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# Lambda-shiftings for the secondary sources in the Young's experiment allow to rebuild patterns looks like piston/tilt

J. Salinas-Luna
*Instituto de Ecología, Universidad del Mar,*
*Campus Pto. Ángel, UMAR, 70987, Pto. Ángel, Oax., México.*

J. Mentado Morales
*Instituto de Industrías, Universidad del Mar,*
*Campus Pto. Ángel, UMAR, 70987, Pto Ángel, Oax., México.*

J. M. Nuñez-Alfonso
*Instituto de Astronomía, Universidad Nacional Autónoma de México,*
*22160 Ensenada Baja California, México.*

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In this work, we perform the mathematical theory used by Born and Wolf to rebuild a geometric characterization for piston and tilt or a combination of both surface errors by shifting the secondary sources in the classical Young's experiment. The last is accomplished making a comparison between the generated patterns of classical Young' experiment and Young's patterns when the secondary sources are shifted axially of the order of $\lambda$. Images with effects looks like to surface errors by piston or tilt are obtained and this give us a good idea on how could be the co-phasing of an optical flat surface in a real experiment.

*Keywords:* Young's experiment; piston; tilt; phase measurement.

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## 1. Introduction

It is well-Known that the Young's experiment has been perfectly well explained and can be solved without difficult by means diffraction integrals, which are applied to the observation plane. Although, there is enough information even for complicated illumination beams and free software to generate changes in the behavior phase, we consider that a geometric approximation can be made by relating the displacements of the secondary sources in the Young's experiment; very similar patterns simulating surface errors produced by piston or tilt effects can be obtained. According to the reported bibliography here, but never in an any prior case, small longitudinal increases of the secondary source's position of the order of $\lambda$ or fractions of that has been used, this would be worthy to be analyzed. Therefore, without fear that this may appear an interferometry exercise or a diffraction routine, rather it is other way, where we try that our analysis starts from what is already known on the geometric part of the Young's experiment, thus, to go gradually constructing a final pattern, when changes produced by the axial shiftings of the secondary sources can be observed, in addition, the behavior for the phase or interference function, and for diffraction function (modulating the phase) are obtained, and so, in the next sections, it will be shown the treatment followed in each case.

The last mentioned, comes from the motivation when asking us on how complex co-phasing corrections have been spectacularly solved regarding pistons and tilts in the mirror segments of the James Webb Telescope of the NASA, since the new stellar images that we are currently viewing depends on this correction, the same way, we ask ourselves, if the effects of piston or tilts errors can also be viewed by using a division wavefront interferometer as is the case of the Young's experiment. Firstly, we have started analyzing the performance of Meslin's experiment and of Fresnel's mirrors with slit source [1], the possibility to produce a fringe's pattern with characteristics of segmentation, where adjoining patterns can be mutually developed and compared with reference patterns, as it was demonstrated for a misalignment of the optical surface for an offset or piston and a tilt [2]. Also based on initial numerical-experimental phase measurement experiments for two adjacent surfaces, [3], where changes in the fringes frequency for piston were obtained, we consider that by means Young experiment, it is possible to compare patterns free of surface errors with piston or tilt patterns. For this, we propose small position changes of the order of $\lambda$ for the secondary sources in the Young experiment, and thus, to characterize piston and tilt, see Fig. 1. In this scheme, our analysis indicates the introduction of an offset that, it would depends on the secondary sources' position, so that, whether it occurs, the OPD must also produce a variation in the phase behavior, this is our initial hypothesis.

In Fig. 1, there are two types of the Young experiments, one where is a reference Young experiment united to Young experiment with an axially displacement for his respective sources. In order to illustrate the generation of an offset, both sources are joined by an imaginary vertical straight line, this is as, to generate a flat segmented surface to be co-phased.

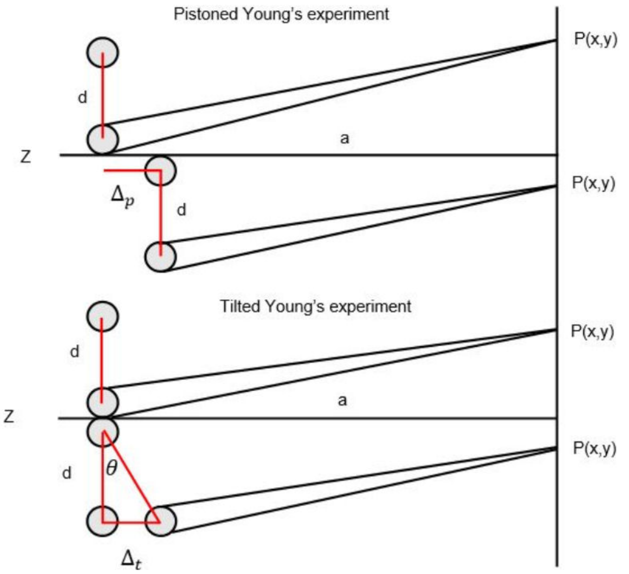


FIGURE 1. Sources performance as actuators, displacing $\Delta p$ and $\Delta t$ to generate piston and tilt, respectively.

