The relativistic connection between harmonic bidimensional electrostatic and magnetostatic fields

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The antecedent of this contribution is [1], which constructed the harmonic bidimensional expansions for the electrostatic and magnetostatic potentials, produced by a straight line with uniform charge and uniform current distributions, respectively, in Cartesian, and cylindrical circular, elliptic and parabolic coordinates. For the successive geometries, the sources are confined in the respective cylinders containing the source line, plus induced sources in two grounded flat, elliptical and parabolic plates; the potentials are continuous at the source cylinder and vanish at the grounded plates. In the electrostatic case, the electric intensity field is evaluated as the negative of the gradient of the potential; in the magnetostatic case, the magnetic induction field is the rotational of the axial potential. Both potential and force fields are bidimensional, and the equipotential surfaces and force fields are orthogonal. The normal components of the electric field at the source cylinder show a discontinuity, which according to Gauss's law is a measure of the surface charge distribution; in contrast, the tangential components are continuous due to the conservative character of the electrostatic force. The normal components of the induction field are continuous due to its solenoidad character; its tangential components show a discontinuity which by Ampere's law is a measure of the linear current intensity. Figures 1-4 illustrate the equipotentials on the left and electric field lines on the right; and the magnetic field lines on the left and the equipotentials on the right, exhibiting also their respective orthogonalities. The differences between the electric and magnetic multipoles are recognized, but we can still ask if there is a connection between them. The answer is given here in terms of the Lorentz transformations of the four-vector potentials and sources, and of the antisymmetric force field four-tensor.

Keywords: Bidimensional; harmonic; electrostatic; magnetostatic; force fields; sources; potentials; Lorentz transformations.

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1. Introduction

The article in [1] involved the ideal case of sources uniformly distributed along an infinite straight line parallel to the axis of circular, parabolic and elliptic cylinders, with the consequence that the associated electrostatic or magnetostatic potential and force fields are independent of the axial direction, and bidimensional in the circular, parabolic, elliptic and Cartesian coordinates, in each transverse plane. In the Cartesian case there are two flat parallel grounded plates, and in the cylindrical elliptic and parabolic there are also grounded plates of the respective shapes. The analysis starts from the harmonic expansion of the logarithmic potentials Eqs. (19), (21) satisfying the boundary conditions at the respective plates, and distinguishing between the inner and outer harmonic components in the source circle, ellipse, parabola or plane, as well as their continuity at the same boundary. In the electrostatic case, the electric intensity field is the negative of the gradient of the scalar potential Eqs. (22E): its normal components at the source boundaries are discontinuous by Gauss's law Eqs. (23E), yielding the harmonic surface charge densities Eqs. (24E); its tangential components are continuous due to the conservative character of the force field. The same boundary conditions and connections apply for the grounded plates Eqs. (24E). In the magnetostatic case, the magnetic induction field is the rotational of the axial magnetostatic potential Eqs. (22M): its normal components at the source boundaries are continuous due to its solenoidal character; its tangential components are discontinuous by Ampere's law Eqs. (23M), yielding the harmonic linear current densities Eqs. (24M). The same boundary conditions and connections apply at the grounded plates Eqs. (24M). Figures 1-4 illustrate graphically the above connections for both electrostatic and magnetostatic cases for the lower multipolarities $m = 0, 1, 2, 3$. In the electrostatic case, the left side corresponds to cross sections of scalar equipotentials and the right side to electric intensity field lines ending or starting perpendicularly to the grounded plates. In the magnetostatic case, the left side corresponds to closed magnetic induction field lines tangential at the grounded plates, and the right side to cross sections of vector equipotentials. Notice in both cases the orthogonality of each pair of Figures, due to different and complementary reasons. In any case, it may be ascertained that by crossing the electrostatic (magnetostatic) field with the axial unit vector $\hat{k}$ leads to the magnetostatic (negative electrostatic) field, and the natural question is: What is the reason for this?

Electrostatics and Magnetostatics were originally investigated separately. Electromagnetism as discovered and developed by Oersted, Ampere, Biot and Savart was the first step to identify their connection. The additional steps by Faraday, Lenz and Henry describing time dependent magneto-electric induction; and by Maxwell on the also time dependent electromagnetic induction led to the formulation of the Dynamical Theory of the Electromagnetic Field, with its prediction of the existence of electromagnetic waves [2–6].

On the other hand, Einstein in his 1905 article *On the Electrodymamics of Moving Bodies* [7] developed the theory of Special Relativity, recognizing that Maxwell equations are valid in inertial frames, that the ether hypothesis is superfluous, and the speed of light is the same for all observers in inertial frames; this allowed him to deduce the space-time Lorentz transformations, and their kinematic, dynamic and electrodynamic consequences. In 1907, Minkowski recognized the four- dimensionality of space-time, the four-vector character of the current and charge densities as sources of the four-vector (trivector and scalar) electromagnetic potentials, and the anti-symmetric four-tensor character of the electromagnetic force fields, in his article *Space and Time* [8]. See also [9].

