Modeling damped spring vibration using python to train students' critical thinking and scientific reasoning

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Programming has been carried out to model the vibration of a mass spring system with no damping and with damping variations using `Python` programming language coding. This research aims to simulate the simple harmonic vibration of a mass spring (without damping) and the vibration of a damped mass spring. The programming is designed after formulating the equations of motion of damped mass spring vibrations that behave as second-order differential equations and analyzing the numerical formulation of the Feynman-Newton algorithm arrangement. The method used is experimentation using `Python` to simulate the vibration of the spring. Simulation by varying the damping constant ($c = 0$, $c < \sqrt{4mk}$, $c = \sqrt{4mk}$, and $c > \sqrt{4mk}$). The simulation results show various vibration graphs, namely simple harmonic vibration, under damped vibration, critically damped vibration, and over damped vibration. The shape of the vibration graph is influenced by the mass, spring constant, and damping constant. The greater the damping constant, the less the maximum speed of vibration. This research has succeeded in visualizing the simple harmonic vibration of a massless spring (without damping) and the vibration of a damped massless spring. This modeling can help the physics learning process in high school in understanding the concept of spring vibration to obtain critical reasoning in accordance with physical phenomena. Modeling of damped spring vibrations using `Python` can be used as a physics learning media on vibration material to train scientific and critical reasoning skills, and as a learning innovation because it is a new thing where the curriculum and high school learning in Indonesia are not used to being delivered.

Keywords: Simple harmonic vibration; under-damped; critically damped; over-damped.

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1. Introduction

The spring system is a system consisting of a series of springs loaded with mass, in experiments generally using a spring system vertically, namely the spring is hung on a stative and the lower end is given a load of a certain mass. To produce spring vibrations or oscillations by giving a downward deviation to the associated mass and then released. Spring oscillation systems can be made with many variations, including variations in the mass of the load, variations in the arrangement of springs in series, variations in the arrangement of parallel springs, variations in the arrangement of springs combined in series and parallel, variations in load and spring arrangement can produce spring vibrations that produce equations of motion so that simulations or modeling can be made using certain programs including spreadsheet, `MATLAB`, and `Python` as has been researched by [1–5] including variations in spring geometry shapes involving spring diameter variables [6].

Variations of spring system vibrations can be divided into free spring systems or harmonic vibrations (without dampers), and variations of spring systems with dampers (damping) without friction and friction. The spring system variation system with damping consists of underdamped, critically damped, and overdamped. As a spring vibration damper, it can be in the form of mass, and friction from a liquid such as oil [7] which can be applied to shock absorber technology [8], manned aircraft landing gear movement [9] construction of high-rise structures and buildings, bridge vibration control with distributed mass dampers (DMD) and tuned mass dampers (TMD) [10], as structural vibration control under earthquakes and other external dynamic forces it seems [11], using the utilization of vibration damping systems can be utilized in life related to technology, namely by engineering vibration damping vibrations including vibration engineering in the fields of acoustics, building physics, industrial fields, vibrations in structures, sound absorption measurements, and environmental noise.

As a medium of learning and theoretical studies that are relevant to reality, the vibration of the spring system can be simulated using graphs that are very useful in learning literacy that can sharpen reasoning and improve the ability to think at the that can sharpen reasoning and improve high-level thinking skills (HOTS), high order thinking skills (HOTS) and as a fun teaching material [12, 13] both at high school and college level, learning at the high school and university levels. To obtain a simulation or modeling using a graph on a spring vibration system first must first determine the equation of motion of the oscillation. There are several analytical and numerical methods that can be used to determine spring vibration modeling, among others Rungge-Kutta Gill method [14], Adams-Basforth-Moulton method, the Milne method [15], the Shooting method, the Galerkin method, Euler's equation [16], Hamming's method, finite difference method [17], Hamming's method, the finite difference method [18], Laplace transform, Feynman-Newton method, and others.