In the same Fig. 1, can be changed only the position of one source, and thus, the behavior of a tilted segmented flat surface is obtained, notice that until an inclination angle $\theta$ for a tilt could be obtained, see Fig. 1. Then we do that the sources' position performs as small actuators of a flat segmented surface to be co-phased by using $\Delta_p$ and $\Delta_t$, as will be seen in the next sections to characterize piston and tilt respectively.

According to the aforementioned, we can show the experimental optical configurations required to detect piston or tilt respectively in optical systems with flat segmented surfaces in Fig. 2, notice that, even when spherical wavefronts are interferometrically compared, it would be necessary to use achromatic lens to avoid residual aberrations, and therefore, it is evident the different convergence for the light beams incoming to the detector in each case. Thus in both Figs. 1 and 2, spherical wavefronts are compared in the different detectors used and straight fringes should obtained in the patterns.

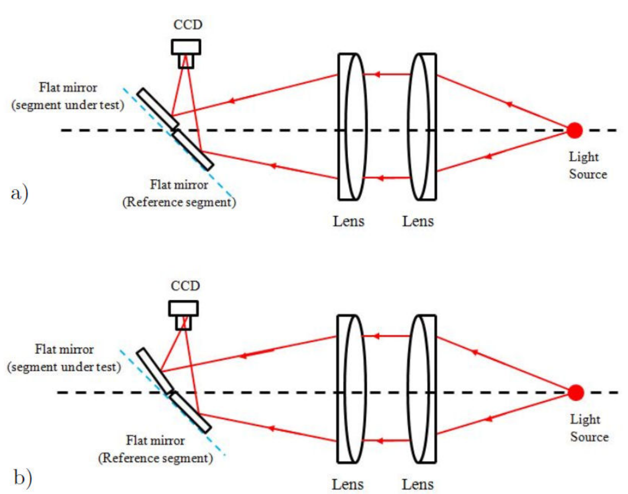


FIGURE 2. Optical configurations to detect surface errors, a) piston, b) and tilt, respectively.

Also, we can assume that, based on the knowledge on optical patterns produced for piston [3,4], by tilt approximation, or a combination of both [5], in this work, we have used the mathematical theory of a contiguous Young's experiment to generate patterns with these surface errors.

As is well-known, in any Zernike or Seidel theory and piston and tilt could be characterized with not problem, however it was founded as a set of pistons added to each point (x,y) of the sagitta surface can produce a tilt [5]. So that, the phase obtention with piston and tilt terms in Young's experiment was developed comparing the fringes in contiguous patterns, following with an analysis to evidence that the results can be used as measurement references for the phases during alignment process, where small tolerances of the order of fractions of $\lambda$ are not required, as is the case of surfaces for solar concentrators, [6] in the environmental engineering area or for clean energy applications, where, small changes in temperature for the panels not incite an examination of fine co-phasing tolerances.

For the case of optical instrumentation carried out for telescopes, the co-phasing must be finest, because the alignment tolerances are of the order of $\lambda/10$ or fraction of that [7], in addition, the alignment of big and heavy segmented surfaces in operation worldwide has turned out to be an important topic, for the scientific community dedicated to the construction of big optical surfaces, because the only way to successfully develop those types of projects, is solving the monetary budget plus the necessary techniques and knowing on how to obtain the required image quality, in case of making adjustments to procedures of tip, tilts and piston parameters, this can be corrected with active [7] and deformable systems [11].

Besides, by the diffraction size, these effects are unavoidable, because they are generated by the light obstruction with an edge of separation between segments, and the segments can suffer fractures in the alignment.

Some authors have said in the co-phasing study, [7] that, if an axial displacement of a segment or piston error in a segmented surface is minimized, probably to get a good alignment, from a mathematical point of view, to characterize a segmented surface turns out to be a very complicated task and in general, the hard work is to reduce mathematical ambiguities or perform an analysis by parts.

This work is distributed as follows. In Secs. 2 and 3 are shown the mathematics of tilt and piston characterization developed in Young's experiment. In Sec. 4, the factors involved in the combined case for the interference component of Young's experiment are found. Section 5 shows the methodology carried out in the simulations with Young's experiment for some cases analyzed in this work, in Sec. 6 the results obtained are discussed, and in Sec. 7 the conclusions and remarks are given.

## 2. Geometric approach for tilt in the Young's experiment

An analysis on variations of the secondary source's position of the order of $\lambda$ in one direction is developed.

It's known that, the Young's experiment uses the separation distance between secondary sources, $d$, distance from secondary sources plane to the observation screen, $a$ and the sources' distance to the observation screen, $(x, y)$.

We have analyzed that when a $\Delta_t$ is axially introduced in $z$-direction on one of the sources, the effect produced in Young's patterns is similar to an imaginary tilted surface for the formed plane with the two sources (see Fig. 1 for the tilted case), this is whereas the other reference plane also imaginary is maintained fixed in the reference Young's experiment, see red lines) and thus a co-phasing experiment is generated, where we could say that the secondary sources perform as mechanical actuators in a co-pashing experiment for two flat surfaces. With that we would expect an effect similar to the produced effect for a shift on the fringes' patterns in a segmented surface, [2–5], so that, a tilt term arises naturally on the Young's experiment, and so, it is possible to use the position for each source in the $z$-axis in according to the theoretical analysis for the Optical path difference, (OPD) in Young's experiment, [1], Fig. 3.