These characteristics and their connections are applicable to electrostatic and magnetostatic sources, potentials and force fields of the same multipolarity. Correspondingly, in Sec. 2 the Lorentz transformations of four-vectors and four-tensors are reviewed. The respective Lorentz transformations are applied in Sec. 3 to the harmonic bidimensional potentials and sources, and electrostatic and magnetostatic force fields in Ref. [1], for inertial frames with axial and transverse motions, illustrating their explicit relativistic connections. In Sec. 4 the results are discussed following Einstein's extended question of How do the electrostatic and magnetostatic fields look in the inertial frames? The Lorentz invariants are also identified.

2. Lorentz transformations

This section presents the coordinate and time transformations of events in the rest frame S and a moving inertial frame S' with constant velocity v along the x axis, with coinciding origins and parallel axes at the initial instant of time $t'=t=0$ [2–9]. The same transformations hold for the respective components of source densities $(I/c,\lambda)$ and the four-vector potentials $(\mathbf{A},\Phi)$. Their explicit forms are respectively:

$$x'=\gamma(x-vt),\qquad y=y',\qquad z=z',$$
$$t'=\gamma\left(t-\frac{v}{c^2}x\right),\quad I_{x'}=\gamma\left(I_x-v\lambda\right),\quad I_{y'}=I_y,$$
$$I_{z'}=I_z,\quad \lambda'=\gamma\left(\lambda-\frac{v}{c^2}I_x\right),\quad A_{x'}=\gamma\left(A_x-\frac{v}{c}\Phi\right),$$
$$A_{y'}=A_y,\qquad A_{z'}=A_z,\qquad \Phi'=\gamma\left(\Phi-\frac{v}{c}A_x\right).\qquad (1)$$

where $\gamma=1\big/\sqrt{1-v^2/c^2}$ is the time dilation factor.

The transverse components remain the same, while the longitudinal component is a superposition of the longitudinal and time components, in the proportion $1:v/c$ and opposite signs; and the time component is the superposition of the time and longitudinal components in the same proportion and opposite signs.

On the other hand, the electric intensity field and the magnetic induction fields as the components of the antisymmetric force field tensor have transverse components as superpositions of their own components and the cross product of the velocity of the moving inertial frame with respect to the rest frame and their companion field, and their components along the velocity remain the same:

$$E_{x'}=E_x,\quad E_{y'}=\gamma\left(E_y-\frac{v}{c}B_z\right),$$
$$E_{z'}=\gamma\left(E_z+\frac{v}{c}B_y\right).$$
$$B_{x'}=B_x,\quad B_{y'}=\gamma\left(B_y+\frac{v}{c}E_z\right),$$
$$B_{z'}=\gamma\left(B_z-\frac{v}{c}E_y\right).\qquad (2)$$

It may be recognized that the cross vector product of the velocity and the transverse fields connects with the last line of the first paragraph in the Introduction.

3. The relativistic connections of potentials, sources and force fields

The application of the Lorentz trasformations to the electrostatic and magnetostatic bidimensional harmonic expansions in Ref. [1] is straightforward and implemented next. For the sake of clarity, two changes of notation are made: 1. The prime index for the source points in the equations in Ref. [1] is replaced by a sub index zero, and in this way the prime index in the moving frame for the Lorentz transformations can be used directly. Additionally 2. for didactic reasons the presentation follows the order of C) Cartesian, c) circular, e) elliptic and p) parabolic geometries. Notice that C) and p) involve open boundaries and the same type of harmonic (exponential and trigonometric) functions; while c) and e) involve closed boundaries, and the ellipses become circles and the hyperbolas become their radial asymptotes, in the limit of a vanishing focal distance. The following subsections illustrate the superpositions of the respective electrostatic and magnetostatic potentials, sources and force fields for axial and transverse motions of the S' inertial frame relative to the rest frame S in Ref. [1], along the z-axis and y-axis, respectively. While in C) the unit vectors are common in S' and S, for the cylindrical coordinates their respective unit vectors must be projected on the Cartesian ones in order to implement the respective Lorentz transformations.

3.1. Axial motion along z-axis

Since the vector potential and current density are along the z-axis, their transverse components in the x-axis and y-axis vanish in both inertial frames S and S', and their longitudinal components and the time component scalar potentials in S' become the respective superpositions of their counterparts in S. Correspondingly, the longitudinal components of the electric intensity and magnetic induction field vanish in both inertial frames S and S', and their transverse components in S' are the superpositions of their own components and the cross product of the velocity vector and their companion force field. These Lorentz superpositions are shown explicitly for the successive geometries.

- Cartesian Coordinates

We start with this case because the Lorentz transformations involve the Cartesian components and the time component of the fields and their sources. Here we take into account the velocity in the z-direction, and Eqs. in A i) and B i) in Ref. [1].

The respective magnetic and electric field sources are:

$$I_{x'} = 0, \qquad I_{y'} = 0, \qquad I_{z'} = \gamma(I_z - v\lambda), \qquad \lambda' = \gamma\left(\lambda - \frac{v}{c^2} I_z\right). \tag{3}$$

The transformed components of the potentials are:

$$\begin{aligned}
A_{x'} &= A_x = 0, \qquad A_{y'} = A_y = 0,\\
A_{z'} &= \frac{4}{c}\gamma\,(I - v\lambda)\sum_{m=1}^{\infty}\frac{1}{m}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
\Phi' &= 4\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m}^{\infty}\frac{1}{m}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},
\end{aligned} \tag{4}$$

where $x_<$ and $x_>$ are the major and minor values of x and x_0 respectively.

