

GUI for conic sections: parabola, ellipse and hyperbola

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Received 5 May 2023; accepted 6 June 2023

In this paper we provide three Excel spreadsheets for the explanation of Parabola, Ellipse and Hyperbola respectively. Our target audience is University science students (Physics/Mathematics/Computer science) and mentors.

Keywords: GUI; teaching mathematics; excel spreadsheets; conic sections.

DOI: <https://doi.org/10.31349/RevMexFisE.21.010203>

1. Introduction

Graphical User Interface (GUI) allows users to interact with a computer or software application using visual elements/pictorial representation such as icons, menus, and buttons, rather than typing commands in a text-based interface. Ivan Sutherland is considered as the pioneer of GUI. In 1963, nearly 60 years ago, in his seminal PhD thesis [1] he presented the idea of human computer interaction and demonstrated that the computer graphics can be used for multi purposes. Since 1963 his thesis is considered as the justification of GUI. Researchers have developed several GUI platforms and have applied to variety of fields [2–10]. Graphical User Interfaces allow university students and mentors, computer users, to move one application to another application. GUI provides a platform to demonstrate abstract and complicated ideas of Physics/Mathematics in pictorial/graphical ways. These ways are very simple and user friendly and can be opted at the university level. Birth of computer revolutionised many fields of sciences. University level teaching has also changed drastically due to this technology. The need of GUI is rapidly increased in case of natural disaster and pandemics. We all are witnessed the recent outbreak of Covid-19 [11–13] when whole world was facing lockdowns and work from home was the only preferred method in most of the universities and educational institutes. Thus, without any doubt, GUI provides an alternate way of learning that can be accessed remotely with its great impact in the understanding of typical mathematical ideas.

First year students and sophomore are very much familiar with Microsoft Excel. They have used this platform in their higher secondary schools and colleges. Thus, we have chosen the environment that would make the understanding and explanation of conic sections more convenient. The idea of conic sections is not new. In fact it was coined in 360–350 B.C when an ancient Greek mathematician Menaechmus used the conic sections in his well known problem *doubling the cube* [14–16].

In this study we provide three Excel spread sheets for the explanation of conic sections that include parabola, ellipse and hyperbola. In Sec. 2 we first demonstrate the conic sections (*i.e.*, parabola, ellipse and hyperbola), their mathematical forms (two forms for each curve) and their important physical features along with the graphical representation. We then provide our experiments for the executions of these conics in Sec. 3.

2. Theory

Definition 1. (Parabola) *A locus of a point which is equidistant from a fixed point and the fixed line is called parabola. The fixed point is called focus and the fixed line is called directrix of parabola. Mid point between the focus and the directrix is called vertex of a parabola. An imaginary line passing through a focus and a vertex is called an axis of parabola or axis of symmetry. See Fig. 1 for more details. Two mathematical forms of parabolas are given below:*

1. Axis of parabola is parallel to x -axis:

The desired equation when axis of parabola is parallel to x -axis is:

$$(y - k)^2 = 4a(x - h)$$

Here, coordinates of vertex = (h, k) , coordinates of focus = $(h + a, k)$, equation of directrix is: $x = h - a$.

2. Axis of parabola is parallel to y -axis:

The desired equation is defined as:

$$(x - h)^2 = 4a(y - k)$$

Here, coordinates of vertex = (h, k) , coordinates of focus = $(h, k + a)$, equation of directrix is: $y = k - a$.

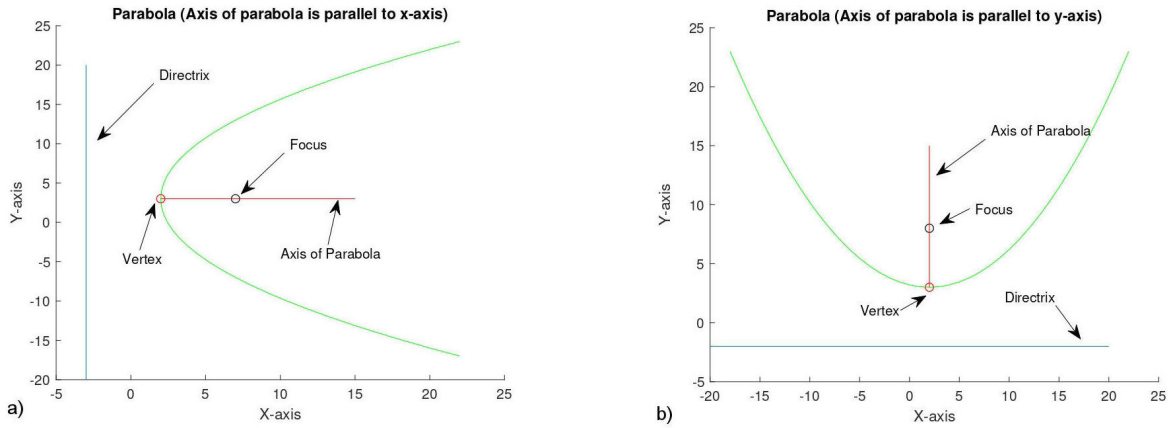


FIGURE 1. Two parabolas are given. a) Axis of parabola is parallel to x -axis. b) Axis of parabola is parallel to y -axis.