In Young's experiment as it is known, the interference function generates fringes by the recombination of the secondary sources on a later observation plane, $(x, y)$, placed on a certain distance $a$, measured from the secondary sources' plane and separated a distance $d$, (Fig. 3) [1].

In order to characterize a tilt term by using Young's experiment, one of the secondary sources would be displaced axially to $\Delta_t$, and we analyze the behavior of the interference function in the observation plane. For the fixed source, following [1], we have,

$$s_1 = \overline{S_1P} = \sqrt{a^2 + y^2 + \left(x - \frac{d}{2}\right)^2}, \tag{1}$$

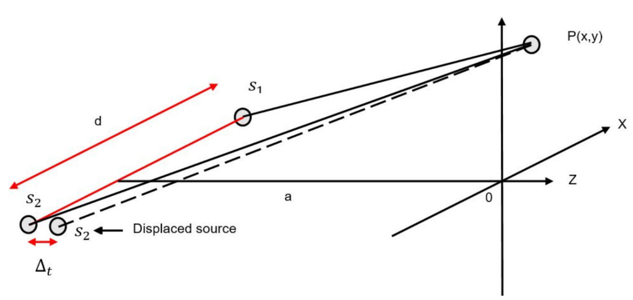


FIGURE 3. Theoretical scheme to characterize tilt with the Young's experiment. $s_1$ and $s_2$ represent sources generated by a primary source $s$ not shown.

whereas for the displaced source, a constant $\Delta_t$ is introduced to generate a tilt, as follows

$$s_2 = \overline{S_2P} = \sqrt{(a + \Delta_t)^2 + y^2 + \left(x + \frac{d}{2}\right)^2}, \tag{2}$$

after some simplifications for (1) and (2)

$$s_2^2 - s_1^2 = \Delta_t^2 + 2a\Delta_t + 2xd, \tag{3}$$

thus OPD is,

$$\Delta s = s_2 - s_1 = \frac{\Delta_t^2 + 2a\Delta_t + 2xd}{s_2 + s_1}. \tag{4}$$

However, according to [1], the fringes can be observed only if $d \ll a$, so, we have,

$$s_2 + s_1 \sim 2a, \tag{5}$$

replacing (5) in (4) we would obtain,

$$\Delta s = s_2 - s_1 = \frac{\Delta_t^2 + 2a\Delta_t + 2xd}{2a}, \tag{6}$$

considering n as the index of refraction of the medium where the experiment is realized, ($n = 1$ in air), the OPD of $s_2$ and $s_1$ towards the point P is,

$$n\Delta s = n\frac{\Delta_t^2 + 2a\Delta_t + 2xd}{2a}, \tag{7}$$

simplifying,

$$n\Delta s = n\left\{\frac{\Delta_t^2}{2a} + \Delta_t\right\} + n\frac{xd}{a}, \tag{8}$$

and therefore the phase difference, $\delta$, is

$$\delta = \frac{2\pi}{\lambda_0}\left(n\left[\frac{\Delta_t^2}{2a} + \Delta_t\right] + n\frac{xd}{a}\right). \tag{9}$$

Analyzing now the behavior of interference function, we can obtain their provisional characteristics for the total irradiance $I$, [1]; after some simplifications, it becomes, as it is well-known, a cuadratic cosenoidal function ($I = 4I_0\cos^2(\delta/2)$). However, this last equation in combination with (9) is the equation for the irradiance in Young's experiment as function of the phase difference $\delta$, considering the tilt term ($\Delta_t$) in this work. Nevertheless in Eq. (9), when $\Delta_t = 0$, we recover the original phase of the classical Young's experiment. Moreover, a second order equation is generated inside (9) when it is solved for $\Delta_t$ , $(1/a)\Delta_t^2 + 2\Delta_t + (2xd/a) = 0, 2\lambda, 4\lambda...$, or if $(1/a)\Delta_t a^2 + 2\Delta_t + (2xd/a) = (\lambda/2), 3(\lambda/2), 5(\lambda/2)...$, so that, we dare to say that, a constructive or destructive interference can respectively be observed for when,

$$\Delta_t = -a \pm \sqrt{a^2 - (2xd - m\lambda)}, \tag{10}$$

or

$$\Delta_t = -a \pm \sqrt{a^2 - \left(2xd - \frac{2m+1}{2}\lambda\right)}, \tag{11}$$

with $m = 0, 1, 2..$, and it can be interpreted as the physical parameters related with $\Delta_t$ for tilt.

