Introduction to the Fourier transform studying the oscillations of a pendulum

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Students of physics, engineering and related majors are generally not aware of how useful it is to transform a signal from the time domain to the frequency domain. The mathematics that makes this transformation possible is well known to senior students but vaguely applied in their experimental work. The basic phenomena studied in elementary lab courses that can provide us with frequency information, the pendulum and the spring, are minimized by focusing only on obtaining the mathematics dictated in books. The pendulum is the most studied physical system taught in laboratories from precollege up to college levels; it is analyzed mathematically in most courses in Physics and Engineering curricula. It is used to introduce wave phenomena. However, this teaching is focused on the plain demonstration that periods are invariant to suspended masses, if and only if the oscillation is within angles not greater than 10 degrees from their normal. The use of technologies, computational and electronic, also focuses on the demonstration of such assertion. In the present work, a mechanical-electrical system was designed that allows to observe, in real time, on the screen of an oscilloscope, the swinging behavior of a pendulum. This system makes evident that the swing movement of a pendulum can be described by a sine function, but also with this same system, and with the help of a digital oscilloscope, it is possible to simultaneously observe the signal generated in the temporal domain and in the frequency domain. This innovation not just breaks the paradigms of teaching but also promotes an alternative to valuable observations that promote understanding.

Keywords: Pendulum; oscilloscope; period; frequency; Fourier transform.

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1. Introduction

"*Why does this magnificent applied science which saves work and makes life easier bring us so little happiness? The simple answer runs because we have not yet learned to make sensible use of it.*" Albert Einstein [1]. Teaching paradigms is often unappealable and immutable; because teaching is generally focused on the repetition or demonstration of mathematical results obtained for long periods of time. Despite today's technology, teaching remains focused on employing systems like those employed in previous centuries.

The pendulum is a phenomenon that has been taught for more than 300 years and pretends to repeat the way in which Galileo demonstrated it. Galileo first became interested in the pendulum movement of a bell in 1684 and applied its motion to the clock in 1640 [2-3]. Today's technology, high-cost electronic systems, and computer simulations are used mainly to demonstrate what the related theory predicts.

In this work, we propose to analyze the swinging motion of a pendulum both in the temporal and in the frequency domains, in order to introduce the concept of Fourier Transform. We start from the premise that such motion generates a periodic signal, which is directly related to sinusoidal signals, necessary and sufficient condition to introduce the mathematical tool known as Fourier Transform [4] and concepts such as period, frequency and phase.

2. Motivation

Three of our senses: sight, touch, and hearing, identify signals in frequency. In other words, our senses do not identify signals in the temporal domain but do so in the frequency domain. Given this assertion, the following question arises: *Why are signals analyzed only in the temporal domain? One of the possible answers is: Because it is "easier" to interpret signals in the temporal domain than to observe signals in the frequency domain.*

The use of the Fourier Transform has been stigmatized for years, to such a degree that despite being known for more than 200 years, today it is still considered as a very complex tool to interpret. The intention is not to minimize the complexity of the mathematical tool that supports the Fourier Transform; but to show that through a very simple phenomenon it is possible to understand a very complex transformation.

It is important to mention that physics students know theoretically the Fourier transform, but do not know how to apply this knowledge to specific situations.

A survey shows that students of the last semesters know the Fourier transform but they have never applied it to time signals obtained in the lab. In a similar survey carried out with teachers, the results were slightly different.

It seems that even though our environment is surrounded by frequency language, in communication, entertainment devices, radio and television, formal education does not con-

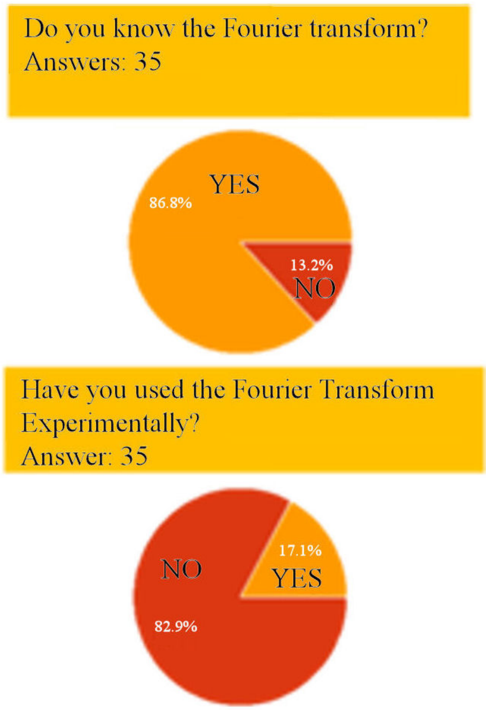

FIGURE 1. Survey carried out on students in the last semesters of the physics curriculum.