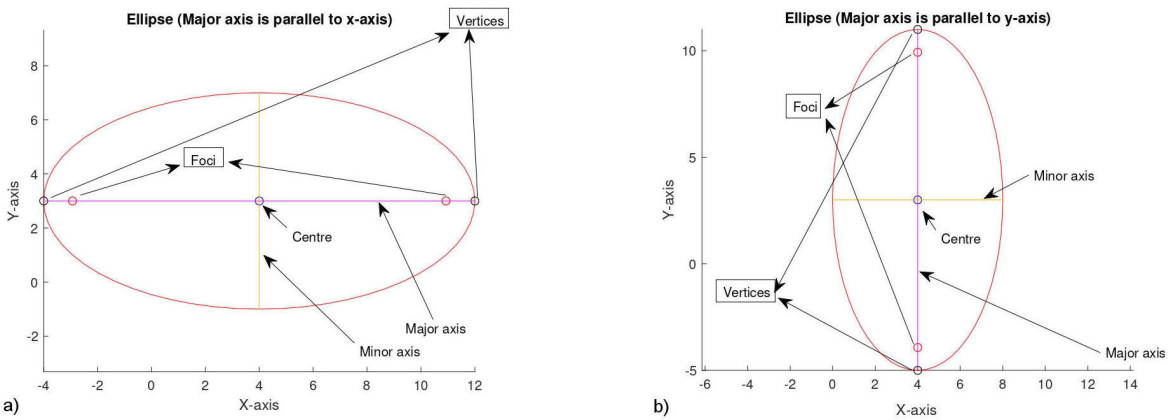


FIGURE 2. Graphical representation of two ellipses are given. a) This ellipse indicates that the major axis is parallel to x -axis. b) Major axis is parallel to y -axis for this ellipse.

Definition 2. (Ellipse) A locus of points such that the sum of their distances from the two fixed points is a constant is called an ellipse. These fixed points are called foci of the ellipse. The longest diameter of an ellipse that passes through both the foci and centre and goes from one end to other end is called major axis of an ellipse. The shortest diameter that goes from one end to other end and passes through centre is called minor axis. Both ends of major axis are called vertices. The complete description can be seen in Fig. 2. Two alternatives forms of ellipse are explained below:

1. Major axis is parallel to x -axis:

When major axis of an ellipse is parallel to x -axis then the desired equation of ellipse is defined as:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \quad a > b.$$

Here, semi major axis = a , semi minor axis = b , coordinates of centre = (h, k) , coordinates of vertices = $(h \pm a, k)$, coordinates of foci = $(h \pm c, k)$, $c = \sqrt{a^2 - b^2}$.

2. Major axis is parallel to y -axis:

The equation of the ellipse when major axis is parallel to y -axis is:

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1, \quad a > b$$

Here, semi major axis = a , semi minor axis = b , coordinates of centre = (h, k) , coordinates of vertices = $(h, k \pm a)$, coordinates of foci = $(h, k \pm c)$, $c = \sqrt{a^2 - b^2}$.

Definition 3. (Hyperbola) A locus of a point which is the closest to a fixed point and farthest to another fixed point with constant amount is called hyperbola. These two fixed points are called foci of the hyperbola. An imaginary line that passes through both the vertices through centre is called transverse axis. An imaginary passing through centre and perpendicular to transverse axis is called conjugate axis. There are several similarities between ellipse and hyperbola and details can be found Fig. 3. Two alternatives forms of hyperbola are explained below:

1. Transverse axis is parallel to x -axis:

If the transverse axis of the hyperbola is parallel to x -axis then the desired mathematical form of hyperbola is defined as:

