Physical reality

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The principle of non-locality, or the existence of systems of particles with properties that relate them even at great distances, is called entangled systems and defies the intuition that requires all relationships to be described by means of energy exchange or by material links. Quantum physics presents non-locality as a consequence of objects being described by a single wave function, and as such, unless they decohere (lose their nonphysical link), the relationship remains, and each cannot be understood separately. Recently, there have been many technological models that can produce entangled systems. In addition to the examples that submicroscopic physics can illustrate, the simplest is spontaneous parametric fluorescence, which requires a laser and a parametric crystal, which has allowed very elaborate experiments to be carried out and shows the relevance of quantum physics and the limitations of our perception. The examples described here have emerged over the years as attempts to make this concept more acceptable and try to guide the imagination to situations where these types of phenomena can be plausible, pointing to human perception with the duality of bringing us closer to nature appreciation, but at the same time, it is the main limitation to understand nature. The relevance of this concept was recognized with the Nobel Prize 2022 in physics, and it can be summarized as the Proof of the Bell inequality using anecdotes from a Nobel non-recipient.

Keywords: Non-locality; non-contextuality; Bell's inequality; entangled states.

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1. Introduction

In considering communication challenges, let's simplify this with two terms: "teaching" and "explaining." When teaching, both the teacher and the learner invest considerable effort, dedication, and time to reach shared conclusions. In contrast, explaining relies more on the recipient's existing knowledge and experience, requiring less effort but with similar comprehension expectations. Disappointment is more probable with explanations since the burden of clarity falls on the speaker, while teaching places responsibility on the learner.

This dynamic becomes particularly pronounced with complex subjects like quantum physics. Even among those well-versed in the field, there can be difficulties in interpreting certain details, especially when they clash with our everyday experiences. This is evident in concepts like locality, which are inherently challenging to grasp outside specialized training.

In the concept of reality, several words should be understood in the context of the dynamics of the processes so that we can identify and explain how a system changes to something else. The ideas that define reality from a classical point of view are rigid and obtained from human experience, which is very limited since humans did not evolve to understand nature but to survive, and these two properties are not always aligned. Since the last century, the question of what physical reality is has given rise to numerous discussions since the behaviour of the submicroscopic world, where one understands the interactions between particles that cannot be seen with an optical microscope, does not fully correspond to the classical sense of the systems, and the difference between the two has been attempted to reconcile by invoking that the physics of the submicroscopic world is incomplete, that it includes not linear phenomena described in the equations, which requires to emphasizing the random aspects, in addition to considering that if the system is closed, the energy is a constant and only change in an open system that communicates with the outside and the unknown details may help to explain the difference with perception. Points of disagreement may include any number of the above arguments.

In a classical sense, reality is local, which involves the concept of space (the force and the response are spatially limited and related); it is causal, which involves the concept of time (the response is posterior to the cause), in addition to assuming that the processes are continuous and that wave and corpuscular phenomena are independent and different.

In the sense of physical reality, quantum physics is the theory that produces predictions that even exceed the limits of one's experimental capacity, but with three inherent drawbacks: First, the interpretation of its meaning is different for different groups of scholars, or the interpretation is even considered irrelevant when the results of the calculations are so precise. Second, every system is associated with a wave function that generally can be complex, which is why the distinction between waves and particles disappears. Everything is represented by a wave function. Third is that it seems that it is only relevant in phenomena that have objects much smaller than those that a human can perceive.

The main concept that makes it difficult to understand the results of quantum physics and somehow converges in one interpretation is the concept of whether two events commute or not. In everyday life, we accept that some events do not commute since experience tells us so. Something like dressing up and taking a shower and hoping that the result is the same as first taking a shower and after dressing up, not all preparations impose this restriction, but clearly, some require an imposed order, and the result is different according to the sequence.

Accepting that the same procedure can produce different results is a good starting point for understanding many phenomena in the submicroscopic world, where atoms and elementary particles like electrons and photons are described, as well as stop thinking that the correlation between systems must be given by something physical that connects them in space or through an interaction of energy (signals at a distance) that connects them with information as well as realizing that it is not obvious what features of the system require attention to understand that relationship and what degree of connection this is. Understanding the measurement process is crucial, especially in contexts where it's typically considered independent of the measured phenomenon. However, when dealing with variables in the submicroscopic realm (like atoms) or requiring high precision (as in interferometers for measuring gravitational waves), the measurement process itself can significantly influence the result. It's important to note that what may seem strange or unusual often occurs when working with very small systems. For instance, quantum behaviour has been observed in objects as large as the 40 kg mirrors used in the LIGO interferometer for measuring gravitational waves [1-3].