The electric and magnetic fields are:

$$\begin{aligned}
E_{z'} &= 0, \quad B_{z'} = 0,\\
E_{x'} &= \frac{4\pi}{a}\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m=1}^{\infty}(\mp)\,e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
E_{y'} &= \frac{4\pi}{a}\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m=1}^{\infty}(-)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\cos\frac{m\pi y}{a},\\
B_{x'} &= \frac{4\pi}{ca}\gamma\,(I - v\lambda)\sum_{m=1}^{\infty}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\cos\frac{m\pi y}{a},\\
B_{y'} &= \frac{4\pi}{ca}\gamma\,(I - v\lambda)\sum_{m=1}^{\infty}(\mp)\,e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a}.
\end{aligned} \tag{5}$$

The difference in signs of the normal components is associated with Gauss's Law in its boundary condition form.

- Circular Coordinates

Sources:

$$I_{x'} = 0,\ I_{y'} = 0,\ I_{z'} = \gamma(I_z - v\lambda),\ \lambda' = \gamma\left(\lambda - \frac{v}{c^2}I_z\right). \tag{6}$$

Potentials:

$$\begin{aligned}
A_{x'} &= A_x = 0, \qquad A_{y'} = A_y = 0,\\
A_{z'} &= \gamma\left(A_0 - \frac{v}{c}\Phi_0\right) - \frac{2}{c}\gamma\,(I - v\lambda)\ln R_> + \frac{2}{c}\gamma(I - v\lambda)\sum_{m=1}^{\infty}\frac{R_<^m}{R_>^m}\frac{\cos m\,(\phi - \phi_0)}{m},\\
\Phi' &= \gamma\left(\Phi_0 - \frac{v}{c}A_0\right) - 2\gamma\left(\lambda - \frac{v}{c^2}I\right)\ln R_> + 2\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m=1}^{\infty}\frac{R_<^m}{R_>^m}\frac{\cos m\,(\phi - \phi_0)}{m}.
\end{aligned} \tag{7}$$

Force Fields:

$$\begin{aligned}
&E_{z'} = 0, \qquad B_{z'} = 0,\\
&E_{x'} = 2\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\cos\phi\cos m(\phi-\phi_0) - \sin\phi\sin m(\phi-\phi_0)\right],\\
&E_{y'} = 2\gamma\left(\lambda - \frac{v}{c^2}I\right)\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\sin\phi\cos m(\phi-\phi_0) + \cos\phi\sin m(\phi-\phi_0)\right],\\
&B_{x'} = \frac{2}{c}\gamma\left(I - v\lambda\right)\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\pm)\sin\phi\cos m(\phi-\phi_0) - \cos\phi\sin m(\phi-\phi_0)\right],\\
&B_{y'} = \frac{2}{c}\gamma\left(I - v\lambda\right)\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\cos\phi\cos m(\phi-\phi_0) - \sin\phi\sin m(\phi-\phi_0)\right].
\end{aligned} \tag{8}$$

- Elliptic Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = 0, \qquad I_{z'} = \gamma(I_z - v\lambda), \qquad \lambda' = \gamma\left(\lambda - \frac{v}{c^2}I_z\right). \tag{9}$$

Potentials:

$$\begin{aligned}
&A_{x'} = A_x = 0, \qquad A_{y'} = A_y = 0,\\
&A_{z'} = \frac{2}{c}\gamma\left(I - v\lambda\right)\left[\frac{(u_2-u_>)(u_<-u_1)}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh m(u_2-u_>)}{\sinh m(u_2-u_1)}\frac{\sinh m(u_<-u_1)\cos m(v-v_0)}{m}\right],\\
&\Phi' = 2\gamma\left(\lambda - \frac{v}{c^2}I\right)\left[\frac{(u_2-u_>)(u_<-u_1)}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh m(u_2-u_>)}{\sinh m(u_2-u_1)}\frac{\sinh m(u_<-u_1)\cos m(v-v_0)}{m}\right].
\end{aligned} \tag{10}$$

Force Fields:

