A simple filter Lorenz electronic circuit

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In this work, an electronic circuit of the Lorenz system is developed. The electronic circuit proposed is one of the easiest to implement. We changed the x-state equation of Lorenz‘s system with a low-pass filter to an RC circuit with the same cutoff frequency. Corron's electronic circuit is used as the basis for electronic design. Simulation results support this proposal.

Keywords: Lorenz systems; phase plane; low-pass filter; electronic design; numerical simulation.

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1. Introduction

Filters have played a very important role in the design of electronic circuits. For example, filters have been used in connection with creating synthesizer modules used in the manufacture of musical instruments, signal processors, automation systems, and more [1]. It is very important that students and researchers have a deep knowledge of the field, both theoretically and experimentally. At a theoretical level, filters are designed according to: the concept of a two-port network [2], modifying the subspace [3], Caprio technique [4], or implementation via structures such as Sallen-Key filters [5], Chebyshev topologies [6], or Butterworth [7]. At the experimental level, there are implementations at the NMOS transistor level [8] or using FPAA devices [9]. When studying physics, the topic of filtering is often covered, with optical or electronic filtering being the most common. This topic is important to understand because you need to filter data collection to perform analysis. A very important application of filters is in communication systems, where the receiving system is tuned to a specific frequency using a filter, thereby effectively capturing the energy radiated by the transmitting circuit at that specific frequency [9]. The concept of synchronization is very important in this way of communications with nonlinear systems [10]. The sending system is called the master and the receiving system is called the slave. Although implementing the sender side of communication system is relatively simple, developing the receiver side has proven difficult. Often, the lack of known fixed basis functions makes it difficult to develop suitable filters. Therefore, receivers depend upon greater complicated strategies to atone for the presence of noise.

Synchronization of systems has been performed using different approaches, *e.g.* forced synchronization, in-phase synchronization, etc. [11]. Numerical implementations by computer or through electronic circuit implementation are used to verify the results. Electronic circuits used for synchronization include Lorenz, Chua, Chen, and so on [12-14]. These circuits in communication systems are used as master and slave systems, where the slave system can be regarded as a filter [15]. Over the past decade, fractional order filters have been implemented. For example, in Ref. [16] a second generation voltage conveyor is used, and in Ref. [17] a filter of order greater than 1 and less than 2 is implemented using a field programmable analog array device, and in Ref. [18] High-order filters with controllable frequencies are implemented using operational transconductance amplifiers. There are many types of filters, but the simplest are lowpass, highpass, bandpass, and stopband filters. These electronically implemented filters can be passive using only resistors R and capacitors C, or they can be active filters that incorporate op amps in addition to RC circuits. Active filters allow you to manipulate the amplitude of the filtered signal.

The work presented in Ref. [15] deals with synchronization phenomena and shows that the slave system behaves like a filter. In this work, the Lorenz system is presented as a filter. For this purpose, the Lorenz system is divided into two subsystems of his, with the equations corresponding to x state being one filter and the equations corresponding to y and z states being the second filter. These two subsystems are cascaded. Therefore, the Lorenz slave system behaves like a filter. A demonstration of this is given in Ref. [15]. Other research on Lorenz systems have been conducted: [19] uses Presnov decomposition to address the synchronization problem of fractional order Lorenz families, [20] uses CMOS technology to synthesize a mathematical Lorenz model, and [21] analyzes the system through rotation. In this investigation, we use the work [15] to replace the Lorenz electronic circuit according to the equation of x state. In Ref. [22], a low-pass filter similar to the x state equation is also used to reconstruct the information signal in the slave system. If the cutoff frequency is not sufficient, synchronization between master and slave will not be achieved. This means that the received signal cannot be reconstructed correctly. In Refs. [23,24] it is also shown that the use of filters is important for constructing communication based chaotic systems. If the cutoff frequency is good, you will get correct decoding.

The aforementioned work is based on the Lorenz system. In this research, the modifications made to the electronic circuit are based on replacing the equation of x state with a passive low-pass filter RC. To achieve this, we use a Lorenz system in the filtering approach proposed in Ref. [15] and decompose it into two cascaded subsystems. The first subsystem corresponds to the equation of state x, and the second subsystem corresponds to the remaining equations y and z. The equation for subsystem 1 (state x) expresses duality with the equation for the first-order passive low pass filter RC, so we replace state x in the electronic circuit with a passive filter RC and check whether the new electronic circuit can generate chaotic oscillations. There are various works in which Lorenz's electronic circuits are modified, such as [23,24] replacing the resistor connecting z state and y state. Other suggestions for electronic circuits come from [14,25]. This paper modifies one of the simplest proposals ever made, made by Dr. Ned Corron in 2010 [26].