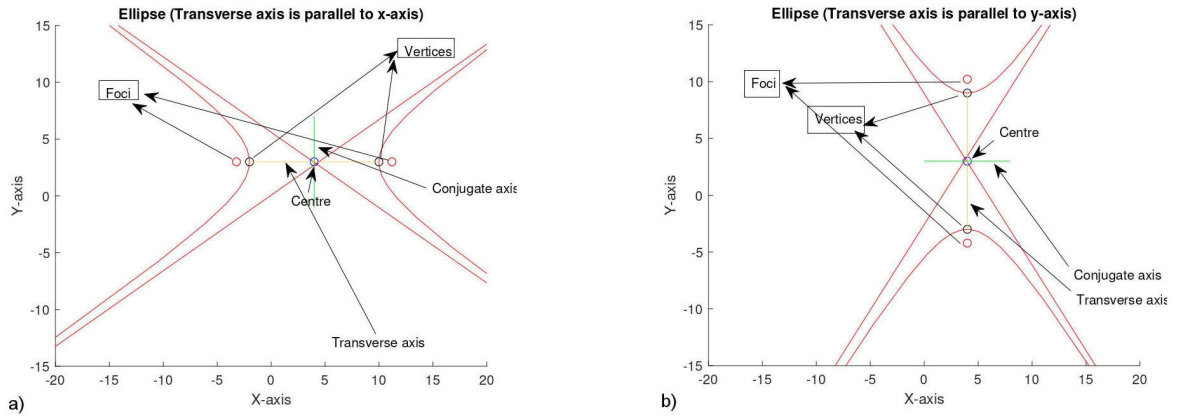


FIGURE 3. Graph of two hyperbolas (with complete descriptions) are given. a) Transverse axis is parallel to x -axis. b) Transverse axis is parallel to x -axis.

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1,$$

Here, semi transverse axis = a , semi conjugate axis = b , coordinates of centre = (h, k) , coordinates of vertices = $(h \pm a, k)$, coordinates of foci = $(h \pm c, k)$, $c = \sqrt{a^2 + b^2}$.

2. Transverse axis is parallel to y -axis:

Mathematical expression of hyperbola, when the transverse axis is parallel to y -axis, is:

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

Here, semi transverse axis = a , semi conjugate axis = b , coordinates of centre = (h, k) , coordinates of vertices = $(h, k \pm a)$, coordinates of foci = $(h, k \pm c)$, $c = \sqrt{a^2 + b^2}$.

Note 1.

1. If the vertex of a parabola is at origin, i.e., $(h, k) = (0, 0)$, then the corresponding mathematical forms are called standard forms of parabola.
2. If the centre of an ellipse/hyperbola is at origin, i.e., $(h, k) = (0, 0)$, then these conics are called in the standard forms.
3. If $a = b$ then the ellipse will become circle and hyperbola is called a rectangular hyperbola.

3. Experiments

In this section we provide three spreadsheets for the explanation parabola, ellipse and hyperbola respectively.

3.1. Spread sheet 1: Parabola

We provide an excel spreadsheet for the explanation of parabola. The minimum requirement that user needs to specify is coordinates of vertex (h, k) and focal length a of a

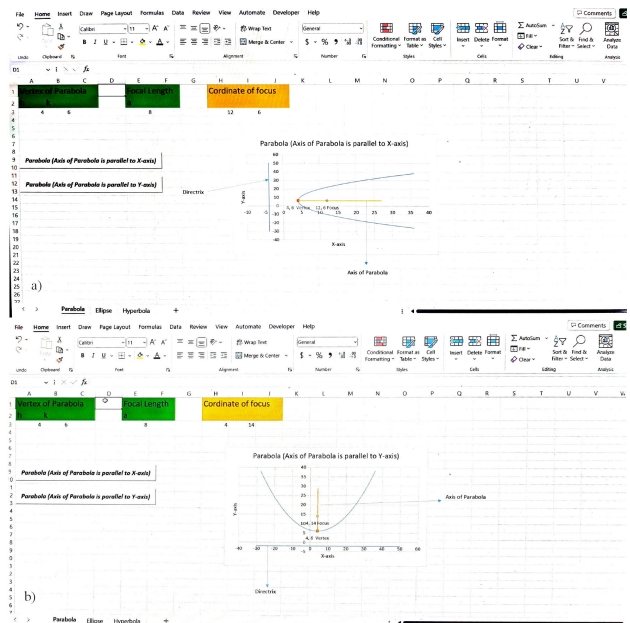


FIGURE 4. Screen shot of two spreadsheets for the explanation of parabola are given. a) Axis of parabola is parallel to x -axis and b) axis of parabola is parallel to y -axis. Two execution buttons are also provided for the generation of corresponding Parabolas.

parabola. These information are provided in cell A3, B3 and E3 respectively. We provide two execution buttons in the cells A9 and A12. These execution buttons will generate the coordinates of focus and corresponding parabolic shape, i.e., button at A9 will generate a parabola whose axis of symmetry/parabola is parallel to x -axis and button at A12 will generate a parabola when its axis is parallel to y -axis. See Fig. 4 for more details.