$$\begin{aligned}
&E_{z'} = 0, \quad B_{z'} = 0\\
&E_{x'} = \gamma\frac{2f}{h_u^2}\left(\lambda - \frac{v}{c^2}I\right)\left\{\sinh u\cos v\frac{\begin{pmatrix}-(u_2-u_0)\\ u_0-u_1\end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh\begin{pmatrix}m(u_2-u_0)\\ m(u_0-u_1)\end{pmatrix}}{\sinh m(u_2-u_1)}\right.\\
&\qquad\left.\times\left[\sinh u\cos v\cosh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\cos m(v-v_0) + \cosh u\sin v\sinh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\sin m(v-v_0)\right]\right\},\\
&E_{y'} = -\gamma\frac{2f}{h_u^2}\left(\lambda - \frac{v}{c^2}I\right)\left\{\sinh u\cos v\frac{\begin{pmatrix}-(u_2-u_0)\\ u_0-u_1\end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh\begin{pmatrix}m(u_2-u_0)\\ m(u_0-u_1)\end{pmatrix}}{\sinh m(u_2-u_1)}\right.\\
&\qquad\left.\times\left[\cosh u\sin v\cosh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\cos m(v-v_0) - \sinh u\cos v\sinh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\sin m(v-v_0)\right]\right\},\\
&B_{x'} = \gamma\frac{2f}{ch_u^2}\left(I - v\lambda\right)\left\{\sinh u\cos v\frac{\begin{pmatrix}-(u_2-u_0)\\ u_0-u_1\end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh\begin{pmatrix}m(u_2-u_0)\\ m(u_0-u_1)\end{pmatrix}}{\sinh m(u_2-u_1)}\right.\\
&\qquad\left.\times\left[\sinh u\cos v\sinh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\sin m(v-v_0) - \cosh u\sin v\cosh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\cos m(v-v_0)\right]\right\},
\end{aligned}$$

$$B_{y'} = \gamma \frac{2f}{ch_u^2}(I - v\lambda)\left\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2 - u_0) \\ u_0 - u_1 \end{pmatrix}}{u_2 - u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2 - u_0) \\ m(u_0 - u_1) \end{pmatrix}}{\sinh m(u_2 - u_1)} \right.$$
$$\left. \times \left[\sinh u \cos v \cosh \begin{pmatrix} m(u - u_1) \\ m(u_2 - u) \end{pmatrix} \cos m(v - v_0) + \cosh u \sin v \sinh \begin{pmatrix} m(u - u_1) \\ m(u_2 - u) \end{pmatrix} \sin m(v - v_0) \right] \right\}. \tag{11}$$

- Parabolic Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = 0, \qquad I_{z'} = \gamma(I_z - v\lambda), \qquad \lambda' = \gamma\left(\lambda - \frac{v}{c^2} I_z\right). \tag{12}$$

Potentials:

$$A_{x'} = A_x = 0, \qquad A_{y'} = A_y = 0,$$
$$A_{z'} = \frac{4}{c}\gamma(I - v\lambda)\sum_{m=1}^{\infty}\frac{1}{m} e^{\frac{-m\pi(\xi_> - \xi_<)}{(\eta_1 - \eta_2)}} \sin\frac{m\pi(\eta_0 - \eta_1)}{\eta_2 - \eta_1} \sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1},$$
$$\Phi' = 4\gamma\left(\lambda - \frac{v}{c} I\right)\sum_{m=1}^{\infty}\frac{1}{m} e^{\frac{-m\pi(\xi_> - \xi_<)}{(\eta_1 - \eta_2)}} \sin\frac{m\pi(\eta_0 - \eta_1)}{\eta_2 - \eta_1} \sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1}. \tag{13}$$

Force Fields:

$$E_{z'} = 0, \qquad B_{z'} = 0,$$
$$E_{x'} = \gamma\left(\lambda - \frac{v}{c^2} I\right)\frac{4\pi}{h_\xi(\eta_2 - \eta_1)}\sum_{m=1}^{\infty} e^{-\frac{m\pi(\xi_> - \xi_<)}{\eta_2 - \eta_1}} \sin\frac{m\pi(\eta_0 - \eta)}{\eta_2 - \eta_1}\left[(\mp)\frac{\eta}{h_\xi}\cos\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1} - \frac{\xi}{h_\xi}\sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1}\right],$$
$$E_{y'} = \gamma\left(\lambda - \frac{v}{c^2} I\right)\frac{4\pi}{h_\xi(\eta_2 - \eta_1)}\sum_{m=1}^{\infty}(-) e^{-\frac{m\pi(\xi_> - \xi_<)}{\eta_2 - \eta_1}} \sin\frac{m\pi(\eta_0 - \eta)}{\eta_2 - \eta_1}\left[(\mp)\frac{\xi}{h_\xi}\cos\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1} + \frac{\eta}{h_\xi}\sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1}\right],$$
$$B_{x'} = \gamma(I - v\lambda)\frac{4\pi}{ch_\xi(\eta_2 - \eta_1)}\sum_{m=1}^{\infty} e^{-\frac{m\pi(\xi_> - \xi_<)}{\eta_2 - \eta_1}} \sin\frac{m\pi(\eta_0 - \eta)}{\eta_2 - \eta_1}\left[(\mp)\frac{\xi}{h_\xi}\cos\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1} + \frac{\eta}{h_\xi}\sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1}\right],$$
$$B_{y'} = \gamma(I - v\lambda)\frac{4\pi}{ch_\xi(\eta_2 - \eta_1)}\sum_{m=1}^{\infty} e^{-\frac{m\pi(\xi_> - \xi_<)}{\eta_2 - \eta_1}} \sin\frac{m\pi(\eta_0 - \eta)}{\eta_2 - \eta_1}\left[(\mp)\frac{\eta}{h_\xi}\cos\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1} - \frac{\xi}{h_\xi}\sin\frac{m\pi(\eta - \eta_1)}{\eta_2 - \eta_1}\right]. \tag{14}$$

For the four geometries, the vanishing of the transverse x' and x and $y'$and y components of sources and potentials, as well as the Lorentz-transformation mixings of their axial and scalar components, in the first set of equations are common. In turn, the last set of equations show the common vanishing axial components of the force fields, and the Lorentz-transformations of their transverse components. The cross product of the axial velocity with the other field connects with the remarks about the cross product of the axial unit vector at the end of the first paragraph in the Introduction, and at the end of Sec. 2, thus answering the corresponding question. Notice also the common mixing proportions in the sources appearing in the potentials, and in the force fields, allowing for the changes in sign associated with the cross products. The transverse force fields in the axially moving frame S' share the same distributions as in the S frame, and their magnitudes are determined by the magnitudes of the Lorentz transformed sources $I_{z'}$ and λ' for all geometries.