## 3. Geometric approach for Piston in The Young's experiment

In order to generate an offset or the piston effect, we can add or subtract a constant term $\Delta_p$ of the order of $\lambda$ to both secondary source positions, with this, small increases or decreases of $a$ are produced, such effect represents the generation of other contiguos Young's experiment, so that, now the magnitude between the observation plane distance to the sources' plane suffers an increase or decrease. This hypothesis is tested by applying a $\Delta_p$ in the position of both sources at the same time, as follows,

$$s_1 = \overline{S_1 P} = \sqrt{(a+\Delta_p)^2 + y^2 + \left(x - \frac{d}{2}\right)^2}, \qquad (12)$$

and,

$$s_2 = \overline{S_2 P} = \sqrt{(a+\Delta_p)^2 + y^2 + \left(x + \frac{d}{2}\right)^2}, \qquad (13)$$

simplifying, $s_2^2 - s_1^2$, we have

$$s_2^2 - s_1^2 = -2xd, \qquad (14)$$

where the sign means that the phase will produce an effect in the opposite sense to $\Delta_p$. And as expected, it is a new Young's experiment obtained for piston, where we would have for the interference function,

$$I = 4I_0 \cos^2\left(-n\frac{\pi}{\lambda_0}\frac{2xd}{2a}\right), \qquad (15)$$

where "a" can be expressed simply as an " a-pistoned", and moreover

$$a = a_p, \qquad (16)$$

thus simply,

$$I = 4I_0 \cos^2\left(n\frac{\pi}{\lambda_0}\frac{xd}{a+\Delta_p}\right), \qquad (17)$$

so $a_p$ in Eq. (15) can be replaced without problem by $a+\Delta_p$ in the interference function for piston (17). This last equation is equivalent to adding a constant $\Delta_p$ at each secondary source in the piston case, as it was mentioned above.

Figure 4 shows the behavior for the phases obtained in the equations (17) $\delta_p = n(\pi/\lambda_0)(xd/a+\Delta_p)$ and (9) $\delta_t = (2\pi/\lambda_0)(n[\{\Delta_t^2/2a\} + \Delta_t] + n[xd/a])$, respectively. As it is shown, there are some points on the graph where the tilted phase and pistoned phase coincide, it indicates that both piston and tilt can have the same value in these points. Moreover, the curve with maximum slope is for the combined phase behavior ($\delta_t + \delta_p$), (18), therefore, we can assure that the combined case would produce a different pattern for the same parameters considered for $\delta_p$ and $\delta_t$ respectively to be observed in the followings images.

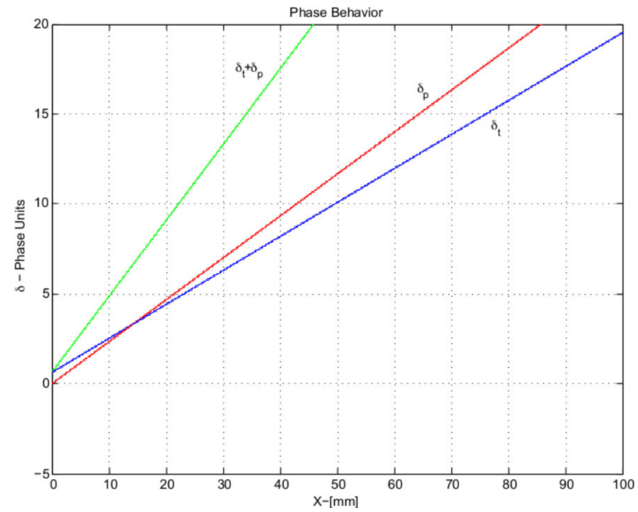


FIGURE 4. Phase Behavior for $\delta_p$ of (17), $\delta_t$ of (9) and $\delta_t$+$\delta_p$ of (18), with $\Delta_t = 0.1\lambda$ and $\Delta_p = 30\text{x}10^4\lambda$ a value quite elevated, this only to appreciate a change in the slope.

## 4. Combined effect

A combined effect for the generation of piston and tilt can be proposed for the final irradiance in Young's experiment in the plane (x,y), it could be as follows, after some simplifications, we have,

$$I = 4I_0 \cos^2\left(\frac{\pi}{\lambda_0}\left[n\left\{\frac{\Delta_t^2}{2a} + \Delta_t\right\} + n\frac{xd}{a} + n\frac{xd}{a+\Delta_p}\right]\right), \qquad (18)$$

here, the contribution of the piston and tilt can be seen separately, (18), however, in the case when $\Delta_t = \Delta_p$ in the piston term (18), and after simplifying the argument of the cosine function, we have,

$$I = 4I_0 \cos^2\left(\frac{\pi n}{\lambda_0}\frac{1}{2}\left[\frac{\Delta_t + 2a}{a}\right]\left[\Delta_t + \frac{2xd}{a}\right]\right), \qquad (19)$$

where however, if $\Delta_t = 0$ (case for zero tilt), we would expect only the piston effect, so that, the last equation becomes,

$$I = 4I_0 \cos^2\left(\frac{\pi n}{\lambda_0}\left[\frac{2xd}{a}\right]\right) \qquad (20)$$

meaning the factor 2 in the term $xd/a$ that a change in frequency of the patterns for piston in the classical Young's experiment is obtained, which is consistent with our assumptions of a change of plane for the piston effect of the Sec. 3.