This work uses National Instruments Multisim to run the Lorenz system and supports the proposal realized in order to relate the first equation of the Lorenz system to a passive RC lowpass filter. The research will be treated as follows: In Sec. 2, the x state equation is related to the passive RC circuit and the relationship of the cutoff frequency parameter σ is related to the occurrence of this state. A Bode plot is generated. Section 3 gives the electronic implementation of the circuit. The circuit proposed by Corron is modified. Section 4 shows the effect of different cutoff frequencies on the Lorenz system. Finally, in Sec. 5 presents conclusions.

2. Lorenz's x state as RC filter

Consider the Lorenz system represented by

$$\begin{aligned}\dot{x} &= \sigma(y - x),\\ \dot{y} &= rx - xz - y,\\ \dot{z} &= xy - bz.\end{aligned} \tag{1}$$

The following parameters make the system chaotic: $\sigma = 10$, $b = 8/3$, and $r = 30$. According to [15], the Lorenz system works as a filter. This is because the x state equation is the dual of the RC equation passive low pass filter. A diagram of an RC low-pass filter is shown in Fig. 1. This gives the output voltage as a function of the input voltage.

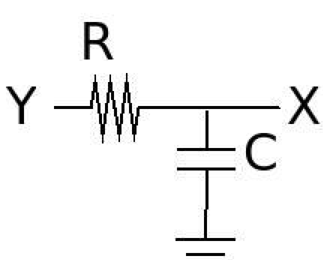

FIGURE 1. Passive RC filter.

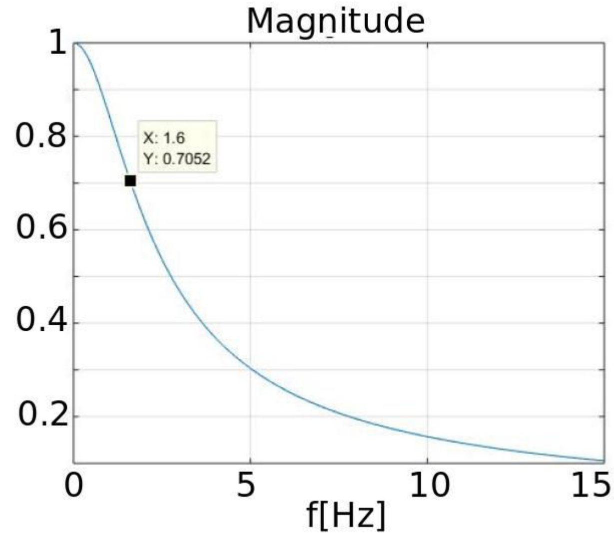

FIGURE 2. The Bode plot for $\sigma = 10$.

$$\dot{x} = \frac{1}{RC}(y - x). \tag{2}$$

Comparing the x state equation of (1) with (2), (2) is the dual of (1) and $1/RC$ plays the role of σ, so that the cutoff frequency is $\omega_c = 1/RC = \sigma$. The Bode's x plot in Eq. (1) for $\sigma = 10$ is shown in Fig. 2, checking the lowpass filter response.

Electronically implemented Lorenz systems typically use integrator circuit with x state equation [14,25]. As we have already seen, (2) is the dual of (1) corresponding to x state. The next question then arises: can one still obtain chaotic oscillations in a Lorenz circuit by electronically replacing the x-state integrator with an passive RC lowpass filter?

3. Modifying Corron's electronic circuit

To investigate the possibility of replacing the electronic op-amp integrator with an passive RC low-pass filter, we implemented the electronic circuit proposed by Corron [26]. It consists of two active filters, an adder, and an integrator with an op-amp and two ad633 multipliers. The Corron's electronic schematic is shown in Fig. 3.