In real-world events, the oscillations that occur rarely follow true simple harmonic motion, but there is friction that acts to dampen the motion so that the vibrations either stop immediately, or require more force to continue vibrating. An example of damped harmonic motion is when a guitar string stops vibrating a few seconds after being plucked. A child playing on a swing, to keep swinging must continue to push the swing against the vibration dampening force, a suspension system with a vehicle shock absorber that provides comfort due to uneven road surfaces that cause vibrations but with the right time interval back to silence by dampening vibrations, or car shock absorbers. Although we can often make friction and other non-conservative forces small or negligible, for example in a vacuum, truly undamped motion is rare. But the high school physics learning curriculum in Indonesia on the subject of vibration only presents simple harmonic vibration material and no one has discussed damped vibrations, this will greatly affect the mindset of students and physics concept maps so that understanding of physics concepts is not in accordance with actual symptoms or phenomena. Therefore, the author conducts research by modeling damped spring vibrations using numerical formulations with the Feynman-Newton algorithm method.

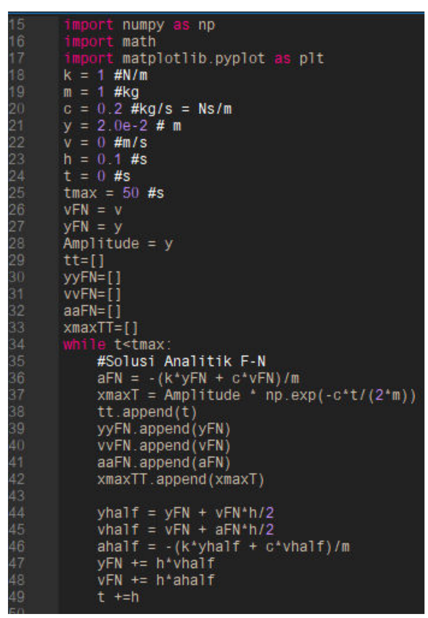
The author chose the Feynman-Newton method because this method tries to improve the Euler method and other classical methods by using the derivative value in the middle between the two final interval points (half-step method) so as to produce smoother graph visualization and more accurate results [19, 20]. The focus of vibration modeling is to compare variations without damping, with several damping constants, with critically damped, over damped, and under damped. The modeling simulation uses `Python` coding, a high-level programming language that has now become standard in the world of scientific computing. `Python` allows us to quantitatively or qualitatively explain complex material [21]. `Python` is a multi-platform open source programming language that can be used on a variety of operating systems (`Windows`, `Linux`, and `MacOS`). In addition, `Python` is also a programming language that is flexible and easy to learn.

Based on research by [22] the utilization of the `Python` programming language can be used as a medium that can encourage students to learn new things in physics. Simple experiments using free `Tracker` software at low cost, and easy-to-find materials can be used in high school physics learning to direct students in developing conceptual understanding of the material being studied [23]. Based on the background of the problem, this research aims to simulate the simple harmonic vibration of a mass spring (without dampers) and the vibration of a damped mass spring. The programming is designed after formulating the equations of motion of damped mass spring vibrations that behave as second-order differential equations and analyzing the numerical formulation of the Feynman-Newton algorithm arrangement.

2. Method

The method used is experimentation using `Python` to simulate the vibration of the spring. After doing analysis of mathematical equations, to create a simulation of the vibration of the spring system with dampers, the stages of research carried out include determining vibration variables include: $k = 1,0$ N/m, $m = 1,0$ kg, $c = 0.2$ kg/s or Ns/m, $y = 2,0 \times 10^{-2}$ m, v(initial) $= 0$ m/s, h(iteration) $= 0,1$ s, t(initial) $= 0$ s, $t_{\max} = 50$ s, then create a spring system simulation program using `Python` as shown in Fig. 1.

After completion, the program is run so as to produce graphs that can be interpreted, as well as by changing the damping constant parameter (c), including without damping constant ($c = 0$), with damping referring to three types of damping, namely under damped ($c < \sqrt{4mk}$) two variations,



```
import numpy as np
import math
import matplotlib.pyplot as plt
k = 1 #N/m
m = 1 #kg
c = 0.2 #kg/s = Ns/m
y = 2.0e-2 # m
v = 0 #m/s
h = 0.1 #s
t = 0 #s
tmax = 50 #s
vFN = v
yFN = y
Amplitude = y
tt=[]
yyFN=[]
vvFN=[]
aaFN=[]
xmaxTT=[]
while t<tmax:
    #Solusi Analitik F-N
    aFN = -(k*yFN + c*vFN)/m
    xmaxT = Amplitude * np.exp(-c*t/(2*m))
    tt.append(t)
    yyFN.append(yFN)
    vvFN.append(vFN)
    aaFN.append(aFN)
    xmaxTT.append(xmaxT)

    yhalf = yFN + vFN*h/2
    vhalf = vFN + aFN*h/2
    ahalf = -(k*yhalf + c*vhalf)/m
    yFN += h*vhalf
    vFN += h*ahalf
    t +=h
```