The space-time Lorentz Transformations Eq. (1) being linear are also valid for the three differential displacements and time. For the axial displacement ẓ is Lorentz contracted to $ẓ' = \gamma ẓ$, for $ṭ' = 0$, simultaneously in S'. The consequences in the axial current intensity and linear charge sources, and in the respective potentials, as four-vectors are recognized in their respective Lorentz transformations in the first pair of equations for each geometry. Their transverse components in x and y are zero in S, and also in S'. The Lorentz transformations of the force fields include: the vanishing of their axial z components in both S and S'; and the superposition of their own corresponding components with the γ time dilatation factor and the other transverse component of the companion field with the γ and v/c factors, as well as the combinations of the current and charge sources of their respective potentials. Their transverse distributions are the same in S' and in S, for each component and in the four geometries, with the primed sources of the first equations for each geometry.

3.2. Transverse motion along y-axis

We recognize the parity properties under the exchange of x by - x in the Figures in Ref. [1], which are preserved for the direction of motion chosen in this Subsection. Consequently, the y component of the vector and the scalar potential, as the time component in S are superposed in their respective Lorentz transformations, yielding their primed counterparts in S'; additionally, $A_{z'} = A_z$ and $A_{x'} = A_x = 0$. The same holds for the respective components of the four-vector current-charge densities, including $I_{z'} = I_z$ and $I_{x'} = I_x = 0$. On the other hand, for the force fields their components in the direction of motion remain the same $E_{y'} = E_y$ and $B_{x'} = B_x$, while their transverse components become the superpositions of themselves and the cross products of v in the x-direction and their companion force field. The corresponding results are also obtained for the successive geometries.

- Cartesian Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = -\gamma v\lambda, \qquad {I_z}' = I_z, \; \lambda' = \gamma\lambda. \tag{15}$$

Potentials:

$$\begin{aligned}
A_{x'} &= 0,\\
A_{y'} &= -\gamma\frac{v}{c}4\lambda\sum_{m=1}^{\infty}\frac{1}{m}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
A_{z'} &= \frac{4I}{c}\sum_{m=1}^{\infty}\frac{1}{m}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
\Phi' &= \gamma 4\lambda\sum_{m}^{\infty}\frac{1}{m}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a}.
\end{aligned} \tag{16}$$

Force Fields:

$$\begin{aligned}
E_{y'} &= \frac{4\pi\lambda}{a}\sum_{m=1}^{\infty}(-)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\cos\frac{m\pi y}{a},\\
E_{z'} &= \gamma\frac{v}{c}\frac{4\pi I}{ca}\sum_{m=1}^{\infty}(-)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\cos\frac{m\pi y}{a},\\
E_{x'} &= \gamma\frac{4\pi\lambda}{a}\sum_{m=1}^{\infty}(\mp)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
B_{y'} &= \frac{4\pi I}{ca}\sum_{m=1}^{\infty}(\mp)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
B_{z'} &= \gamma\frac{v}{c}\frac{4\pi\lambda}{a}\sum_{m=1}^{\infty}(\mp)e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\sin\frac{m\pi y}{a},\\
B_{x'} &= \gamma\frac{4\pi I}{ca}\sum_{m=1}^{\infty}e^{-\frac{m\pi(x_> - x_<)}{a}}\sin\frac{m\pi y_0}{a}\cos\frac{m\pi y}{a}.
\end{aligned} \tag{17}$$

- Circular Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = -\gamma v\lambda, \qquad I_{z'} = I_z, \qquad \lambda' = \gamma\lambda. \tag{18}$$

Potentials:

$$A_{x'} = 0,$$
$$A_{y'} = -\gamma\frac{v}{c}\left[\Phi_0 - 2\lambda\ln R_> + 2\lambda\sum_{m=1}^{\infty}\frac{R_<^m}{R_>^m}\frac{\cos m\,(\phi-\phi')}{m}\right],$$
$$A_{z'} = \gamma\left[A_0 - \frac{2I}{c}\left(\ln R_> + \sum_{m=1}^{\infty}\frac{R_<^m}{R_>^m}\frac{\cos m\,(\phi-\phi_0)}{m}\right)\right],$$
$$\Phi' = \gamma\left[\Phi_0 - 2\lambda\left(\ln R_> + \sum_{m=1}^{\infty}\frac{R_<^m}{R_>^m}\frac{\cos m\,(\phi-\phi_0)}{m}\right)\right]. \quad (19)$$