3.2. Spread sheet 2: Ellipse

In this spread sheet we display the important features of ellipse along its graphical representation. User needs to specify centre (h, k) , major axis a and minor axis b of the required ellipse. We provide two buttons for execution. Button at A7

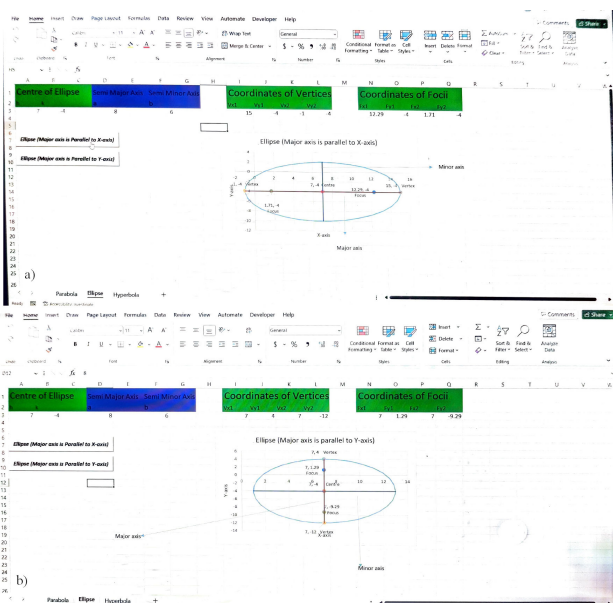


FIGURE 5. Spreadsheets of two ellipse are given. a) When major axis is parallel to x -axis and b) when major axis is parallel to y -axis. Two execution buttons can also be seen.

will generate the ellipse when its major axis is parallel to x -axis. Whereas, button at A9 will generate the ellipse when the major axis is parallel to y -axis. Both buttons also generate the corresponding coordinates of vertices and foci. Description can be seen in Fig. 5.

3.3. Spread sheet 3: Hyperbola

In this last spreadsheet we again provide two execution buttons. Button at A6 generates the hyperbola when the transverse axis is parallel to x -axis and push button at A9 generates the hyperbola that has its transverse axis parallel to y -axis. Alike ellipse user needs to enter coordinates of centre (h, k) , transverse axis a and conjugate axis b in cells A3, B3, D3 and F3 respectively. This execution also generates the corresponding coordinates of vertices and foci. Complete details are provided in Fig. 6.

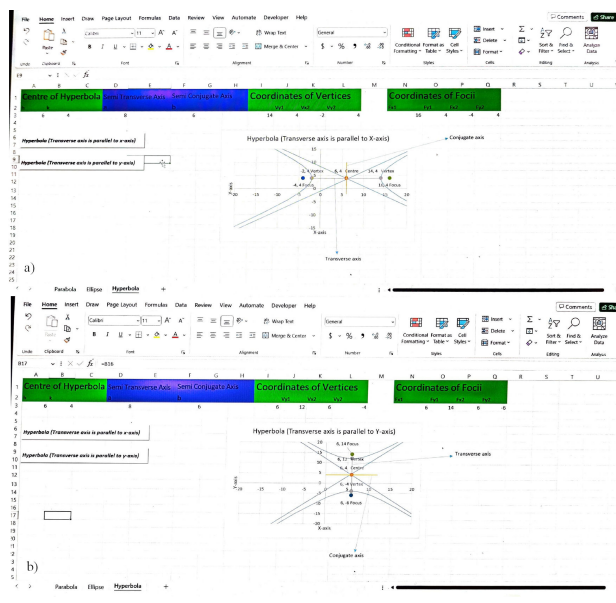


FIGURE 6. Two hyperbolas are shown in the spreadsheets. a) Transverse axis is parallel to x -axis and b) transverse axis is parallel to y -axis. Two execution buttons are also provided for the generation of corresponding hyperbolas.

4. Conclusion

In this manuscript we provide three excel spreadsheets for the explanation of conic sections to undergraduate students of mathematics/physics and computer science. We believe that university students are very much familiar with Microsoft Excel (MSEXCEL) as they have learnt working with MSEXCEL in their higher secondary schools and at college level. Thus this study will help university mentors to explain conic sections to their students in more graphical and appropriate way. On other hand students will have no difficulty in the understanding. Video of the program is available at:

https://drive.google.com/drive/folders/1g3dTse65hX5M2DbuTOuQ1WwNtbbU46tj?usp=drive_link (For English listeners)

https://drive.google.com/drive/folders/166xZ91kbc19TA0rUEPwcAvbP9a9vxSdy?usp=drive_link (For Urdu/Hindi listeners)

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