FIGURE 1. Coding of spring vibration programming using `Python`.

```
    vvFN.append(vFN)
    aaFN.append(aFN)
    xmaxTT.append(xmaxT)

    yhalf = yFN + vFN*h/2
    vhalf = vFN + aFN*h/2
    ahalf = -(k*yhalf + c*vhalf)/m
    yFN += h*vhalf
    vFN += h*ahalf
    t +=h
#create figur
fig,ax = plt.subplots()
#plot
ax.plot(tt,vvFN, color="red",marker="",label='v')
#set x_axis label
ax.set_xlabel("t(s)",fontsize=14)
#set y_axix label
ax.set_ylabel("v(m/s)", color="red",fontsize=14)
ax.legend(loc='upper left')

#twin plot
ax2=ax.twinx()
ax2.plot(tt, yyFN,color="b",marker="",label='x')
ax2.plot(tt, xmaxTT,'--', label=r'$y_{max}(t)=Ae^{-ct/2m}$')
ax2.set_ylabel("x(m)",color="b", fontsize=14)
ax2.legend(loc='upper left')

ax = plt.axes(projection='3d')
ax.plot3D(tt,tt,vvFN, color="red",marker="",label='v(m/s)')
ax.plot3D(tt,tt,yyFN, color="blue",marker="",label='x(m)')
ax.plot3D(tt,tt,xmaxTT, label=r'$y_{max}(t)=Ae^{-ct/2m}$')

ax.legend(loc='upper left')
ax.set_xlabel('t(s)')
ax.set_ylabel('t(s)')
#ax.set_zlabel('x(m)');
```

FIGURE 2. Coding of 2-dimensional and 3-dimensional plot programming in Python.

critically damped ($c = \sqrt{4mk}$), and over damped ($c > \sqrt{4mk}$) three variations. In this research, graph visualization is also made in 2-dimensional and 3-dimensional forms. The detailed coding for 2-dimensional and 3-dimensional plots is shown in Fig. 2.

3. Results and Discussion

After determining the parameters of the spring vibration system variables and simulating it using Python, the results of several graphs such as Figs. 3a), b), c), d), e), and f) are obtained.

Based on Fig. 3a), it appears that the vibration of a spring without a damping constant ($c = 0$) produces simple harmonic vibrations, namely vibrations that produce a fixed amplitude along t. according to the simulation using a spring constant $k = 1.0$ N/m, a load mass $m = 1.0$ kg, and the first spring deviation upwards as far as $y = 2.0 \times 10^{-2}$ m $= 0.02$ m, so that a fixed amplitude of 0.02 m is vibrating up