Force Fields:

$$E_{y'} = 2\lambda\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\sin\phi\cos m(\phi-\phi_0) + \cos\phi\sin(\phi-\phi_0)\right],$$
$$E_{z'} = \gamma\frac{2Iv}{c^2}\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\sin\phi\sin m(\phi-\phi_0) + \cos\phi\cos m(\phi-\phi_0)\right],$$
$$E_{x'} = \gamma 2\lambda\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\cos\phi\cos m(\phi-\phi_0) - \sin\phi\sin m(\phi-\phi_0)\right],$$
$$B_{y'} = \frac{2I}{c}\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\cos\phi\cos m(\phi-\phi_0) - \sin\phi\sin m(\phi-\phi_0)\right],$$
$$B_{z'} = \gamma\frac{2v}{c^2}\lambda\sum_{m=1}^{\infty}\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\cos\phi\cos m(\phi-\phi_0) - \sin\phi\sin m(\phi-\phi_0)\right],$$
$$B_{x'} = \gamma\frac{2I}{c}\sum_{m=1}^{\infty}(-)\begin{pmatrix}R^{m-1}/R_0^m\\ R_0^m/R^{m+1}\end{pmatrix}\left[(\mp)\sin\phi\sin m(\phi-\phi_0) + \cos\phi\cos m(\phi-\phi_0)\right]. \quad (20)$$

- Elliptic Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = -\gamma v\lambda, \qquad I_{z'} = I_z, \qquad \lambda' = \gamma\lambda. \quad (21)$$

Potentials:

$$A_{x'} = 0,$$
$$A_{y'} = -2\gamma\frac{v}{c}\lambda\left[\frac{(u_2-u_>)(u_<-u_1)}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh m(u_2-u_>)}{\sinh m(u_2-u_1)}\frac{\sinh m(u_<-u_1)\cos m(v-v_0)}{m}\right],$$
$$A_{z'} = \frac{2I}{c}\left[\frac{(u_2-u_>)(u_<-u_1)}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh m(u_2-u_>)}{\sinh m(u_2-u_1)}\frac{\sinh m(u_<-u_1)\cos m(v-v_0)}{m}\right],$$
$$\Phi' = \gamma 2\lambda\left[\frac{(u_2-u_>)(u_<-u_1)}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh m(u_2-u_>)}{\sinh m(u_2-u_1)}\frac{\sinh m(u_<-u_1)\cos m(v-v_0)}{m}\right]. \quad (22)$$

Force Fields:

$$E_{y'} = \frac{2\lambda f}{h_u^2}\left\{\sinh u\cos v\frac{\begin{pmatrix}-(u_2-u_0)\\ u_0-u_1\end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty}\frac{\sinh\begin{pmatrix}m(u_2-u_0)\\ m(u_0-u_1)\end{pmatrix}}{\sinh m(u_2-u_1)}\right.$$
$$\left.\times\left[\cosh u\sin v\cosh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\cos m(v-v_0) - \sinh u\cos v\sinh\begin{pmatrix}m(u-u_1)\\ m(u_2-u)\end{pmatrix}\sin m(v-v_0)\right]\right\},$$

$$E_{z'} = \gamma \frac{v}{c} \frac{2If}{ch_u^2} \Bigg\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2-u_0) \\ u_0-u_1 \end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2-u_0) \\ m(u_0-u_1) \end{pmatrix}}{\sinh m(u_2-u_1)}$$
$$\times \left[\sinh u \cos v \sinh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \sin m(v-v_0) + \cosh u \sin v \cosh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \cos m(v-v_0) \right] \Bigg\},$$

$$E_{x'} = -\gamma \frac{2\lambda f}{h_u^2} \Bigg\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2-u_0) \\ u_0-u_1 \end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2-u_0) \\ m(u_0-u_1) \end{pmatrix}}{\sinh m(u_2-u_1)}$$
$$\times \left[\sinh u \cos v \cosh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \cos m(v-v_0) + \cosh u \sin v \sinh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \sin m(v-v_0) \right] \Bigg\},$$

$$B_{y'} = \frac{2If}{ch_u^2} \Bigg\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2-u_0) \\ u_0-u_1 \end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2-u_0) \\ m(u_0-u_1) \end{pmatrix}}{\sinh m(u_2-u_1)}$$
$$\times \left[\sinh u \cos v \cosh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \cos m(v-v_0) + \cosh u \sin v \sinh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \sin m(v-v_0) \right] \Bigg\},$$

$$B_{z'} = \gamma \frac{v}{c} \frac{2\lambda f}{h_u^2} \Bigg\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2-u_0) \\ u_0-u_1 \end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2-u_0) \\ m(u_0-u_1) \end{pmatrix}}{\sinh m(u_2-u_1)}$$
$$\times \left[\sinh u \cos v \cosh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \cos m(v-v_0) + \cosh u \sin v \sinh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \sin m(v-v_0) \right] \Bigg\},$$

$$B_{x'} = \gamma \frac{2If}{ch_u^2} \Bigg\{ \sinh u \cos v \frac{\begin{pmatrix} -(u_2-u_0) \\ u_0-u_1 \end{pmatrix}}{u_2-u_1} + 2\sum_{m=1}^{\infty} \frac{\sinh \begin{pmatrix} m(u_2-u_0) \\ m(u_0-u_1) \end{pmatrix}}{\sinh m(u_2-u_1)}$$
$$\times \left[\sinh u \cos \sinh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \sin m(v-v_0) + \cosh u \sin v \cosh \begin{pmatrix} m(u-u_1) \\ m(u_2-u) \end{pmatrix} \cos m(v-v_0) \right] \Bigg\}. \quad (23)$$