To implement the electronics, Corron had to adjust the system parameters to stay within the $\pm 10V$ supply range. To do this, we changed the variables in the state equation of system (1) as follows: $x = X/\sqrt{ar}$, $y = Y/\sqrt{ar}$, and $z = Z/ar$. If $a = 1/3$, then the power limit requirement is satisfied and the system of equations becomes.

$$\begin{aligned}\dot{X} &= \sigma(Y - X),\\ \dot{Y} &= rX - arXZ - Y,\\ \dot{Z} &= XY - bZ.\end{aligned} \tag{3}$$

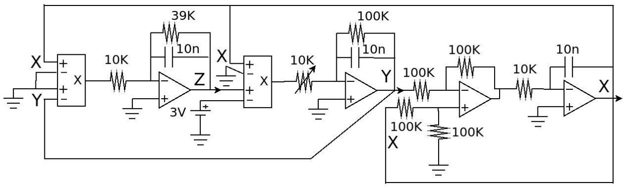

FIGURE 3. Corron's electronic schematic.

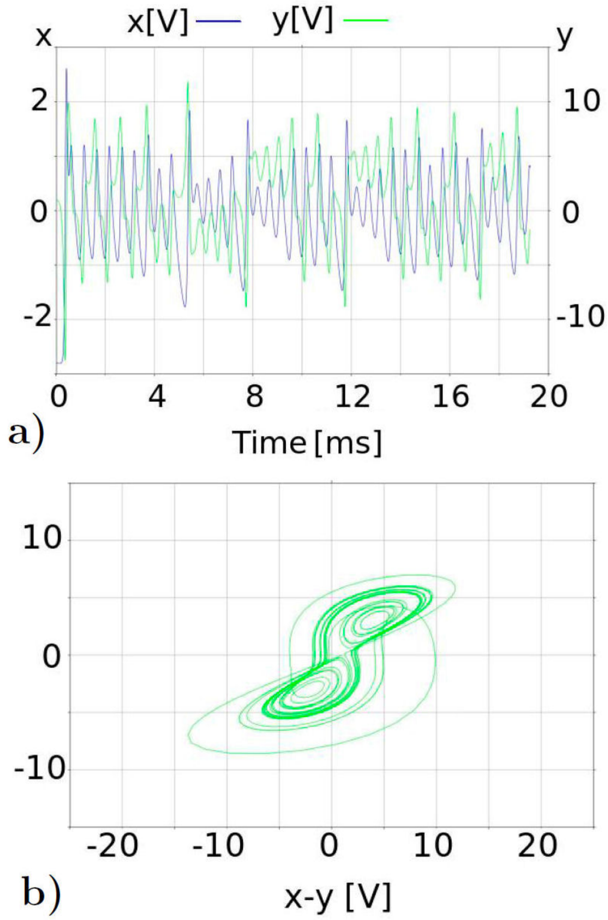

FIGURE 4. Corron's electronic circuit Multisim response. a) x and y states versus time, b) $x-y$ phase portrait.

After changing the variables, Corron implements the system (3) shown in Fig. 3. A Multisim simulation of the electronic circuit Fig. 3 in the x and y states, and the $x-y$ phase portrait over time is shown in Fig. 4. His Kirchhoff equations for electronic circuits are decomposed hereafter, see [26].

The circuit equations in Fig. 3 are obtained by applying the nodal equations for the configuration shown in Fig. 5. These figures will help you better understand how to find the state equations x, y, z for the Corron's electronic circuit.

For the circuit shown in Fig. 5a), we have: *adder* $A = x - y$; *integrator* $\dot{x} = -1/(R_1C_1)A$, thus