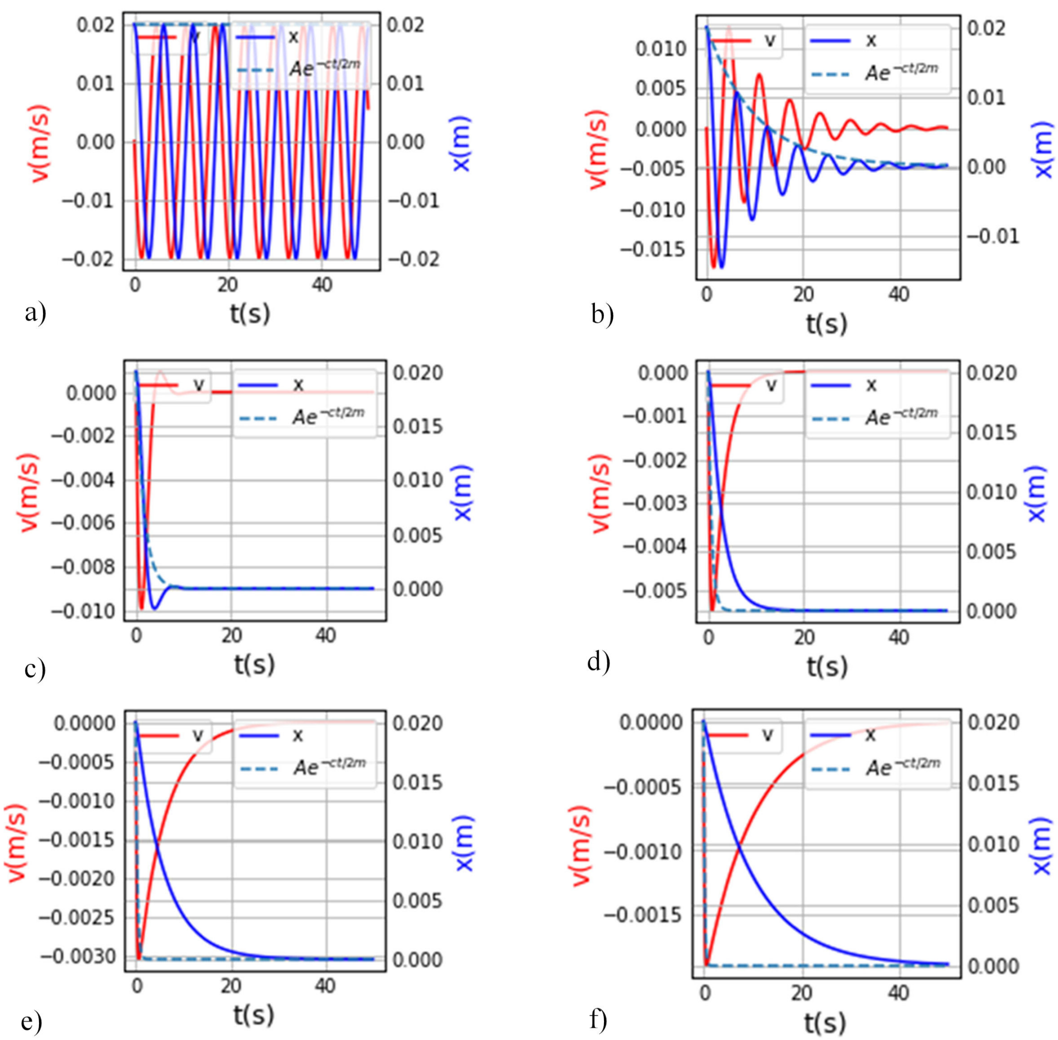


FIGURE 3. Simulation results of spring vibration using Python with different damping constants a) $c = 0$, b) $c = 0,2$ kg/s $< \sqrt{4mk}$, c) $c = 1,2$ kg/s $< \sqrt{4mk}$, d) $c = 2$ kg/s $= \sqrt{4mk}$, e) $c = 6$ kg/s $> \sqrt{4mk}$, and f) $c = 10$ kg/s $> \sqrt{4mk}$

and down around the equilibrium point (0.00). Based on the five graphs show that the direction of velocity v (red) is always opposite to the direction of deviation x (blue), this indicates that v is in accordance with the direction of the spring restoring force which is always against the direction of its deviation. The maximum velocity is 0.02 m/s.

Figures 3b) to f) by filling in the damping constant from small to large. Figures 3b) and c) respectively $c = 0.2$ kg/s and $c = 1.2$ kg/s both $< \sqrt{4mk}$ meet the under damped graph (low damping) *i.e.*, the object oscillates several vibrations then stops, the greater the c looks the less the spring vibrations produced. Figure 3b) has more vibrations, namely as many as 6 vibrations in 40 seconds, with the amplitude getting smaller, decreasing based on $x(t) = A\exp(-c/2m)(\cos\omega t + \theta)$ according to the blue dotted line, this is also shown by decreasing the vibration speed with the maximum speed decreasing from 0.02 m/s to around 0.015 m/s. Whereas Fig. 3c) with a larger c of 1.2 kg/s shows that the oscillation is only half vibration and immediately comes to an equilibrium position at the second before reaching the 10th s with the maximum velocity decreasing to 0.010 m/s.

