Teaching of tension force: a massive rope in equilibrium

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A massive rope in equilibrium, with fixed extremes, takes a particular shape as a consequence of both, the weight and tension on each small rope segment. An analysis of forces acting on each rope infinitesimal section under static equilibrium conditions, results in a differential equation with a solution that provides the general shape of the rope and from which, the tension at each point of the rope is obtained. From this theoretical treatment, a set of three rules is proposed. They allow creating and to solve a variety of theoretical and experimental problems dealing with tension force applied to massive ropes under static equilibrium conditions. The teaching experience with first-year engineering students shows that they deduced the rules and solved different problems. Also, it provides to the teacher possibilities to teach tension force in massive rope at the basic physics level. The first approach to test the didactic-proposal was developed into a collaborative learning framework rooted from a constructivist perspective, supported by Ausubel's (1983) theory of meaningful learning. However, it can be configured to suit pedagogical needs.

Keywords: Tension force; massive rope; equilibrium; basic level.

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1. Introduction

Tension force plays a fundamental role, and is discussed with some detail on several physical systems [1-7]. For example, in the Atwood's machine the ropes are considered as necessary objects to establish forces among different bodies, which is the main purpose of this application. In the simple pendulum, the force exerted by the rope to a point mass in addition to the weight and the force of the air results in a damped harmonic movement of the system. In variable mass problems, examples of falling ropes or chains are shown. Considering standing mechanical waves, their propagation speed, refraction and reflection phenomena and so on, again tension forces are still present. Due to that the tension force action is somehow frequent, and it plays its key role in many physical systems, it should be mandatory to pay attention to the way it is taught in basic physics courses. Nevertheless, the strong efforts of several authors [1-7], there are relatively few works devoted to tension force teaching. In some general physics texts, traditionally used in basic physics courses [8-10], the tension force is treated in massive vertical ropes, and in some of the proposed problems are involved hanging massive ropes in equilibrium, from which it can be said that, comparatively with other topics, it is paid little attention to the tension force analysis. Feynmann [5], French and Klepner [11] and Kolenkow [12] discuss about the fundamental origin of tension force. Indeed, they explain the forces acting along a rope considering its mass and frictional forces, and propose some problems, which cannot be solved without a deep understanding of the tension force. In specialized texts of engineering [4,13,14], or mathematical physics [15], implying a strong background of differential and integral calculus and differential equations for their reading, rigorous and accurate treatments of tension force in massive ropes in equilibrium are done. In advanced physics courses for architecture, direct applications of tension force in ropes with mass are observed. They need again deep knowledge of mathematics and sometimes, the use of mathematical software. However, in order to deal with the topic of forces in basic physics courses for first year students of physics and engineering, teaching of tension force on massive ropes is a challenge, due to the exigent background in mathematics the authors propose [14,15]. The teacher can verify by its own experience that usually students have many questions with a high conceptual component related to tension force. In particular, it is natural that students ask if the tension value is the same at both ends of the rope. On the other hand, in the daily environment it can be appreciated several situations that imply tension forces on ropes: energy line, hanging bridges, cableway, etc., that demand from an engineer a deep knowledge of tension force. This work demonstrates the potential of utilizing tension force to teach and develop skills related to physics, mathematics, numerical methods, and laboratory experiments. This approach facilitates a deeper understanding of knowledge through meaningful learning [16], allowing students to leverage their prior knowledge to devise practical strategies for problem-solving. In the following, it is shown a theoretical review of tension force on massive hanging ropes and on a rope with beads in equilibrium, that allow to point out equations suggesting to students a simple but rigorous presentation of tension force. Then, topics involving deduction of formulas and experiments, with the goal of simplifying the comprehension of tension forces acting on massive ropes, are illustrated. Finally, a comment about the results of the teaching experience of this topic with first year engineering students is done.

2. Tension force review

Tension force in a rope in equilibrium varies from one point to another. The way it changes depends on how high its ends are located and on the linear mass density. To calculate the tension value it is required to solve a differential equation resulting from the static equilibrium condition of a differential segment of the rope, which implies challenging mathematical work. This differential equation is obtained in an identical way for a rope with beads in equilibrium, when the mass of each bead is higher than the rope mass (Fig. 2).