$$\dot{x} = -\frac{1}{R_1C_1}(x-y). \tag{4}$$

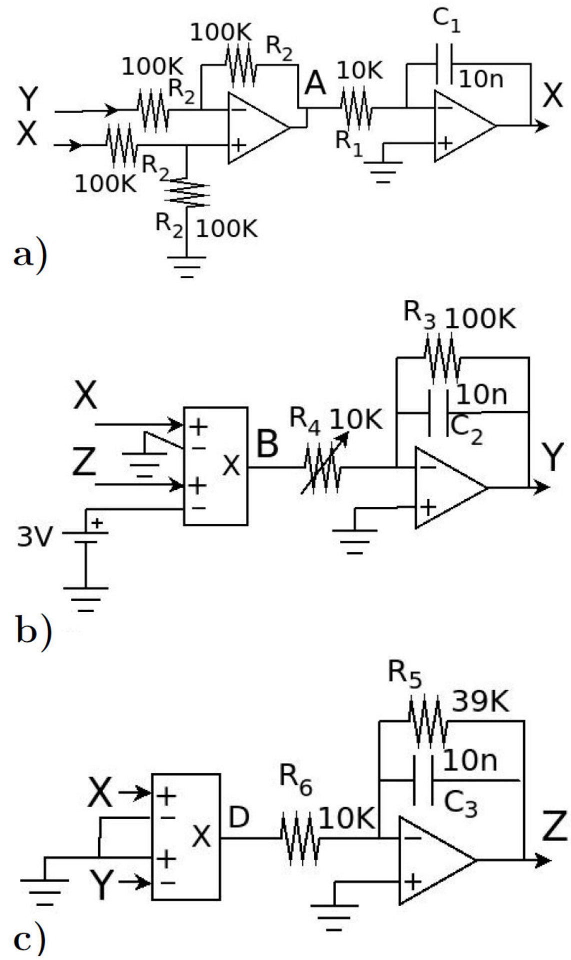

FIGURE 5. Corron's electronic configurations: a) integrator plus adder, b) filter plus multiplier, c) filter plus multiplier.

For the circuit shown in Fig. 5b), we have: *multiplier* $B = x(z-3)/10$; *filter* $\dot{y} = -(1/R_3C_2)y - (1/R_4C_2)B$, *i.e.*

$$\dot{y} = \frac{3}{10R_4C_2}x - \frac{1}{10R_4C_2}xz - \frac{1}{R_3C_2}y. \tag{5}$$

For the circuit shown in Fig. 5c), we have: *multiplier* $D = -xy/10$; *filter* $\dot{z} = -(1/R_5C_3)z - (1/R_6C_3)D$, thus

$$\dot{z} = \frac{1}{10R_6C_5}xy - \frac{1}{R_5C_3}z. \tag{6}$$

Using the resistance and capacitance electronic circuit values in Fig. 3, the equations of states are:

$$\begin{aligned}\dot{x} &= 10K(y-x),\\ \dot{y} &= 30Kx - 10Kxz - 1Ky,\\ \dot{z} &= 1Kxy - 256Kz.\end{aligned} \tag{7}$$

Observing the x state in Eq. (7), we find that the cutoff frequency corresponds to 1.59 KHz (10 Krad/s), and, the proposed passive RC lowpass filter is tuned with $R = 10$ KΩ and $C = 10$ nF to get the cutoff frequency corresponding to the x state.

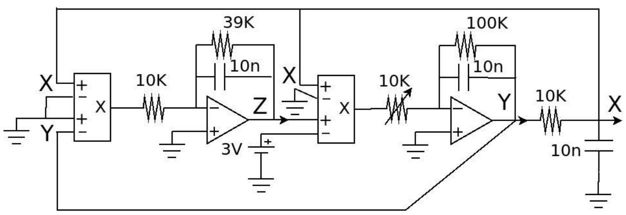

FIGURE 6. Lorenz's electronic schematic proposal.

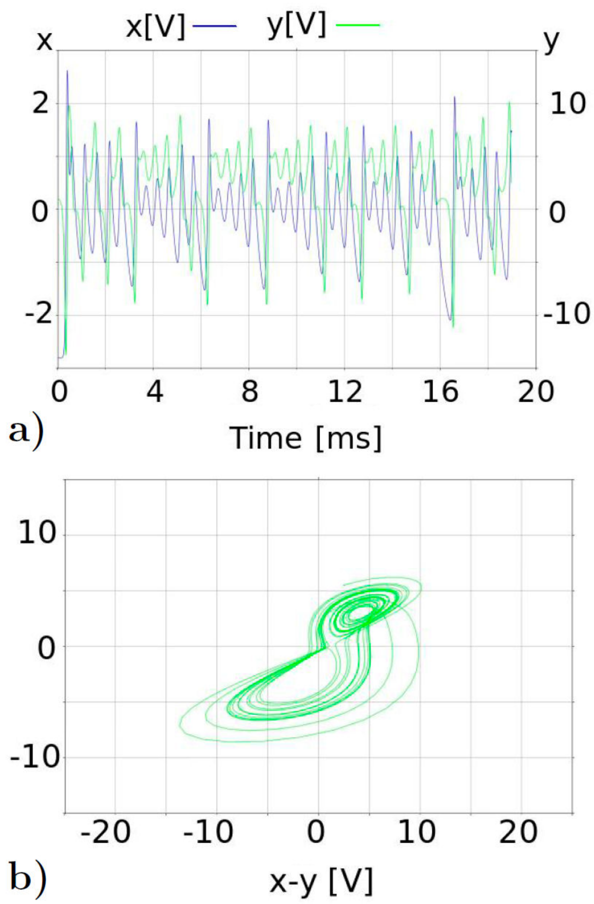

FIGURE 7. Lorenz electronic circuit Multisim response. a) x and y states versus time, b) $x-y$ phase portrait.