Complementary, non-commutation, uncertainty principle, entanglement, and contextual properties are intricate concepts that lead to surprising results, exemplified by phenomena like Young's double slit experiment with particles [4]. These ideas highlight the human difficulty in comprehending complex or non-linear processes, as illustrated by the anecdote about the game of chess and grains of rice [5]. This text will explore the concept of non-locality, using various examples to convey its complexity. It will delve into the importance of context using simple probability and relate it to understanding clinical diagnoses. The discussion will then transition to non-locality demonstrated through simple operations with surprising outcomes, followed by an analogy involving an example of quantum-baking reminiscent of Sterns and Gerlach's proposed experiments. Lastly, a straightforward experiment conducted by Professor Fry from TAMU will be described, showcasing the didactic nature of understanding non-local properties in nature, ultimately convincing the reader about the non-local properties of nature.

2. Diagnosis through clinical analysis

Before discussing some examples of non-locality and context in the realm of physics, let's first discuss an example of a medical diagnosis. This example can be used to understand the role of context and probability in determining the state of a system. In the medical diagnosis scenario, the probability of an individual being sick or not sick is determined based on the entire population's context and the test's properties. Similarly, in quantum physics, the properties of entangled particles are determined not only by their individual states but also by their contextual relationship, regardless of the distance between them. This analogy underscores the idea that understanding a system's state relies on contextual information and probabilities in both scenarios, reflecting the non-local nature of quantum phenomena.

In the context of understanding the results of a medical diagnosis [6,7], consider the following hypothetical situation: the universe of the population is divided into sick and not sick people $U = \{S, NS\}$, and let us assume it has been determined that for every 10,000 people in this universe one is sick, indicating that the probability that an individual is sick is $P(S|U) = 0.0001$, with the probability that an individual is not sick is given by the complementary part $P(NS|U) = 1 - P(S|U) = 0.9999$.

Additionally, a clinical study detects the disease when the patient is sick 90% of the time $P(+|S) = 0.9$ (analogous to the case when the culprit is convicted). In addition to indicating that the patient is not sick 99.9% of the occasions $P(-|NS) = 0.999$ (the innocent is released). Moreover, due to the partition of the patients, it must be fulfilled that: $P(+|NS) = 1 - P(-|NS) = 0.001$ (false positive, the innocent is convicted) and $P(-|S) = 1 - P(+|S) = 0.1$ (false negative, the culprit is released).

The relevant information for the patient is not the prevalence of the disease in the population nor any other conclusion about the clinical assay performance. What matters to the subject is to know the probability of being really sick in case the result of the clinical test was positive $P(S|+)$ or any practical conclusion derived from such test; not sick despite a positive result $P(NS|+)$, sick despite negative result $P(S|-)$ or non-sick with negative result $P(NS|-)$. Let us see this invoking again total probability:

$$\begin{aligned} P(+|U) &= P(+|S)P(S|U) + P(+|NS)P(NS|U) \\ &= (0.9)(0.0001) + (0.001)(0.9999) \\ &= 0.001089, \\ P(-|U) &= P(-|S)P(S|U) + P(-|NS)P(NS|U) \\ &= (0.1)(0.0001) + (0.999)(0.9999) \\ &= 0.9989101. \end{aligned}$$

Figure 1 summarizes the results of the contextual probability that we have talked about and concludes that 8.2% probability of being sick if the test result is positive is not very reassuring, but it is information to be obtained, and it is not obvious at first glance, results are dependent on the context.

Applying Bayes' rule: $P(a|b)/P(a|U) = P(b|a)/P(b|U)$ to determine the information that is sought