- Parabolic Coordinates

Sources:

$$I_{x'} = 0, \qquad I_{y'} = -\gamma v \lambda, \qquad I_{z'} = I_z, \qquad \lambda' = \gamma \lambda. \quad (24)$$

Potentials:

$$A_{x'} = 0,$$
$$A_{y'} = -\gamma \frac{v}{c} 4\lambda \sum_{m=1}^{\infty} \frac{1}{m} e^{-\frac{m\pi(\xi_> - \xi_<)}{\eta_2-\eta_1}} \sin \frac{m\pi\,(\eta'-\eta_1)}{\eta_2-\eta_1} \sin \frac{m\pi\,(\eta-\eta_1)}{\eta_2-\eta_1},$$
$$A_{z'} = \frac{4I}{c} \sum_{m=1}^{\infty} \frac{1}{m} e^{\frac{-m\pi(\xi_> - \xi_<)}{(\eta_1-\eta_2)}} \sin \frac{m\pi(\eta_0-\eta_1)}{\eta_2-\eta_1} \sin \frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1},$$
$$\Phi' = \gamma 4\lambda \sum_{m=1}^{\infty} \frac{1}{m} e^{\frac{-m\pi(\xi_> - \xi_<)}{(\eta_1-\eta_2)}} \sin \frac{m\pi(\eta_0-\eta_1)}{\eta_2-\eta_1} \sin \frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}. \quad (25)$$

Force Fields:

$$E_{y'} = \frac{4\pi\lambda}{h_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}(-)e^{-\frac{m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[\frac{\eta}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}(\mp)\frac{\xi}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right],$$

$$E_{z'} = \gamma\frac{v}{c^2}\frac{4\pi I}{h_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}(-)e^{-\frac{m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[\frac{\eta}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}(\mp)\frac{\xi}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right],$$

$$E_{x'} = \gamma\frac{4\pi\lambda}{h_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}e^{-\frac{m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[(\mp)\frac{\eta}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}-\frac{\xi}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right],$$

$$B_{y'} = \frac{4\pi I}{ch_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}e^{\frac{-m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[(\mp)\frac{\eta}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}-\frac{\xi}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right],$$

$$B_{z'} = \gamma\frac{v}{c}\frac{4\pi\lambda}{h_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}e^{-\frac{m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[(\mp)\frac{\eta}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}-\frac{\xi}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right],$$

$$B_{x'} = \gamma\frac{4\pi I}{ch_\xi(\eta_2-\eta_1)}\sum_{m=1}^{\infty}e^{-\frac{m\pi(\xi_>-\xi_<)}{\eta_2-\eta_1}}\sin\frac{m\pi(\eta_0-\eta)}{\eta_2-\eta_1}\left[\frac{\eta}{h_\xi}\cos\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}(\mp)\frac{\xi}{h_\xi}\sin\frac{m\pi(\eta-\eta_1)}{\eta_2-\eta_1}\right]. \quad (26)$$

For the motion in the transverse direction y of S' relative to S, the x' and z' components of the current and vector potential remain as their counterparts in S, the first one being zero. The y components are also zero, and the y' component of the current is $-\gamma v\lambda$, and the λ' is $\gamma\lambda$, the first equation for each geometry; and likewise for the y' component of the vector potential and the scalar potential. For the force fields their y' and y components are the same, and their x and z components are mixed in their Lorentz transformations in their primed components; since their z components are zero, $E_{z'} = -\gamma\frac{v}{c}B_x$, $E_{x'} = \gamma E_x$, $B_{z'} = \gamma\frac{v}{c}E_x$, $B_{x'} = \gamma B_x$.

In the last set of equations for each geometry, the respective primed sources can be identified, as well as the primed sources of the companion field due to the motion of S' relative to S with velocity v in the y direction.

4. Discussion

In the Introduction a comparison of the electrostatic and magnetostatic harmonic bidimensional fields of [1] was made, with emphasis in their differences and complementarities, leading to the question about their connection. Recognizing the connections of the electric intensity and magnetic induction fields under the cross product with the unit axial vector, and asking about its reasons. Naturally, the titles of the Article and of Sec. 2 emphasize that the Lorentz transformations of Special Relativity provide the connections between the space and time components of the electromagnetic potentials, sources and force fields, including the electrostatic and magnetostatic ones.