2.1. Massive rope

Consider a massive rope in static equilibrium fixed by its extremes (Fig. 1). The rope shape is parametrized by means of its Cartesian coordinates $(x, y(x))$ that indicate the position of each rope segment. The horizontal and vertical components of the equilibrium forces equation yield to:

$$\frac{T(x)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = T_{0x}, \tag{1}$$

and

$$T_{0x}\left(\frac{d^2y}{dx^2}\right) = g\lambda(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}, \tag{2}$$

where $\lambda(x)$ is the linear mass density, $T(x)$ the tension magnitude, g the gravity acceleration and T_{0x} is the horizontal tension component, which is constant along the rope. Additionally, for Eq. (1), the tension vertical component $T_y(x)$ can be deduced and is given by:

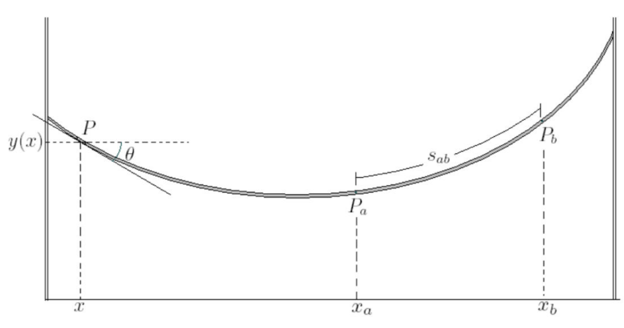

FIGURE 1. Massive rope.

$$T_y(x) = T_{0x}\frac{dy}{dx} = T_{0x}\tan\theta, \tag{3}$$

where θ is the angle between the tangent line to the rope in the $(x, y(x))$ point and the horizontal. With the last expression Eq. (3) and considering that the horizontal tension component is constant, it is possible to create and solve problems in a very simple way. In addition, it offers many possibilities of performing experiments for laboratory lectures. For instance, the constant value of the horizontal tension component T_{0x}, in the case of a uniform linear mass density $\lambda(x) = \lambda$, can be verified by

$$|T_{0x}| = \lambda g \frac{s_{ab}}{\left|\frac{dy(x_a)}{dx} - \frac{dy(x_b)}{dx}\right|}, \tag{4}$$

where s_{ab} is the rope length between P_a and P_b, and $\tan\beta_a = dy(x_a)/dx$ and $\tan\beta_b = dy(x_b)/dx$ being the tangent of the angles between the tangent lines in the positions P_a, P_b and the horizontal (Fig. 1).

2.2. Ropes and beads

Consider a set of N massive beads in static equilibrium, fixed by ropes with negligible mass compared to the bead mass. The magnitude of Cartesian tension components T_{ix} and T_{iy} corresponds to the $i-th$ segment as a function of the tension magnitude T_0 of the first segment. Here, the angle θ_0 between the segment and the horizontal, and the angle θ_i between the $i-th$ segment and the horizontal (Fig. 1), are given by:

$$T_{ix} = T_0\cos\theta_0 = T_{0x}, \tag{5}$$

and

$$T_{iy} = T_{0x}\tan\theta_i, \tag{6}$$

the expressions (5) and (6), yield:

$$|\vec{T}_i| = \frac{T_{0x}}{\cos\theta_i}. \tag{7}$$

It is pointed out that the horizontal tension component T_{0x} is constant. Moreover, the vertical tension component T_{iy} and the tension magnitude of each $i-th$ segment, can be calculated with two parameters: T_{0x} and θ_i.

3. Didactic-proposal

This work was successfully developed with first-year engineering students from three universities (UGC,PUJ, and UPTC) in two cities (Bogotá and Tunja).

The first challenge to overcome is the literature on tension force. There is a robust theory related with [11,12,14,15,17,18]. However, to first-year engineering students this kind of bibliography could be too difficult to understand. Furthermore, it is missing in standard general physics textbooks [8-10]. An option could be to wait for students to acquire skills and knowledge to address this topic. We prefer a different approach.

Besides, the process to develop skills and knowledge about tension force in massive ropes, proposed here, offers an excellent way to go from discrete system to continuum one, the application of the derivative concept and tangent line to a rope. In addition, to face a new set of problems with basic tools.

It is presented a teaching-proposal by a set of class moments, which could be adapted to the pedagogical model that the professor is following. The way it is asking a team of students is the key to get an excellent learning strategy. Next, the class moments are described.