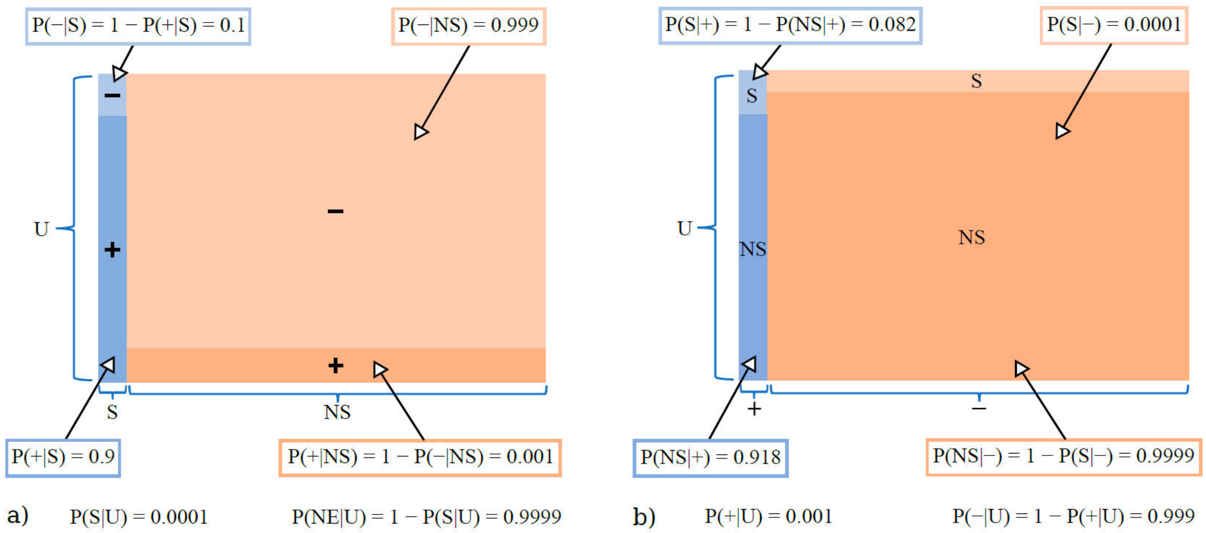


FIGURE 1. Contextual probabilities: a) relative to sick and healthy people, b) relative to positive and negative results in the clinical test.

and that it is not evident from the information available at first hand.

$$P(S|+) = \frac{P(+|S)P(S|U)}{P(+|U)} = \frac{(0.9)(0.0001)}{0.0010899} \approx 8.2\%,$$

$$P(NS|+) = \frac{P(+|NS)P(NS|U)}{P(+|U)} = \frac{(0.001)(0.9999)}{0.0010899} \approx 91.8\%,$$

$$P(S|-) = \frac{P(-|S)P(S|U)}{P(-|U)} = \frac{(0.1)(0.0001)}{0.9989101} \approx 0.001\%,$$

$$P(NS|-) = \frac{P(-|NS)P(NS|U)}{P(-|U)} = \frac{(0.999)(0.9999)}{0.9989101} \approx 99.999\%.$$

3. Game of surprises

Let's define a game that can be deceivingly simple and still guide us to confusion. It is a game of chance of finding objects in adjacent boxes [8], and to write a simple formal way to express the results and to leave with the impression we understand very little or again, if one ignores the context, the results are deceiving. The game consists of 5 boxes, one next to another, at the vertices of a pentagon. It turns out that an observer can open two adjacent boxes and observe their contents, which can be full or empty, and the rules that we are going to follow, the definition of reality is:

1. It is true that when box B is full, box A is empty, and it is also true that when box B is empty, box C is full; mathematically, this can be expressed as $P(0,1|\text{A,B}) + P(0,1|B,C) = 1$. Also, this rule can be expressed as: the boxes A and B cannot be full simultaneously, nor can boxes B and C be empty at the same time, $P(1,1|\text{A,B}) = P(0,0|\text{B,C}) = 0$.

2. It is also true that when box D is full, box C is empty, and it is also true that when box D is empty, box E is full, $P(0,1|\text{C,D}) + P(0,1|\text{D,E}) = 1$, which can also be said that boxes C and D cannot be full simultaneously nor can boxes D and E be empty at the same time $P(1,1|\text{C,D}) = P(0,0|\text{D,E})$.

The first two rows in Table I correspond to rule 1, and the last two rows correspond to rule 2. From this table, we construct the summary in Table II with the information that does not contradict each other; the first row is formed from rows 1 and 3 of Table I, the second row is formed from rows 1 and 4 of Table I, and the third row is formed from rows 2 and 4 of Table I, and It is not possible to group the rules to construct any row different from those indicated in Table II. The boxes

TABLE I. The rules of the game of boxes.

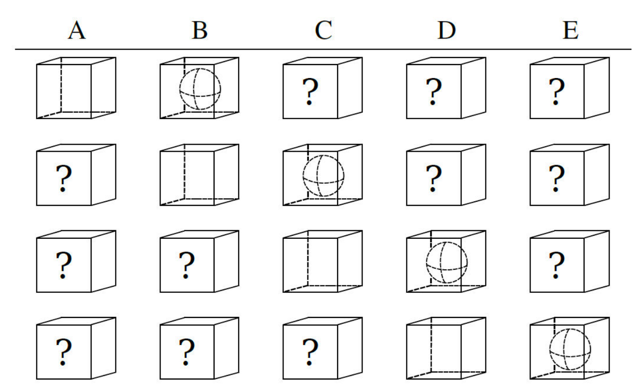
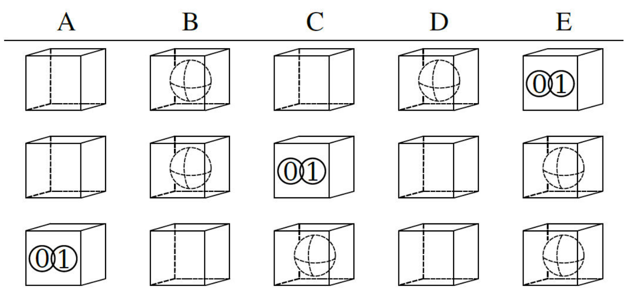


TABLE II. All the possible scenarios in the game.



with the 0 and 1 symbols in Table II indicate any of the two possible cases: the box is full or empty. For all the boxes, there are only six possible scenarios, two scenarios for each row in Table II.