Then to answer the question, is it possible to obtain chaotic oscillations in the Lorenz system by electronically replacing the op-amp used in the x state with a passive RC filter? the Lorenz electronic circuit shown in Fig. 6 is proposed.

The electronic proposal replaces two op-amps in subtractive and integral configurations (Fig. 3) with an passive RC low-pass filter circuit (Fig. 6). To corroborate this change in the circuit, we simulate the circuit in Multisim to obtain the response to the x and y states over time, these are shown in Fig. 7a). And Fig. 7b) also shows the $x-y$ phase portrait.

As shown in Fig. 6, the proposed electronic circuit of the Lorenz system can generate chaotic oscillations by replacing the x state with an equivalent passive RC low-pass filter.

TABLE I. Parameters.

R (Ω)	C (F)	frequency (Hz)	Figure
1 K	10 n	15915.45	a), b)
4.5 K	10 n	3536.76	c), d)
10 K	10 n	1591.54	e), f)
18.8 K	10 n	846.56	g), h)
30 K	10 n	520.51	i), j)
100 K	10 n	159.15	k), l)
1 M	10 n	15.91	m), n)
10 M	10 n	1.59	o), p)

4. RC filter tuning

To study the behavior of the Lorenz system, the resistance R of the filter takes on different values, and the capacitance is kept constant. The operation is shown in the following images. The images are arranged horizontally from the top of the sheet to the bottom of the sheet in the following order from the highest cutoff frequency (15915.45 Hz) to the lowest cutoff frequency (1.59 Hz). Table I shows the resistance and capacitance values used and the cutoff frequency in Hz, and the figure to which it belongs.

Taking as a reference point to Fig. 8e)-f), it corresponds to $R = 10$ kΩ and $C = 10$ nF . The cutoff frequency is 1.59 kHz, which corresponds to the circuit proposed by Corron. At these values of R and C, chaotic oscillations occur. Increasing the cutoff frequency is the opposite of decreasing the resistance of the lowpass filter in the RC network. At the cutoff frequency $fc = 3.53$ KHz in Fig. 8c)-d), the oscillations are chaotic. Counting the x state crossings near 0 V in the corresponding figure as an index of frequency, there are 24 of these crossings for a 20 ms time interval, compared to 22 for $fc = 1.59$ KHz. It can be regarded as a measure of the oscillation velocity of the Lorentz circle between the lobes ($1st$ and $3rd$ quadrants). Also, at $fc = 15.91$ KHz, which is ten times higher than the Fig. 8e)-f), the oscillation is suppressed as shown in Fig. 8a)-b). From the above observations, there seems to be an upper limit to the cutoff frequency at which the circuit will still oscillate chaotically.

Now bring the fundamental cutoff frequency fc back to 1.59 KHz, Fig. 8e)-f), and start lowering the cutoff frequency. For example, $fc = 846.56$ Hz, Fig. 8g)-h) sustains chaotic oscillations. But by lowering to lower cutoff frequencies the filter; for example, at cutoff frequencies $fc = 520.51$ Hz Fig. 9i)-j), at $fc = 159.15$ Fig. 9k)-l), at $fc = 15.91$ Hz Fig. 9m)-n) and for $fc = 1.59$ Hz Fig. 9o)-p), the oscillations in the Lorenz circuit are extinguished. We can also infer that in these cases there is a lower cutoff frequency limit that allows the circuit to oscillate chaotically, as can be seen from the pictures.