TABLE I. Parameters used in the Young's experiments.

| # fringes | $\lambda_0$ [mm] | d [mm] | a[mm] | n | x[mm] |
|---|---|---|---|---|---|
| $\approx 7^a$ | $0.000632^b$ | 0.019 | 1000 | 1 | $100^c$ |
| $\approx 7$ | 0.000550 | 0.017 | 1000 | 1 | 100 |
| $\approx 7$ | 0.000475 | 0.014 | 1000 | 1 | 100 |

[a]In the central diffraction order. [b]From a geometric point of view used in the simulations. [c]Value equivalent to 512 pixels in the images.

## 5. Diffraction effect

As it is well-known, for the diffraction part, the interference pattern performing the classical Young's experiment is modulated by a diffraction function that, it contains the geometric characteristics of the apertures used. Therefore, the modulating function can be expressed as: $D = \sin(\pi\Delta l/\lambda_0)/(\pi\Delta l/\lambda_0)$, where, $\Delta l = n\Delta s$ with $\delta = n(k)(\Delta l) = n(k)(\Delta s)$, for this work. By this way, we have obtained a geometric approximation of characterization of some surface errors, where we attempt to generate patterns as close to reality as possible for the mathematical equations of diffraction.

With the initial conditions shown (Table I) and with the involved math in Secs. 2 , 3 and 4, images in grey levels of $512 \times 512$ pixels were generated, where the half of each image was considered for the classical Young's experiment and the other half for the different cases analyzed. The Figs. 5b, 5d, 6b and 6d were obtained for the rows 128 and 384 in the simulated images, because they are at the midpoint and far from the interface.

In Fig. 5a) a comparison of stripe shifting for the classical Young's experiment of reference and when the sources has not been moved are shown. As can be also viewed in Fig. 5b), the respective curves for both fringes on the behavior phase appears aligned, therefore it indicates that Fig. 5a) can be considered as a reference pattern.

Figure 5c) shows a comparison of stripe shifting for the classical Young's experiment and when only one of the secondary sources, $s_1$, has been displaced $\Delta_t = 0.5\lambda$ to generate tilt (similar effects should be obtained in the opposite direction for when the secondary source $s_2$ is displaced). In