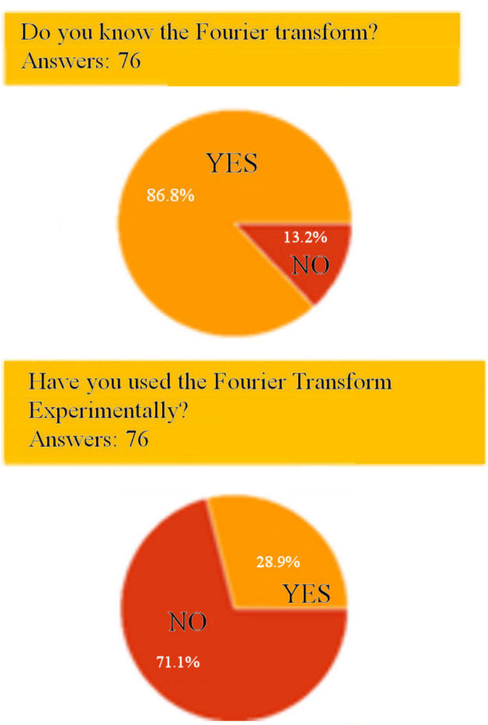

FIGURE 2. Survey made to professors and researchers during a meeting of experimental colleagues.

sider these notions important. A pendulum is an adequate device to introduce students to the world of the Fourier transform.

The system proposed in this work helps, to visualize its sinusoidal behavior in real time, which can be seen on the screen of an oscilloscope. Given this innovative way of graphing the swinging motion of a pendulum in real time, it is also possible to measure its period. These measurements are essential to introduce students to observe signals in the frequency domain [5-7].

Without a doubt, a very important aspect, is the use of a digital oscilloscope to introduce students to the *Fourier Transform*. This important tool can be used, at basic levels, with the simple idea that there are two ways of observing a sinusoidal signal: Time domain and frequency domain.

3. The theoretical pendulum

Most of the revised literature focuses on the analysis of the movement of a pendulum by reproducing the Simple Harmonic Motion, a concept that comes from the analysis of the elongation and compression of a spring which is governed by Hook's law, [5-13].

A pendulum is integrated by a suspended mass and a thread (Fig. 3). When the mass is separated from its equilibrium position, a swinging motion starts.

Under the premise that the mass m is released at an angle ϕ less than $15°$, it's possible to get Eq. (1), [5-13].

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1}$$

The study and analysis of a pendulum focuses on observing and obtaining results from the swinging motion of a mass released at an angle $\phi \leq 10°$, the purpose of which, is usually to check Eq. (1). Although it is worth mentioning that there are works that predict the period of a pendulum for larger angles [14,15].

With the system proposed in this work, given its stability, it is possible to experiment and demonstrate Eq. (1). In addition, with the electromechanical system described below, it is possible to show, in real time, the sinusoidal behavior that governs a swinging motion, since it is drawn on the screen of an oscilloscope. In addition, it is possible to measure directly on the screen the period of the oscillating mass. All this information was only possible to visualize by simulations or

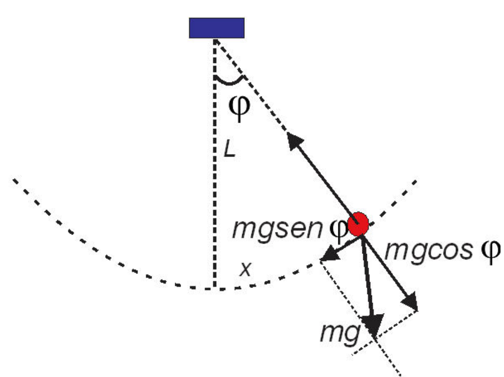

FIGURE 3. Forces that interact in the pendular motion.

graphing the acquired data. These systems are very expensive in the market, and [16-18] require other means to gather data to process and show the sinusoidal behavior. We present a low-cost system that allows to observe, in real time, the sinusoidal behavior.