If we ask about the probability of finding box E empty and box A full, the answer is zero $P(0,1|\text{E,A}) = 0$, since the first row of Table II produces E empty, but A is also empty, and the third produces A full, but E full too.

The idea is to propose a systematic strategy to describe the imposed reality, which faithfully reproduces it and allows the last question to be posed and answered so that the conditions imposed by the game are met.

One can image probability as represented by the dot product, using a 3-element basis $|\eta\rangle = (1/\sqrt{3})(1,1,1)^T$, the properties described can be represented by projections of vectors representing the boxes. The first goal is to invent the vectors that represent the boxes and fulfill the description of reality by the dot product. The projection of the vectors represents the probability that the box is full or empty as follows:

$$|v_\text{A}\rangle = \frac{1}{\sqrt{3}}(1,-1,1)^T, \qquad |v_\text{B}\rangle = \frac{1}{\sqrt{2}}(1,1,0)^T,$$
$$|v_\text{C}\rangle = (0,0,1)^T, \qquad |v_\text{D}\rangle = (1,0,0)^T,$$
$$|v_\text{E}\rangle = \frac{1}{\sqrt{2}}(0,1,1)^T.$$

As can be seen, the vectors are orthogonal with their neighbours; in the case of boxes B to E, the definitions are obvious, and box A is obtained from the cross product of box B and box E. The definition is arbitrary and developed to fulfil the definitions of reality.

TABLE III. Limit values for the reactor design variables.

	$\|\eta\rangle$	$\|v_\text{A}\rangle$	$\|v_\text{B}\rangle$	$\|v_\text{C}\rangle$	$\|v_\text{D}\rangle$	$\|v_\text{E}\rangle$
$\langle\eta\|$	1	$\frac{1}{3}$	$\sqrt{\frac{2}{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$
$\langle v_\text{A}\|$	$\frac{1}{3}$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	0
$\langle v_\text{B}\|$	$\sqrt{\frac{2}{3}}$	0	1	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\langle v_\text{C}\|$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	1	0	$\frac{1}{\sqrt{2}}$
$\langle v_\text{D}\|$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	1	0
$\langle v_\text{E}\|$	$\sqrt{\frac{2}{3}}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	0	1

In such a way that the projections can be obtained by the dot product, as listed in Table III.

Using this mathematical frame, it is possible to reproduce the results defined by the rules of the game:

$$P_{|\eta\rangle}(0,1|\text{A,B}) = 0\langle\eta|v_\text{A}\rangle^2 + 1\langle\eta|v_\text{B}\rangle^2 = \frac{2}{3},$$
$$P_{|\eta\rangle}(0,1|\text{B,C}) = 0\langle\eta|v_\text{B}\rangle^2 + 1\langle\eta|v_\text{C}\rangle^2 = \frac{1}{3},$$
$$P_{|\eta\rangle}(0,1|\text{C,D}) = 0\langle\eta|v_\text{C}\rangle^2 + 1\langle\eta|v_\text{D}\rangle^2 = \frac{1}{3},$$
$$P_{|\eta\rangle}(0,1|\text{D,E}) = 0\langle\eta|v_\text{D}\rangle^2 + 1\langle\eta|v_\text{E}\rangle^2 = \frac{2}{3}.$$

The problem comes when we calculate the probability of finding the E box empty and the A box full, which is zero according to the scenarios in Table II.

$$P_{|\eta\rangle}(0,1|\text{E,A}) = 0\langle\eta|v_\text{E}\rangle^2 + 1\langle\eta|v_\text{A}\rangle^2 = \frac{1}{9}.$$

Confronting the opposing result, one observes the absence of an observer for boxes A and E in the first preparation. The second preparation is more systematic and describes in more detail the interaction and the context that produces the difference. The previous examples introduce the concept of context as a possibility for confusion. The goal is to understand non-locality in a broader sense because it is essential in quantum physics; John Bell [9] is credited for expressing the importance of context in a mathematical way, and the Nobel Prize in 2022 to Aspect, Clauser, and Zeilinger [10]. Although Edward Fry should have been included in the time between Clauser's experiment and Aspect, it was the most forceful experiment (Clauser invited Fry to the Nobel Prize ceremony).