Figures 3d) with a value of $c = 2$ kg/s $\sqrt{4mk}$ is the limit value of the critical damping constant (critically damped) *i.e.*, it appears that the spring system is not isolated, but after the object is released from its deviation the object returns to its equilibrium position, it appears that before the 20th second it is in an equilibrium position and stops vibrating with a maximum speed drop to 0.005 m/s.

While Figs. 3e) and f) with damping constants exceeding the critical damping c value of 6 kg/s and 10 kg/s, respectively, so that the graph meets over damped. It appears that both graphs do not oscillate and after being released from the deviation, the object goes to the equilibrium point, namely in Fig. 3e) after passing the 20th second with the maximum speed decreasing to 0.003 m/s, and in Fig. 3f) after passing the 40th second with the maximum speed decreasing to 0.002 m/s. The time required to go to the equilibrium position in Fig. 3f) is greater than in Fig. 3e) because the friction force (which is influenced by the damping constant c) which inhibits the motion of the object is greater. The results of modeling using `Python` are in accordance with the references presented by [24, 25]. In addition, the researchers also visualized the graph in the 3-dimensional form shown in Fig. 4.

Figure 4 shows the simulation of damped spring vibration using `Python` in 3-dimensional form. The longer, the damping is visible, with the red color showing the velocity variable and the blue color showing the position variable. With the 3-dimensional form, it is hoped that it can better visualize the shape of the damping in detail. It is hoped that with the visualization of the data, critical thinking and scientific reasoning of students can increase.

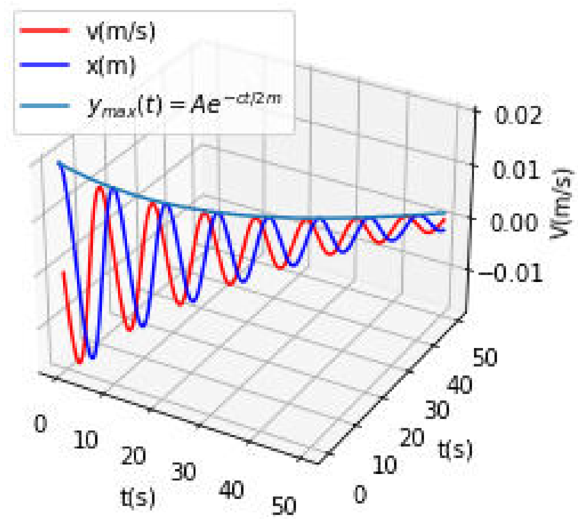


FIGURE 4. Simulation results of spring vibration using `Python` in 3-dimensional form.

4. Critical Thinking and Scientific Reasoning

Critical thinking and scientific reasoning are skills or life skills that must be provided to students in a rapidly evolving and increasingly complex world [26–30]. Critical thinking is the ability of individuals to think and make correct decisions independently to achieve understanding, evaluate different perspectives, and solve problems [31]. There are 5 strategies to build critical thinking skills [32] including: providing motivation to think, time to develop ideas, collaboration and support from a learning community that provides information, feedback and encouragement. Central to the development of critical thinking is reflection and stimulating representation [33] In addition, learning that provides opportunities for student-led inquiry can encourage the development of their creativity and critical thinking [34]. Critical thinking skills can be included in the curriculum, can be taught by incorporating active learning using simulations, formative feedback, and clinical reasoning by building on knowledge gained through student-centered learning, and assessed using validated tests [35, 36]. Based on Bloom's taxonomy, instruments to measure critical thinking skills are not suitable if only at the Lower Order Thinking Skills (LOTS) level which measures the ability of memory (C1), understanding (C2), and application (C3) but it is recommended at the Higher Order Thinking Skills (HOTS) level which measures the ability of analysis (C4), evaluation (C5), and creation (C6) and by using essay questions or open questions [25]. Critical thinking can be expressed through a skillful interpretation and evaluation process of observation, communication, information and argumentation so that students have the ability to explore points of view, reasoning, investigating, comparing, connecting, finding the complexity of a problem, and coming up with new ideas [30].