Digital oscilloscopes are currently very low cost. An oscilloscope that can Fourier transform costs minimum around 200 dollars (approximatel y 4 thousand Mexican pesos. A laptop has a minimum price of 300 dollars. If you want to analyze an oscillatory movement in real time, you must buy the interface and the software. With these systems it is possible to observe the two-way movement. A second software would be required if you want to analyze the sinusoidal signal in the frequency domain. This long journey to study a sinusoidal signal, in the temporal and frequency domain, would not be didactic for teaching the Fourier Transform. not including the spectral analysis software, although the software is free to use, the hardware is required to acquire the signal from the pendulum.

The simulation of a pendulum is limited to observing the behavior in the time domain; in general, either due to ignorance of the existence of the FFT or because of how complicated it would be to simulate the FFT.

4. Mechanical-electrical system

The system that we present is low cost and uses easily accessible materials. It consists of a voltage splitter [19-21] (Fig. 3). The splitter was built using a potentiometer, whose power supply is placed at its ends. V_{out} is measured at the center. This device and the potentiometer are placed inside a mechanical system (Fig. 5); the splitter circuit is powered by a 9 Vols battery.

Resistance changes caused by swinging motion provide voltage changes. These voltage variations are recorded and plotted on the oscilloscope, see Fig. 6.

The complete experimental setup of the electromechanical system is shown in Fig. 7. It is worth mentioning that in some bibliographies schematic drawings are exposed over the sinusoidal behavior of the pendulum, [11,12]. Figure 8, it is important to note that these diagrams do not represent the actual period of the mass oscillation, it is only a representation to demonstrate the sinusoidal behavior of a pendulum.

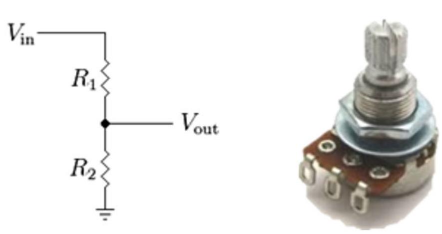

FIGURE 4. Electronic Circuit Voltage Splitter, the right side shows the potentiometer used.

Support point for m and L variations

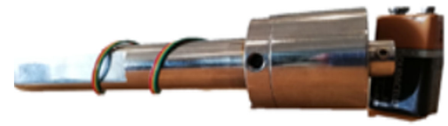

FIGURE 5. Mechanical system where the voltage splitter is installed; experiment with the different masses m and lengths L.

FIGURE 6. Sinusoidal signal of the pendulum motion, on this oscilloscope screen, you can observe the period of that signal $|\Delta x| = 1.46s$.

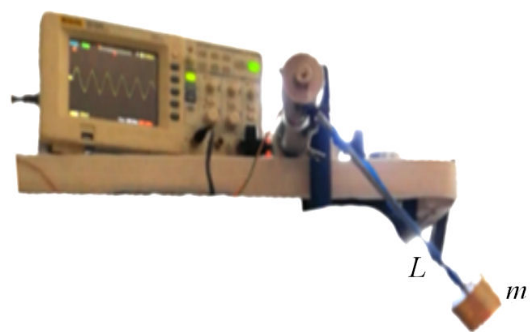

FIGURE 7. Experimental setup. Oscilloscope connected to the electromechanical system, to study the motion of a pendulum.

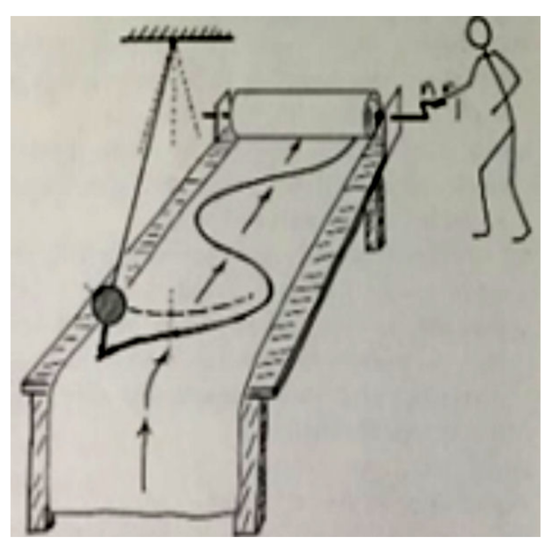

FIGURE 8. Systems, rarely used, to record or draw the oscillating motion produced by a pendulum or spring.

5. Exercise

Several masses were used for demonstration purposes (Fig. 9).

The obtained results showed that, for a length $L = 0.57$ m, and for different masses 72.5 g, 161.9 g, 517.7 g and 545.2 g, an invariant period measure and independent of the mass is measured; obtaining an experimental value $T_{(\rm exp)} = 1.521$ s, while the theory predicts, applying Eq. (1), $T_{(\rm Teo)} = 1.52$ s. Our goal is to show that this system can be used to introduce students to the Fourier Transform.