4. Baking of quantum cakes

Let's start this section with a distraction to the Stern and Gerlach experiment [11]. In essence, it can be used to discuss the main principles of quantum physics, quantization, non-commutation, and entanglement. We will accept the first and third and concentrate on the second. We will conclude with the quantum cakes that focus on the third, accepting the second as a fact.

Observations by Stern and Gerlach (S-G), indicate that if a neutral particle is sent in a uniform magnetic flux density ($\vec{B}$ = constant) its trajectory is not modified ($F_z = \mu_z[dB/dz]$), the force will be zero. If a charged particle is sent, the trajectory is modified to produce spirals ($q(\vec{E} + \vec{v} \times \vec{B})$ as the electron path in an electron microscope, if the particle is quantum and neutral in a non-uniform field $\vec{B}$ ($F_z = g[e\hbar/2m][s/\hbar][dB/dz]$) the force will deflect according to the spin and for an electron the effect independent of the Lorentz force will be ($F_z = 2[e\hbar/2m](\pm[1/2\hbar])[dB/dz]$), that is similar to silver neutral atoms.

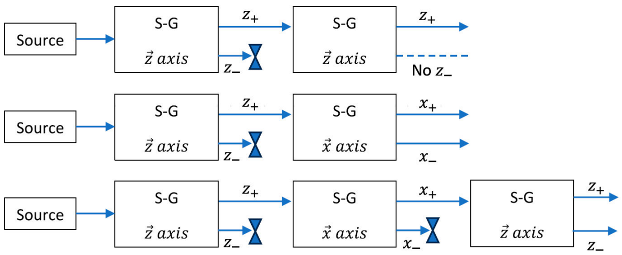


FIGURE 2. Observation de Stern y Gerlach after a series of analyzers, here one particle is analyzed after a sequence of S-G devices, pay attention to the third sequence where the no commutation is evident.

Sequential experiments of (S-G) type produce the experimental results exhibited in Fig. 2, which are analogous to using a sequence of polarizers for light. In the first experiment, a beam for which we do not know the information of the spin projection passes through a (S-G)$_z$ and the "measurement" produces two spots that correspond to the information of the beam, half are orientated in each direction, one of them passes through a second (S-G)$_z$ and only confirms the information we had, another deviation consistent with the previous one. In the second experiment, the first (S-G)$_z$ is identical to the previous observation, which measures only the information in Z, then it goes through a (S-G)$_x$ that now measures the information in X of the sample for which you knew the information in Z, and measures equal proportion in the resulting sports. The conclusion is that we cannot know both information simultaneously. The projection in Z and in X, only one of them, and the other is erased; the projections do not commute.

As a preparation for the final experiment, the quantum cake emphasizes entanglement, the property to share information from the origin, which is a paraphrase from the S-G observation but with two production lines that are related. This example is another opportunity to appreciate the importance of context, again assuming that only one observable is measured and it is random. In Measurable #1: a cake is observed in the middle of the process (rise (R) or collapses (N)), and in Measurable #2 the cake is tasted at the end of the process (good (G) or bad (B)). A line of cakes is defined with two exits, left and right, with the following measurements:

- The cake can come out good ($|G_L\rangle$) or bad ($|B_L\rangle$) on the far left.
- The cake may be raised (spongy) ($|S_L\rangle$) or collapsed ($|N_L\rangle$) when looking at it halfway through the process on the left.
- The cake can come out good ($|G_R\rangle$) or bad ($|B_R\rangle$) on the far right.
- The cake may be raised ($|S_R\rangle$) or collapsed ($|N_R\rangle$) when looking at it halfway through the process on the right.

This physical system that could describe this process [12] produces the following results:

1. The probability of observing both cakes raised is 9%, $P(|S_L\rangle \& |S_R\rangle) = 0.09$.
2. If $|S_L\rangle$ is observed then $|G_R\rangle$ is produced.
3. If $|S_R\rangle$ is observed then $|G_L\rangle$ is produced.
4. The probability that both cakes are good is zero, $P(|G_L\rangle \& |G_R\rangle) = 0$.

The contradiction with reality is that even setting out to achieve such observations, they cannot be achieved; at least 9% of the time, we would expect both cakes to be good.

A state that meets the imposed conditions is:

$$|\psi\rangle = \frac{1}{2}|B_L\rangle|B_R\rangle - \sqrt{\frac{3}{8}}(|B_L\rangle|G_R\rangle + |G_L\rangle|B_R\rangle) + 0|G_L\rangle|G_R\rangle.$$

A quarter of the time, it produces both bad cakes, the first term at right, and three-quarters of the time, it produces only one good one, but never both good ones.

