rolling friction force versus normal force (the linear and parabolic fit of the experimental results).

that is a quadratic function of the velocity, or nonidealities in the spring that used in this experiment, lead to the deviation from the linear function of rolling friction vs. speed. The analysis of these effects goes beyond the framework of learning physics in high school because of that is left to other investigators to explore.

To be efficient for learning of the proposed method, it must obtain the magnitude value with its corresponding absolute error and correct units. In addition, the calculated value of the magnitude must be within the error interval. In this regard, the absolute errors of the length of the spring x, distance s and mass m, we used are $\Delta x = 0.0001$ m, $\Delta s = 0.001$ m and $\Delta m = 10^{-5}$ kg, respectively. The accuracy of the density measurement was calculated Eq. (4) and it is obtained that the relative uncertainty expressed as a percentage is 2%. Hence, the experimental result with the correct significant figures which we obtain for the rolling friction coefficient is

$$\mu_r = (0.057 \pm 0.001).$$

4. Teaching friction: notes for didactic choices

Before performing the proposed experiment, there are a few sequences for teaching of rolling friction they can be summarized as follows: (a) introduce friction as an omnipresent set of phenomena crucial for most everyday activities, (b) giving examples in which friction is presented as an important phenomenon, (c) emphasize the crucial role of friction in establishing equilibrium after a stress or motion, (d) avoid an overemphasis on situations with horizontal friction forces, which can favor the identification of normal force with weight, (f) formal models conceived in terms of functional relations are inadequate for the students' needs of understanding. These sequences including: 1. introductory experiments and observations; 2. vertical friction force: definition of descriptive quantities and first qualitative relations; 3. static and kinetic friction and phenomenological laws; 4. topography of surfaces and mechanisms producing friction; and 5. friction phenomena from the point of view of energy.

5. Conclusion

In this paper, we have presented a new method for the measurement of rolling friction coefficient. Our method is based on the measurements of mass, compression of the elastic spring, and the distance that the laboratory wheeled cart goes before stopping. It is observed that the value of the rolling friction coefficient is usually very small and that the mass of the objects will affect the friction force but will have no effect on the coefficient of rolling friction. With two measurements in the laboratory of a General Physics course, which means without large devices or excessive complications, we have obtained the value of the magnitude of rolling friction coefficient with a relative error of 2%. The design and construction of the measuring setup in this study are carried out with materials that are simple and easily obtained. The experiment described here can be useful for teaching school students the concept of rolling friction. Practically, the same experiment can be useful for teaching movements with constant acceleration and to prove Newton's law of mechanics. In order for the laboratory to be efficient for learning, it must obtain the magnitude value with its corresponding absolute error and correct units.

The proposed experiment should encourage a wider and more critical view of different types of friction phenomena, as well as reflections on the characteristics and possible explanations of these phenomena. We believe that such a lab would be a good learning experience for introductory student and the students' reactions to the activities would be extremely positive.

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i. In our experiment, the mass of the one of wheel is $m_w = 5.67$g The rotational kinetic energy is $(1/2)I\omega^2 = 4 \cdot (1/2) \cdot (1/2)m_w r_w^2 (v_0/r_w)^2 = m_w v_0^2$. The ratio $([1/2]m_w v_0^2/[1/2]mv_0^2) = (m_w/m) \ll 1$ so the rotational kinetic energy is much smaller than kinetic energy of translation.

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