With the sine wave observed directly on the screen of an oscilloscope it is possible to obtain, with the same instrument, The Fourier transform, in other words, observe the signal in the frequency domain (Fig. 10).

Figure 10, picture on the left, shows the record of a swinging motion produced by a pendulum. A zoom on the oscilloscope screen, right image, shows the values obtained, period $T = 1.12$ s yellow line and the frequency $f = 893$ mHz blue line, from the oscillation, if we were to apply the well-known relationship to obtain the frequency from the period we would obtain [22,23].

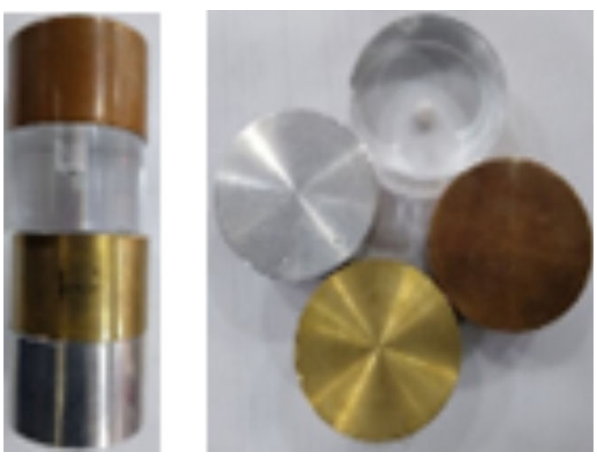

FIGURE 9. Masses used for the demonstration of Eq. (1).

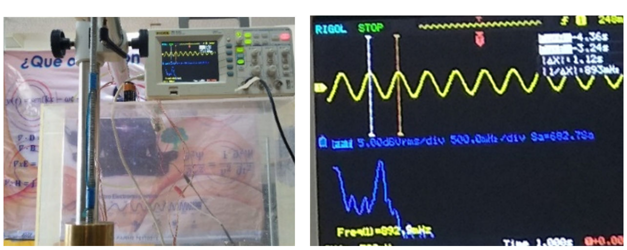

FIGURE 10. Experimental system, measurement and observation of the sinusoidal signal, yellow line, and its corresponding Fourier transform, blue line $f = 1/T = 1/1.12\ \ {\rm s} = 0.8928 \approx 0.893 = 893 \times 10^{-3}$ Hz.

6. Time domain or frequency domain

A very important feature of a periodic signal is its oscillation frequency or its period. Under this premise, it can be said that: There are two different ways of observing a periodic signal:

1.- Observe its evolution over time.

2.- Display the signal in the frequency domain.

The latter is possible thanks to Joseph Fourier [4,24-29]; who developed a tool capable of observing the signal, or signals, that give a certain appearance to a signal observed in the time domain. This tool is better known as the Fourier Transform.

The impact that Fourier's analysis has had in mathematics, physics and engineering is beyond doubt and, in fact, we could say, without fear of mistake, that modern science and technology would not have had the advancement that is now without the development of Fourier's ideas. Their importance lies t in the information they provide when applied to sinusoidal or periodic signals.

7. Two different domains, two different graphs, two different types of information and only one signal

Without doubt one of the better known natural phenomena is the rainbow, which occurs when it just rained on a sunny day (Fig. 10). Rainbows are more than a natural wonder or a physical process of filtration of sunlight by water droplets, it's a different way of seeing the white light!

From this example we can understand better what we mean by analyzing a phenomenon in two different domains.

Figure 12 shows the decomposition of white light as it passes through a prism.

This is the precisely what the Fourier Transform does on a physical signal: white light represents the temporal domain,

FIGURE 11. Double rainbow over Ciudad Universitaria.

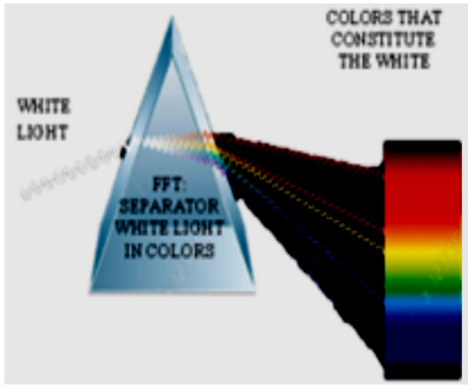

FIGURE 12. Analog interpretation to understand the frequency domain, colors, and temporal domain, white light.