1. When the furnaces are checked halfway, only 9% of the times produces $|S_L\rangle$ and $|S_R\rangle$.
   Replaced intermediate basis in all terms of $|\psi\rangle$ with expressions in Fig. 3. This procedure gives 9% of the time, both cakes are found to be rising.

$$\begin{aligned}|\psi\rangle &= \frac{1}{2}\left(\sqrt{0.4}|N_L\rangle + \sqrt{0.6}|S_L\rangle\right)\\ &\times\left(\sqrt{0.4}|N_R\rangle + \sqrt{0.6}|S_R\rangle\right)\\ &-\sqrt{\frac{3}{8}}\left(\sqrt{0.4}|N_L\rangle + \sqrt{0.6}|S_L\rangle\right)\\ &\times\left(-\sqrt{0.6}|N_R\rangle + \sqrt{0.6}|S_R\rangle\right)\\ &\times|\psi(|S_L\rangle|S_R\rangle)\rangle,\\ |\psi\rangle &= \left(\frac{1}{2}0.6 - \sqrt{\frac{3}{8}}\sqrt{0.6}\sqrt{0.4} - \sqrt{\frac{3}{8}}\sqrt{0.4}\sqrt{0.6}\right)\\ &\times|S_L\rangle|S_R\rangle = (-0.3)|S_L\rangle|S_R\rangle.\end{aligned}$$

2. If $|S_L\rangle$ (the cake on the left is observed to rise), then $|G_R\rangle$ (the cake on the right is good). Replaced intermediate basis in all terms of $|\psi\rangle$ with expressions in Fig. 3. The third term has the information from result #2.

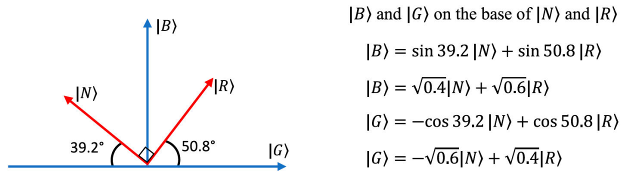


FIGURE 3. Projection of the base Rise and Non-Rise over the base Good and Bad used for the calculation in the example of the cakes.

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{2}\left(\sqrt{0.4}|N_L\rangle|B_R\rangle + \sqrt{0.6}|S_L\rangle|B_R\rangle\right) \\
&\quad - \sqrt{\frac{3}{8}}(\sqrt{0.4}|N_L\rangle|G_R\rangle + \sqrt{0.6}|S_L\rangle|G_R\rangle \\
&\quad - \sqrt{0.6}|N_L\rangle|B_R\rangle + \sqrt{0.4}|S_L\rangle|B_R\rangle), \\
|\psi\rangle &= \left(\frac{1}{2}\sqrt{0.4} + \sqrt{\frac{3}{8}}\sqrt{0.6}\right)|N_L\rangle|B_R\rangle \\
&\quad - \sqrt{\frac{3}{8}}\left(\sqrt{0.4}|N_L\rangle|G_R\rangle\right) - \sqrt{\frac{3}{8}}\left(\sqrt{0.6}|S_L\rangle|G_R\rangle\right).
\end{aligned}
$$

3. If $|S_R\rangle$ (the cake on the right is observed to rise), then $|G_L\rangle$ (the cake on the left is good). Replaced intermediate basis in all terms of $|\psi\rangle$ with expressions in Fig. 3. The third term has the information from result #3.

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{2}|B_L\rangle\left(\sqrt{0.4}|N_R\rangle + \sqrt{0.6}|S_R\rangle\right) \\
&\quad - \sqrt{\frac{3}{8}}\Big[|B_L\rangle\left(-\sqrt{0.6}|N_R\rangle + \sqrt{0.4}|S_R\rangle\right) \\
&\quad + |G_L\rangle\left(\sqrt{0.4}|N_R\rangle + \sqrt{0.6}|S_R\rangle\right)\Big], \\
|\psi\rangle &= \left(\frac{1}{2}\sqrt{0.4} + \sqrt{\frac{3}{8}}\sqrt{0.6}\right)|B_L\rangle|N_R\rangle \\
&\quad - \sqrt{\frac{3}{8}}(\sqrt{0.4}|G_L\rangle|N_R\rangle) - \sqrt{\frac{3}{8}}(\sqrt{0.6}|G_L\rangle|S_R\rangle).
\end{aligned}
$$

4. Both cakes never turn out good. It is obtained from the definition of $|\psi\rangle$, $P(|G_L\rangle|G_R\rangle) = 0$.

Once again, the definition of $|\psi\rangle$, for both cakes and the context given by the projection, the intermediate measurements, make the results less intuitive, the ultimate judgment is predictions and experimental corroborations, and the last section is going to illustrate just that.

5. Formal test by Professor Fry

The equation of conservation and the intuition given by the previous section is enough to give a formal test to the idea of non-locality. A system that is split in two must conserve its properties accordingly. If a system with spin zero is split, half will have spin $1/2$, and the other half will necessarily have spin $-1/2$ no one knows which one.