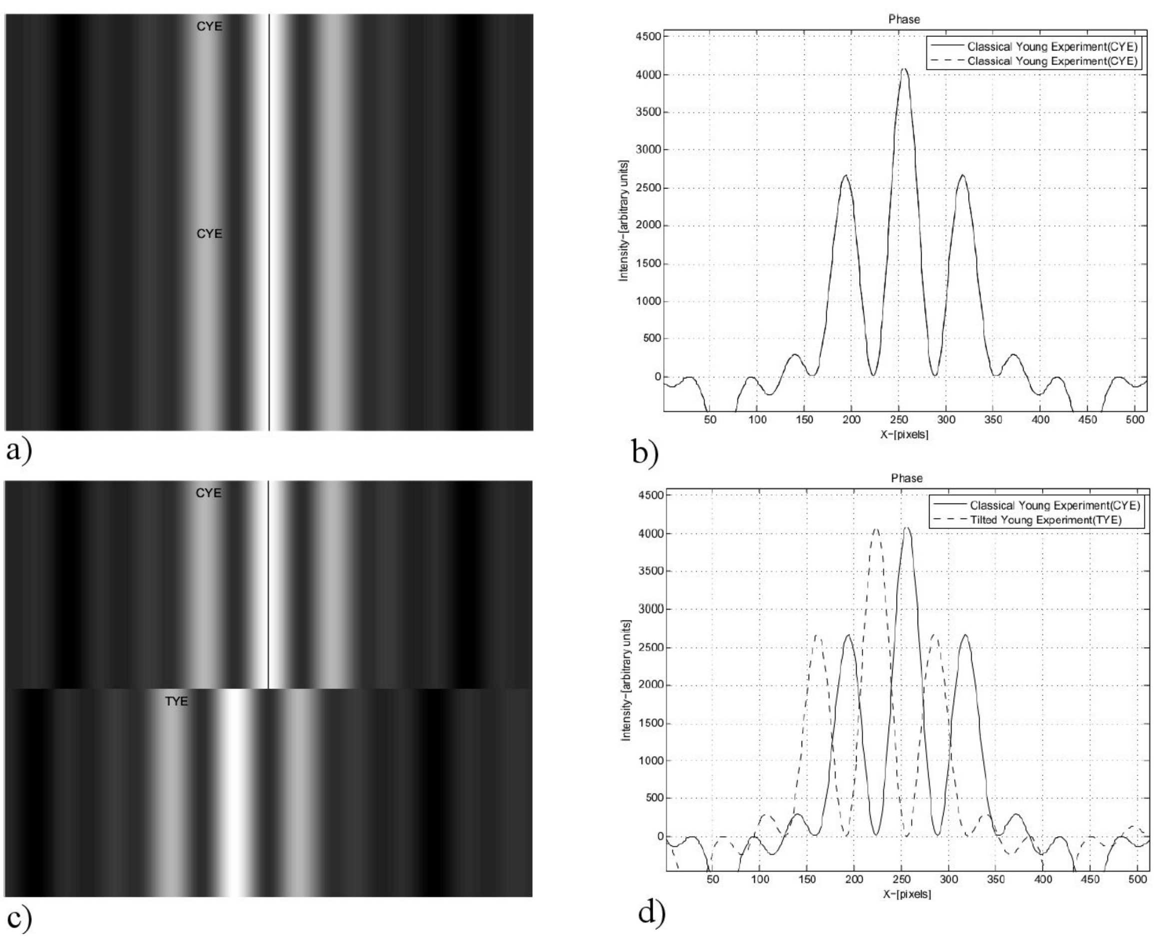


FIGURE 5. a) CYE: Classical Young's experiment in both middle images. b) Alignment case $\Delta_t = 0$, $\Delta_p = 0$ and $\lambda$ =0.000632 mm. c) CYE: classical Young's experiment and TYE: Tilted Young's experiment. d) Phase shifting for tilt, with $\Delta_t = 0.5\lambda$, $\Delta_p$ =0 and and $\lambda = 0.000632$ mm. Comparison of the phase behavior for the diffraction central order in the double slit experiment.

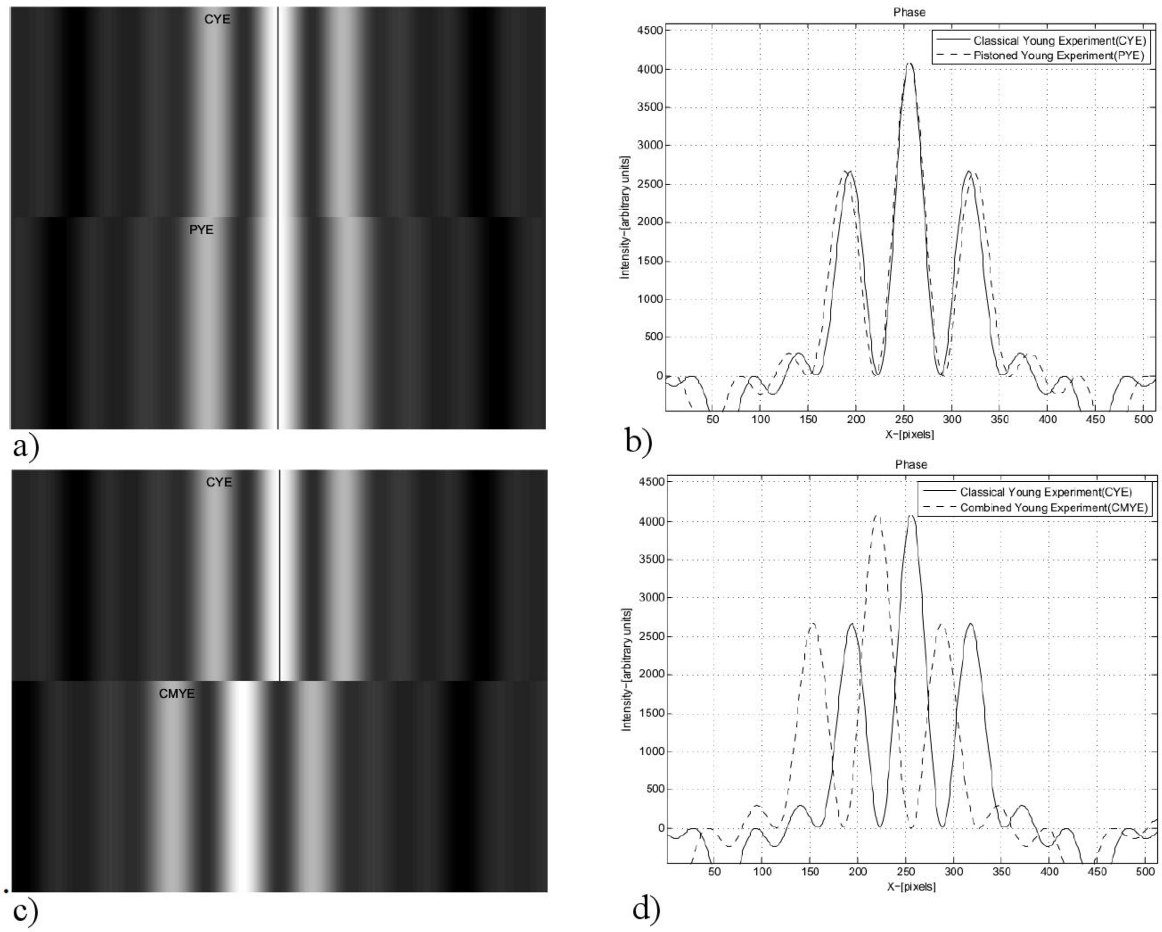


FIGURE 6. a) CYE: classical Young's experiment and PYE: Pistoned Young's experiment. b) Phase frequency Change for piston, with $\Delta_t = 0$, $\Delta_p = 30 \times 10^4\ \lambda$, and $\lambda = 0.000632$ mm. c) CYE: Classical Young's experiment and CMYE: Combined Young's experiment. d) Combined case $\Delta_t = 0.5\lambda$, $\Delta_p = 30 \times 10^4\ \lambda$ and $\lambda = 0.000632$ mm. Comparison of the phase behavior for the diffraction central order in the double slit experiment.

this Figure the striping suffers a shifting towards the image left in presence only of tilt, this fact is checked in Fig. 5d), since the peak of central diffraction order is also displaced.