Section 3 illustrates the explicit forms of such connections for the fields in the successive geometries and boundary conditions for 1. Axial motion along the z-axis, and 2. Transverse motion along the y-axis. In the case of axial motion, the Lorentz Transformations contain directly the reasons for the connections among the components of the force fields including the differences in their signs. It is also appropriate to point out that the connections between the electric intensity field and the magnetic induction field are provided, in general, by Faraday's magnetoelectric induction and Maxwell's electromagnetic induction laws, including the difference in signs and their orthogonality. The physical interpretation of the superpositions of the longitudinal and time components of potentials and sources, and of the transverse components of the fields is straightforward for axial motion, preserving the same transverse distributions as in the rest frame in each geometry; and with the common superposition coefficients combining the products of the coefficients in the Lorentz transformation and the source intensities: $\gamma(I - v\lambda)$, $\gamma(\lambda - (v/c^2)I)$ for potentials and sources, and the same ones with the appropriate changes in sign for the force fields. In the case of transverse motion along the y-axis the transformed components of sources potentials and force fields are:

$$I_{x'} = 0, \quad I_{y'} = -\gamma v\lambda, \quad I_{z'} = I_z, \quad \lambda' = \gamma\lambda,$$

$$A_{x'} = 0, \quad A_{y'} = -\gamma\frac{v}{c}\Phi, \quad A_{z'} = A_z, \quad \Phi' = \gamma\Phi,$$

$$E_{y'} = E_y, \quad E_{z'} = -\gamma\frac{v}{c}B_x, \quad E_{x'} = \gamma E_x.$$

$$B_{y'} = B_y, \quad B'_z = \gamma\frac{v}{c}E_x, \quad B_{x'} = \gamma B_x. \quad (27)$$

There was no current I_x and $I_{y'}$ is due to the motion of the line of charge from the point of view of S', and likewise for the respective components of the potentials. The respective components of the sources are also identified in the last set of equations for each geometry. Since the vector potential is axial in S and transverse and to the motion, it remains the same in S'. The scalar potential in S' is γ times the scalar potential in S. Since the force fields are transverse in the $x-y$ plane in S, their y'-components remain the same as in

S, and their z' and x' components are the superpositions of their counterparts in S with vanishing z-components. This is the difference of the transverse transformations compared to the axial ones.

The orthogonality of the electric intensity and magnetic induction field is also associated with the Lorentz invariant:

$$\mathbf{E} \cdot \mathbf{B} = \mathbf{E}' \cdot \mathbf{B}'. \tag{28}$$

which has the value zero in this case.

The other invariant is proportional to the Lagrangian density of the electromagnetic field:

$$\mathbf{B} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{E} = \mathbf{B}' \cdot \mathbf{B}' - \mathbf{E}' \cdot \mathbf{E}', \tag{29}$$

which is positive for the magnetostatic field, negative for the electrostatic field and zero for the radiation field. Equations (28) and (29) may be proven by using the Lorentz transformations of the respective fields, or for any of the explicit harmonic bidimensional fields in the successive geometries.

In Sec. 3 we have focused on the Lorentz transformations between the respective potentials, sources and force fields, leaving them in terms of the space coordinates in the rest frame. The task can be completed by taking the additional step of including the Lorentz transformation from the moving frame to the rest frame:

$$\begin{aligned}
&x = x', \quad y = y', \\
&z = \gamma(z' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2} z'\right), \\
&y = \gamma(y' + vt'), \quad z = z', \\
&x = x', \quad t = \gamma\left(t' + \frac{v}{c^2} y'\right),
\end{aligned} \tag{30}$$

for axial and transverse motions, respectively. The most important consequence is to recognize that the electrostatic and magnetostatic fields in the rest frame become time dependent in the moving inertial frame, because the sources are moving away. This is a familiar situation for the Coulomb field.

The force fields components can be obtained from the derivatives of the potentials. The electric intensity field is the negative gradient of the scalar potential and the negative partial derivative of the vector potential ; and the magnetic induction field is the rotational of the vector potential.

From the respective components of the four vector potentials: the rotational of its space component leads to the primed magnetic induction field, and the negative gradient of the time component minus the time derivative of the space component leads to the primed electric intensity field, leading to the same results as in Sec. 3. This is also an example of the covariance of the Maxwell equations in any inertial frame.

In conclusion, this contribution exhibits explicitly the relativistic connection of the bidimensional harmonic electrostatic and magnetostatic sources, potentials and force fields, in inertial frames with axial and transverse motions, in four different geometries.

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1. E. Ley-Koo and A. Góngora-T., M. A. Ortiz *Rev. Mex. Fís.* **37** (1988) 759.
2. E. M. Purcell, *Electricity and Magnetism*, Berkeley Physics Course 2, Second Edition, (McGraw-Hill 1985).
3. L. Eyges, *The Classical Electromagnetic field*, Dover, New York, (1972).
4. J. R. Reitz, F. J. Milford and R. W. Christy, *Foundations of electromagnetic theory*, Addison-Wesley, Reading, (1979).
5. G. Arfken, *Mathematical Methods for Phyisicists*, Second Edition, Academic Press, New York (1970).
6. A. Einstein, *On the Electrodynamics of Moving Bodies*. Annalen der Physik, **17** (1905) 891.
7. Minkowski, H. et al, *The principle of Relativity*, Dover Books, (1923)
8. B.F. Schutz, *A first Course in General Relativity*, Second edition, Cambridge university Press, UK (2009).
9. R. Fyenman *The Feynman Lectures on Physics. Mainly Electromagnetism and Matter*, Addison-Wesley, (1975).