Scientific reasoning is defined, broadly, as thinking skills that include investigation, experimentation, evaluation of evidence, inference and argumentation carried out as an effort for scientific conceptual understanding. Scientific reasoning and critical thinking skills are important learning outcomes in modern science education [31]. Scientific reasoning skills [32] consist of analysis skills such as (i) analyzing basic concepts, variables and study components; (ii) relationships between components; (iii) all components represented as a coherent structure; and synthesis *i.e.* evaluating and coordinating more than one study structure (articles, image components, models, etc.) with others. Training concept knowledge and developing scientific reasoning skills related to cognitive abilities such as critical thinking and reasoning can be developed through learning. Scientific reasoning training also has a long-term impact on students' academic achievement [33, 34]. Based on the exposure to the theory of critical thinking and scientific reasoning, the instrument used to measure success in critical thinking and scientific reasoning training is an essay test (open-ended questions) that measures the level of higher order thinking skills (HOTS) including the levels of analysis (C4), evaluation (C5), and creation (C6).

This modeling has been simulated and applied to the Physics MGMP group at SMAN 1 Piyungan and tested as a learning media for students of class XI MIPA so that it can help the physics learning process in understanding the concept of spring vibration which is influenced by several factors including damping. During the learning and assessment process, students can gain learning experiences that train critical thinking skills and scientific reasoning, namely being able to solve physics problems, especially in material related to undamped and damped vibrations through questions and answers, discussions, and practice questions that measure the level of higher thinking skills (HOTS) including the level of analysis (C4), evaluation (C5), and creation (C6). Students also get an overview of vibration facts or events in accordance with real physics phenomena or symptoms. This result is a learning innovation because it is a new thing where the curriculum and previous learning have never been applied.

5. Conclusion

Based on the results of spring vibration research through simulation and modeling using `Python`, it is concluded that researchers have succeeded in visualizing the simple harmonic vibration of a mass spring (without a damper) and the vibration of a damped mass spring. Simulations using the `Python` program can prove the characteristics of the spring vibration equation both undamped and damped, using analytical solutions and numerical formulations that have been derived.

Vibration a spring without a damping constant produces simple harmonic vibrations, namely vibrations of objects that go back and forth around the equilibrium point with a fixed amplitude or maximum deviation is fixed. Under damped spring vibrations with a low damping constant value $c < \sqrt{4mk}$ is the vibration of a spring that produces a decreasing amplitude and velocity so that after oscillating, it will stop at the set position, oscillation will stop at the equilibrium position. Under damped spring vibration (under damped) with a damping constant value of $c < \sqrt{4mk}$ is a spring vibration that produces amplitude and velocity spring vibrations that produce increasingly smaller amplitudes and velocities so that the oscillation will stop at an equilibrium position. After oscillating will stop at the equilibrium position. Critically damped spring vibration (critically damped) with a damping constant value $c = \sqrt{4mk}$ is a spring system that after experiencing a deviation does not oscillate but immediately returns to an equilibrium position with a time interval to equilibrium faster than the strong damping (over damped). Vibration of a strongly damped (over damped) spring with value of damping constant $c < \sqrt{4mk}$ is a spring system that after experiencing system that after experiencing a deviation does not oscillate but immediately returns to an equilibrium position with a time interval to equilibrium longer than the critically damped. The greater the vibration damping constant of the damping constant, the smaller the maximum speed of the load that deviates or vibrate.

This modeling has been simulated and applied to the Physics MGMP group at SMAN 1 Piyungan and tested as a learning media for students of class XI MIPA so that it can help the physics learning process in understanding the concept of spring vibration which is influenced by several factors including damping. During the learning and assessment process, students can gain learning experiences that train critical thinking skills and scientific reasoning, namely being able to solve physics problems, especially in material related to undamped and damped vibrations through questions and answers, discussions, and practice questions that measure the level of higher thinking skills (HOTS) including the level of analysis (C4), evaluation (C5), and creation (C6). Students also get an overview of vibration facts or events in accordance with real physics phenomena or symptoms. This result is a learning innovation because it is a new thing where the curriculum and previous learning have never been applied.

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