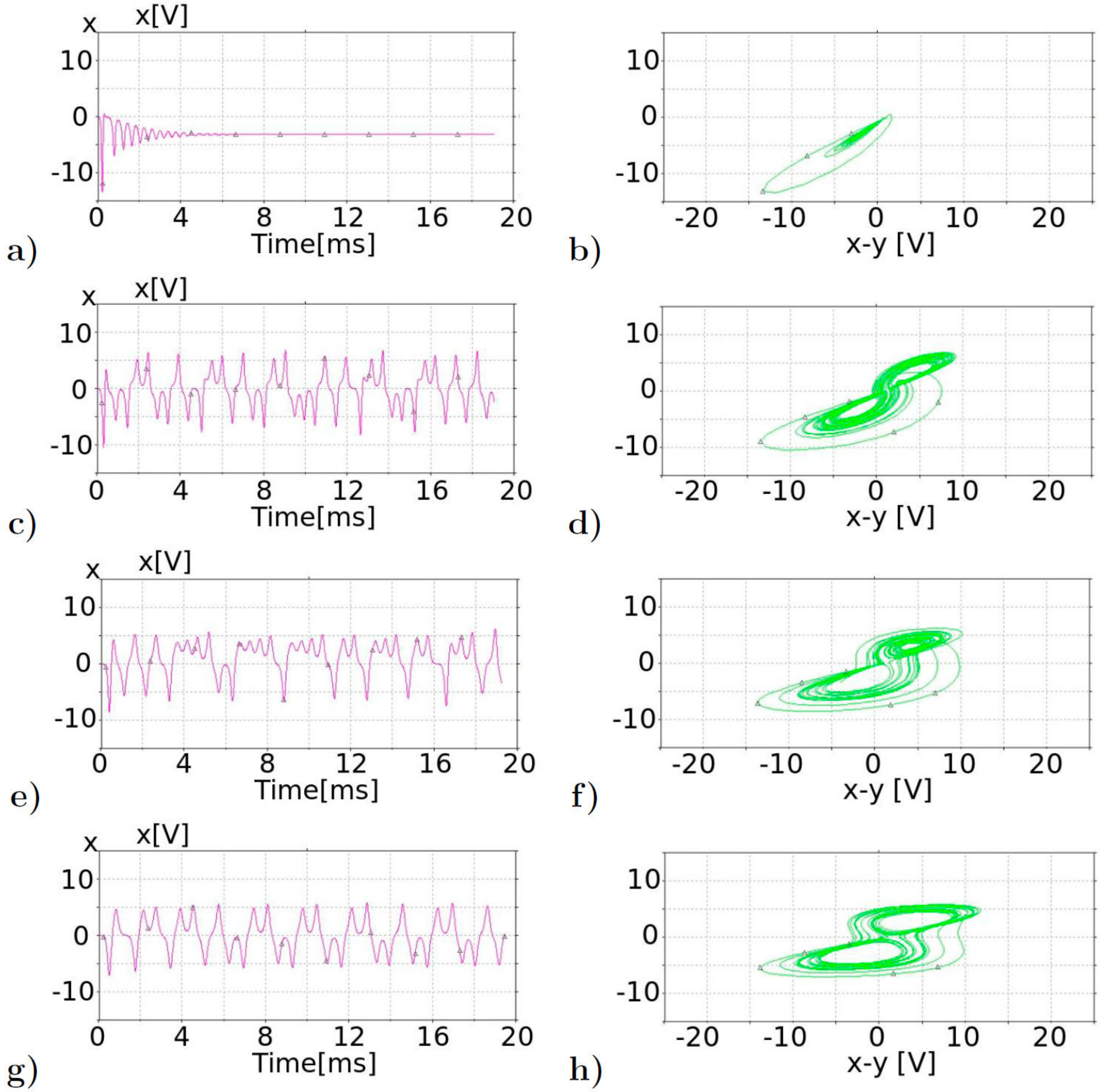

FIGURE 8. Lorenz electronic circuit Multisim response. a) x state versus time, b) $x - y$ phase portrait.

5. Conclusions

In this investigation, we use the equation of x state of Lorentz system. It turns out that the equation for the passive RC low-pass filter is the dual of the x state equation. where the parameter σ corresponds to the cutoff frequency $\omega = 1/RC$. The Bode plot corroborates this duality of both equations as is shown in Fig. 2. The Corron electronic circuit is one of the easiest circuits to emulate the Lorenz system, so it is used as the basis for modification. Based on Corron's scheme, the subtractor-configured and integrator-configured opamps used to construct x state are replaced by a passive RC low-pass filter circuit tuned to the cutoff frequency σ. The proposed circuit in Fig. 6 can generate the chaotic oscillations shown in Fig. 7. This shows that this proposal is one of the simplest ways to emulate a Lorenz system using two multipliers, two amplifiers, resistors, and capacitors. By adjusting the resistance of the RC filter according to Table I, we find that there is a frequency interval in which the chaotic oscillations are maintained, after which the chaotic oscillations cancel. Science students learn that different cutoff frequencies give different behaviors in the modified Lorenz circuit. If you want to vary the value of the R resistor continuously, replace it with a trimpot and the oscilloscope display will show the appearance of chaos and its cancellation at various cutoff frequencies.