As comparison, in Fig. 6a) is the phase behavior for both sources moving in the pistoned Young's experiment (PYE). As can be viewed, the striping suffered a change in frequency, because the pattern's shifting is now in both directions in the dotted curves (see Fig. 6b) towards the left and right, so that, we can appreciate an aligned central fringe in the image for both fringes of reference and with the sources displaced, this is a characteristic already known in the piston patterns, [2–5].

Figure 6c) was obtained moving both sources as in Fig. 6a), after this, a small displacement of one source is realized step by step, and it can be observed that the central fringe in the image begins to suffer a shifting towards the left, as can be viewed in Fig. 6d). Moreover, if we compare the Fig. 5d) with Fig. 6d), we can notice some differences to the behavior of the secondary diffraction orders, *e.i.* the diffraction orders are lapping in different highs, it is attributed to the shifting and frequency change produced at the same time in the fringes pointing out, therefore, the effects in the patterns are not similar to tilt, piston and the combined case viewed separately.

Even more, it is worth mentioning that when $\Delta_t = \Delta_p = m\lambda$ with m even and $\mid$ *piston* $\mid > \mid$ *tilt* $\mid$, a predominant surface error would be obtained, in this case, the piston effect is more notorious.

In the case when $\Delta_t = m\lambda$ with $m$ even, and $\Delta_p = n\lambda$ with $n$ odd, is obtained a predominant pattern for tilt. This last case showed that is independent of $\mid$ *piston* $\mid > \mid$ *tilt* $\mid$ or $\mid$ *piston* $\mid < \mid$ *tilt* $\mid$.

So, all these effects in the patterns could be also called ambiguities of the order of $\lambda$, which has been resolved already by using white light [8] with several wavelengths experimentally [9, 10], where it has been developed co-phasing with white light, by means of a Michelson interferometer and colored fringes has been aligned. Therefore in Fig. 7 and only as illustration, we try to do something approximate to [10] and with this to observe the not coincidence of the colors in the patterns for a geometric approximation in wavelength. Thus, 3 different Young's experiments with different geometric wavelengths, for the blue, 475 $\eta$m Fig. 7a), for the