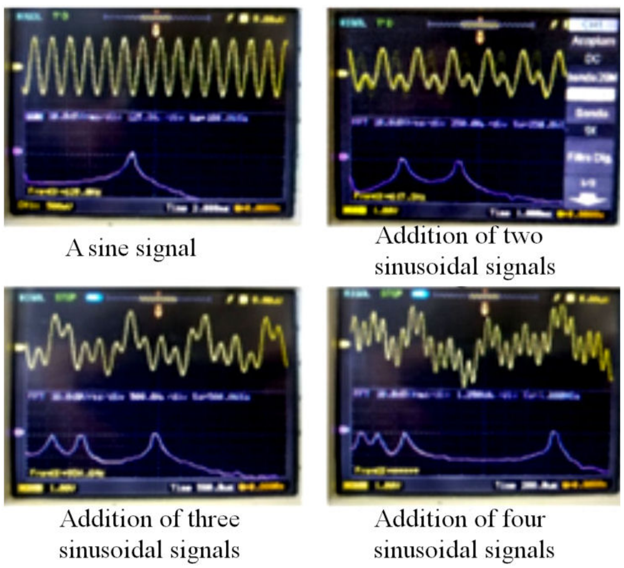

FIGURE 13. Signals in the time domain and their corresponding representation in the frequency domain.

and the colors that constitute it, represent the frequency domain. The prism is the tool that allows to visualize those colors, *i.e*. the frequencies. In other words, a signal can be decomposed in many sinusoidal signals and the Fourier transform allows us to visualize them.

Once the transformation of a sinusoidal signal from the time to the frequency domain is understood, more complicated signals can be analyzed. With the financial support of the project PAPIME PE110216 we built a variety of mechanical devices, instruments, and systems, which would allow the analysis of sinusoidal signals to be studied in the Frequency Domain [26]. One of the examples made with two of these instruments, four-signal generator, and four-signal adder, is shown below. Figure 13 shows the graphs of signals in the time domain, yellow line; while the lower graph, blue line, represents the signal in the frequency domain.

Let's dwell a little on the results shown by the images in Fig. 13. A sinusoidal al signal is represented by a single peak in the frequency domain. A signal in the time domain composed of three sinusoidal signals with different periods and phases would be represented by three peaks in the frequency domain. Students can create more complicated signals in the time domain with a signal generator, watch them in the time domain in the oscilloscope and then watch them in the same instrument. This information can be taught in the most elementary levels of education, and its depth can be increased as students advance in their knowledge of signal analysis according to the school level and the area they are studying. Figure 13 shows the importance of analyzing signals in the frequency domain. Each peak represented in a blue graph in the lower part of each image; represents a sinusoidal signal. The sum of these sinusoidal signals gives the time signal that appears in the upper part of the figure. This is a very simple example to show the students the importance of observing a signal in two different domains, temporal and frequency.

8. Conclusions

One of the core objectives of this work was to present a device that helps on the analysis of a sinusoidal signal in two different domains, time and frequency; by analyzing the swing motion produced by a pendulum, a well known physical system by students of all levels. The paradigms of teaching science and technology are focused on the temporal analysis of physical phenomena. This is due to the erroneous idea that observing or analyzing an oscillating behavior in the frequency domain, implies the mathematical understanding of the Fourier Transform. In a similar way that using a computer does not imply the understanding of the circuitry, and driving of a car does not imply knowing the operation of the engine, students can work in the frequency domain in high school. This work aims to invite teachers of physics and engineering to initiate analysis of sinusoidal signals in both domains described above. On the innovative proposal presented in this paper, the swing movement described by a pendulum is governed by a sinusoidal function, no analogous was found in any of the revised literature up to the date the current work was submitted for publication. To name a few, but in reality there are a wide variety of these pages, better known as blogs where tools and devices are developed for the study and analysis of a pendulum.

The proposal of this work is novel and original due to the way of introducing the *Fourier Transform* using the oscillatory or periodic movement [22]; this being one of the best known phenomenon in nature or mechanics; and with which it is possible to apply and explain the Fourier Transform. Most of the reviewed bibliography introduces the Fourier transform by studying signals and generally analyzing a certain physical phenomenon, [24-28]. With the proposal presented in this article, it is possible to introduce the concept of the Fourier transform by experimentally observing on a screen the graphic behavior of the oscillatory movement in the temporal domain as well as in the frequency domain, as explained in Sec. 7.

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