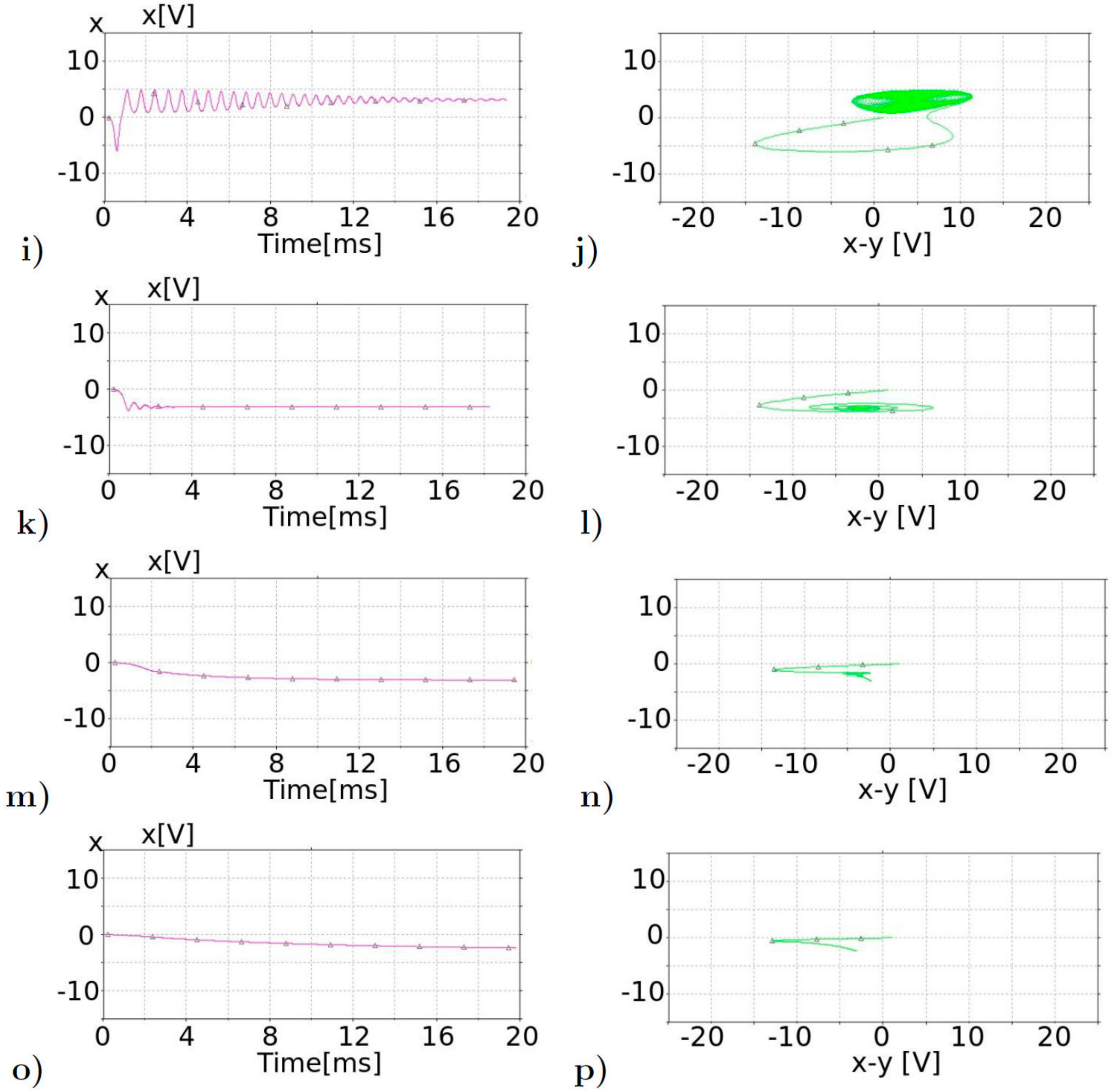

FIGURE 9. Lorenz electronic circuit Multisim response. a) x state versus time, b) $x - y$ phase portrait.

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