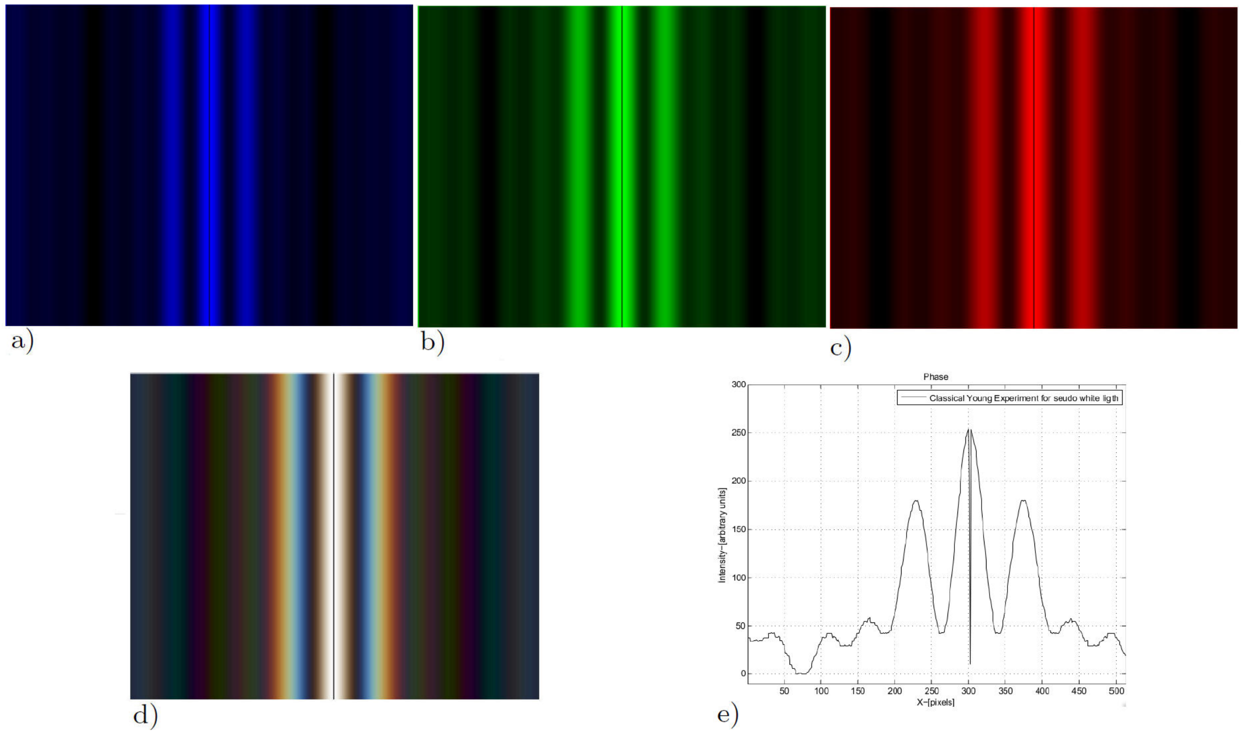


FIGURE 7. a) $\lambda_1$ =475 nm (0.000475 mm). b) $\lambda_2$ =550 nm (0.000550 mm). c) $\lambda_3$ =632 nm (0.000632 mm). d) $\lambda_1 + \lambda_2 + \lambda_3$. e) Crossed-section of Diffraction spectrum of d). 3 different $\lambda$ used in Classical Young's experiment has been assigned to produce RGB images. Notice the tendency towards the white color in the fringes in d) when the 3 images are superimposed and e) Diffraction spectrum of d). In all cases the geometric parameters were mantained.

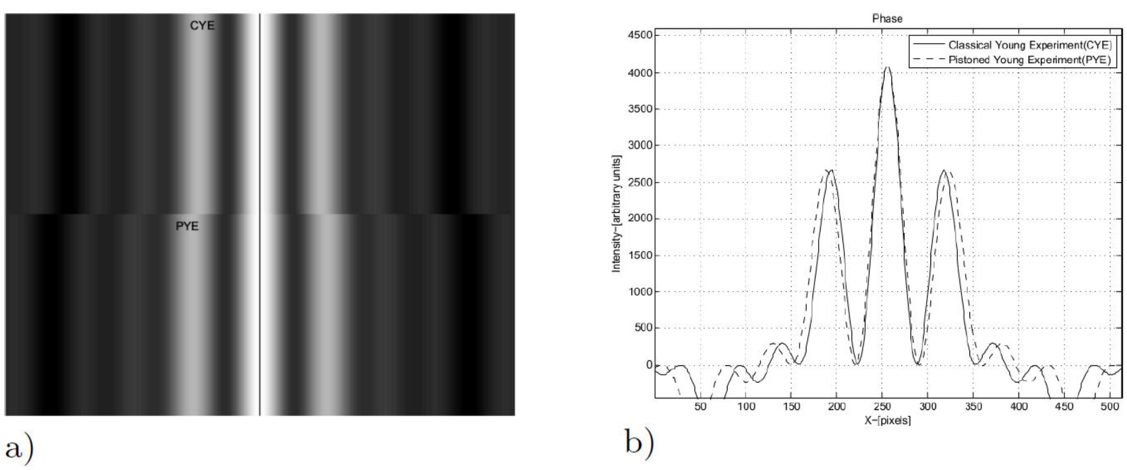


FIGURE 8. a) $\lambda = \lambda_1 + \lambda_2 + \lambda_3$, $\Delta_t = 0.5\lambda$, $\Delta_p = 30 \times 10^4\ \lambda$. b) Crossed section for a). Behavior of the combined effect in the segmented Young's experiment by using seudo-white ligth. It is noticed that a secondary maximum diffraction order achieves to appear in the image.

green, centered in 550 $\eta$m, Fig. 7b) and the for red in 632 $\eta$m, Fig. 7c), have been generated. The idea is simply to superimpose the 3 last images to produce only one RGB pattern, something like when the star image is obtained by using filters, as can be viewed in Fig. 7d), where the fringes have a change frequency, (it is a similar effect produced by piston [3, 4]) in each wavelength , and as expected, the fringes in different RGB color only would coincide for $d = 0$, it is when the secondary sources are superimposed. Moreover, a crossed section for the diffraction spectrum curve in Fig. 8a) was generated, so that by comparing with Fig. 7b), it is possible to appreciate that the maximum peak is found around 300 in x-pixels position on the graph, which shows a shift towards the right, as result we found a produced effect by adding 3 seudo wavelengths, however in all the other figures, this is not so. And in Fig. 8b) appears the behavior of the diffraction orders when piston and tilt are present, so that, we can observe the not coincidence between orders. Also, a second diffraction lobule sneaks on the image, it is an indicative of a piston effect.

## 6. Remarks

Once understood the mathematical theory of Born and Wolf on the Young's experiment, surface errors look like to piston and tilt in a co-phasing numerical experiment by means straight lines were obtained, Fig. 2. This work helps to better to understand the physical parameters involved from a geometric point of view.

As can be viewed in the patterns, for the cases of piston, tilt and for the combined case, tilt and piston can be differentiated one from other, so that a pattern with pure tilt or with pure piston can be analyzed.

In the case of fringe's generation looks like to piston patterns, it is possible to appreciate a inversely proportional behavior for axial shiftings of the secondary sources, so that, for big pistons (of the order millimeters or bigger, not shown) the obtained patterns were easier viewed in the simulations, it is due to tilt kept up a lineal behavior.

Finally, diffraction effects were considered, because in the real Young's experiment, the diffraction function modulates the interferometric phase, at the same time, the aperture geometry for each segment distorts the patterns as is the real case in the James Webb Telescope, where diffraction errors due to the interfaces between segments produce additional effects and also modulate the phase shape.

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