A variation of the above is to carry out the experiment with two particles that have a common origin, which forces them to relate some of their properties by conservation rules. The experiment is carried out with two particles that originate from a singlet (zero spin) and that do not commute their projections.

$$
|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R\right).
$$

This representation maintains zero spin since only complementary orientations are observed, $|\uparrow\rangle_L|\downarrow\rangle_R$ indicates that the left has one orientation and the right has the other, $|\downarrow\rangle_L|\uparrow\rangle_R$ indicates the contrary.

The essence of nonlocality is appreciated by making measurement #1, with (S-G)$_{(Z,L)}$ and (S-G)$_{(Z,R)}$, each time $|\uparrow\rangle_L$ is measured then $|\downarrow\rangle_R$ is also measured and vice versa. But if measurement #2 is made, with (S-G)$_{(X,L)}$ and (S-G)$_{(Z,R)}$, every time L measures, R can only predict the result 50% of the time. How can the measurement of L affect the measurement of R?

In the local reality view [13], measurement #1 is complete since everything is predictable, measurement #2 is incomplete, and information was lost in some way.

If we can orient the (S-G) in three directions, a, b and c and ask for the probability P_{ab} which indicates the probability of having the spin up $(+)$ in the particle that flies to the left in the direction a, simultaneously with the spin up $(+)$ in the particle that flies to the right in the direction b, which can be represented as follows with the sign indicating the information that is known:

$$
\begin{aligned}
P_{ab} &= P(a_L b_L c_L | a_R b_R c_R) \\
&= P\left(+\ ?\ ?\ \middle|\ ?\ +\ ?\right).
\end{aligned}
$$

If the information is completed with the condition of the spin zero at the origin.

$$
P_{ab} = P\left(+\ -\ ?\ \middle|\ -\ +\ ?\right).
$$

If both possibilities are used for the unmeasured direction

$$
\begin{aligned}
P_{ab} &= P\left(+\ -\ +\ \middle|\ -\ +\ -\right) \\
&\quad + P\left(+\ -\ -\ \middle|\ -\ +\ +\right),
\end{aligned}
$$

and doing the same for the other two probabilities:

$$P_{bc} = P\left(\oplus\,\oplus\,\ominus\,|\,\ominus\,\ominus\,\oplus\right) + P\left(\ominus\,\oplus\,\ominus\,|\,\oplus\,\ominus\,\oplus\right),$$

$$P_{ac} = P\left(\oplus\,\oplus\,\ominus\,|\,\ominus\,\ominus\,\oplus\right) + P\left(\oplus\,\ominus\,\ominus\,|\,\ominus\,\oplus\,\oplus\right).$$

Combining the three equations and identifying the probabilities of each term, we have:

$$P_{ab} + P_{bc} = P\left(\oplus\,\ominus\,\oplus\,|\,\ominus\,\oplus\,\ominus\right) + P\left(\ominus\,\oplus\,\ominus\,|\,\oplus\,\ominus\,\oplus\right) + P_{ac}.$$

This can be written without problem as follows, considering the probabilities are always positive and it is referred as the condition for local reality:

$$\delta = P_{ab} + P_{bc} \geq P_{ac}.$$

In quantum mechanics, $P_{ab} = (1/4)\left[1 - \cos(\theta_a - \theta_b)\right]$, and the fix angles used for the measurements are $\theta_a = 0°$, $\theta_b = 45°$ and $\theta_c = 90°$.

The result is not compatible with the local reality since the result opposes the Bell inequality: $\delta = (2 - \sqrt{2}/4) \geq (1/4)$. The previous P_{ab} is for entangle electron spin, for entangle orthogonal photons $P_{ab} = (1/4)\left[1 - \cos(2(\theta_a - \theta_b))\right]$, again the two photons are entangled, now by the polarization, and the optical measurement is produced detecting or not the photon through a polarizer with the rule for polarization defined by the entanglement process, spontaneous parametric down-conversion type I or II produces parallel and orthogonal polarization respectively, for atomic transitions the transition selection rules define the polarization.

One of the simplest proofs of the impossibility of Bell's inequality, after Clauser, is that of Fry and Thompson [14]. In the abstract of the article, it says: "We have measured the linear polarization correlation between the two photons of the cascade $7^3S_1 \to 6^3P_1 \to 6^1S_0$ of Hg-200. The results were used to evaluate Freedman's version of Bell's inequality, $\delta \leq 0$. The result is $\delta_{exp} = +0.046 \pm 0.014$, in clear violation of the inequality and in excellent agreement with the quantum mechanical prediction, $\delta_{QM} = +0.044 \pm 0.007$. An important feature of the experiment was the explicit measurement of the initial density matrix of the cascading atoms."

In the temporal sequence of events, first came Clauser's experiment (it contradicts Bell's inequality), then went Holt's (it agrees with Bell's inequality), subsequently Clauser improved his results and confirmed the contradiction with Bell's inequality, in the fourth experiment Fry and Thompson [14] used the two-photon cascade from Hg-202 shown in the Fig. 4, and it contradicts Bell's inequality. In this experiment, a beam with a natural isotopic abundance of Hg was passed through a solenoid-shaped electron gun where the Hg atoms were excited to the metastable 6^3P_2 state because it is a long lifetime state, and its probability to be occupied increased. Following the path of the ions and using a narrow linewidth laser at 546.1 nm to pump those states only for the Hg-200 atoms to be excited to the state 7^3S_1 to start the cascade of emissions; only cascading photons from the zero nuclear spin isotope of Hg-200 were observed. This was the first experiment in which a laser was used to complete the initial state of the cascade. Polarization correlations between cascade photons at 435.8 nm and 253.7 nm were observed and measured using plate polarizers. The initial state of the cascade had angular momentum J $= 1$ (previous experiments had J $= 0$ in the initial state). Quantum mechanical predictions required measurement of relative populations in substates $m_J = 0, \pm 1$. They observed $\delta = 0.046 \pm 0.04$ in violation of Bell's inequality and in agreement with the quantum mechanical prediction $\delta = 0.044 \pm 0.007$. Data for these outcomes were collected over 80 minutes.

This and all the first-generation experiments had two possible flaws: not all photons were measured, which could lead

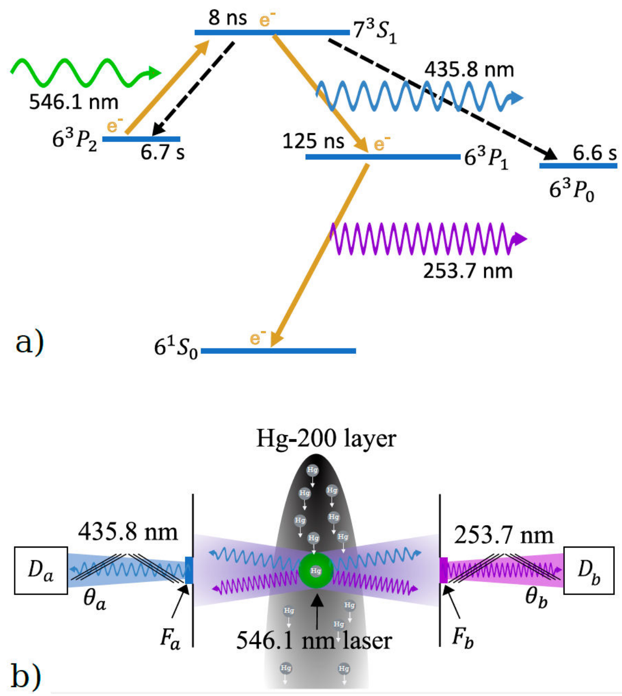


FIGURE 4. a) Energy levels for mercury, the oven pumps electrons from the double occupied $6S$ to the empty $6P$ level, then the laser pumps electrons to the empty $7S$ level to star fast cascade $7^3S_1 \to 6^3P_1 \to 6^1S_0$ and these two photons are analyzed at specific polarizations to test the relation with the Bell inequality. b) Fray's experimental setup, a 546.1 nm laser acts on a Hg-200 layer, photons of 435.8 nm and 253.7 nm are emitted, F_a and F_b are light filters that allow light of 435.8 nm and 253.7 nm to pass respectively, at each side polarizers θ_a and θ_b are placed before the detectors D_a and D_b.

sceptics to propose that luckily only those that gave this result were measured, but lost in the experiments were the others that would prove the contrary, and they did not control the location either, it could be that they gave time for the particles to communicate some information due to the proximity of the detectors. Over time, experiments have been improved to a situation that today it is considered that submicroscopic nature behaves as non-local, which indicates that the particles formed are a single system and, as such, do not require communication or cheating, the properties are joint, they are entangled in a quantum way.

6. Final words

The objective was to present one of the most problematic concepts to understand in quantum physics, the principle of nonlocality or the existence of systems with entangled particles. Recounting the analogies used to convey this idea paved the way to a formal description of a real experiment that proves the case. Recently, many technological models have produced entangled particles, the simplest being spontaneous parametric production that requires only a laser and a parametric crystal. This has allowed very elaborate experiments to be carried out and showed the relevance of quantum physics and the limitations of our perception. The examples described tried to guide the imagination to situations where this type of phenomenon can be plausible and point out human perception with the duality of bringing us closer to appreciating nature, but at the same time, it is the main limitation to understanding it.

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