3.1. Fundamental origin of tension force

It is required to spend some time allowing students to understand of the matter structure, the way it interacts and its range of dominance, because the tension force is the result of distance-dependent-intermolecular-force average, which can be modeled mathematically as illustrated in several books [5,12].

3.2. Class exercise: rope and beads

It is considered a rope, with a set of N beads, hanging from two fixed points in static-equilibrium (Fig. 2). Each angle θ_i ($i = 0, 1, 2, ..., N$), and the tension force T_0 at the first rope segment were measured. Bead masses, m_j ($j = 1, 2, ..., N$), are different from each other, and their values are not known. The idea here is to determine Cartesian components of each tension force, T_{ix}, T_{iy}, and the corresponding magnitude T_i.

At first glance, it seems like a complex and tedious exercise. However, it is possible to confront it with a novel strategy, which finally reveals a hidden symmetry.

So, in that way it could be followed these steps. First of all, to perform a free-body diagram of forces corresponding to mass m_1, then to write a vectorial equation of equilibrium force, next obtain a system of scalar equations (one for each component), and finally to find T_{1x}, T_{1y}, and T_1 in terms of T_0, θ_0 and θ_1. The last step becomes hard due to m_1 is not known. The key here is remembering tension is a force, so it is a vector quantity, then, it is possible to find an additional relationship between T_{1x}, T_{1y}, and T_1 (rectangle triangle for instance). Even though there are N masses, this first solution was made with one free-body diagram.

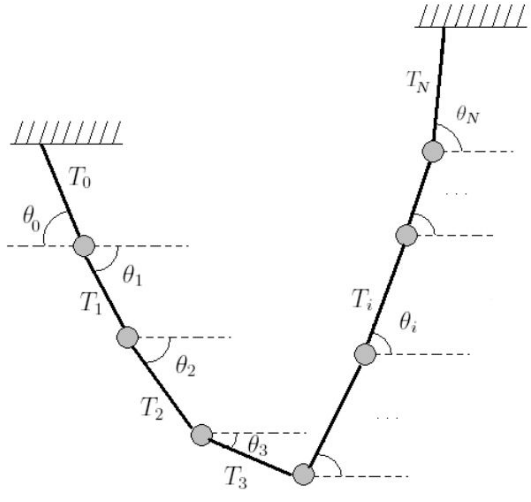

FIGURE 2. Beads and ropes.

Subsequently, it is asking for T_{2x}, T_{2y}, and T_2. To solve it, it could be followed the same procedure with m_2, and to take results obtained before, and so on. Along with this procedure is performing it could be completing a table to summarize outcomes.

From m_3 procedure, even from m_2, it is figured out that the three rules are described by expressions (5), (6) and (7). Therefore, it is possible to write out an expression corresponding to any mass m_i.

3.3. Extrapolation from discrete to continuous and rules to tension force in massive ropes

It is observed that expressions (5) and (6) are the same than (1) and (3), respectively. This is expected because the static equilibrium equations for a bead are identical to those of an infinitesimal element of a massive rope. Due to this, it is natural to extend the result of the beads case under equilibrium conditions to the case of a continuous rope, taking into account a new interpretation that should be done to the terms of the Eqs. (1) and (3): the θ_i angle between the $i-th$ segment and the horizontal corresponds to the angle θ formed by the tangent line to the rope at a particular point P and the horizontal (Fig. 1). In addition, $\tan\theta$ gives the slope of this line or the derivative of $y(x)$ at point P.

As a conclusion, it can be considered three valid equations for the general case of massive ropes (uniform or no uniform linear density mass), hanging of the ends, under equilibrium conditions, with their corresponding interpretation forming the following three rules:

1. $T_x = T_{0x}$: the horizontal tension component is constant along the rope.

2. $T_y = T_{0x}\tan\theta$: the vertical tension component in every point of the rope is equal to the horizontal tension component multiplied by the tangent of the angle (the slope), formed by the tangent line to the rope at that point with the horizontal.

3. $T = T_{0x}/\cos\theta$: the magnitude tension at every point of the rope is equal to the horizontal tension component, which divides the cosine of the angle formed by the tangent line to the rope at that point with the horizontal.

In this way, additional conceptual elements for solving problems, involving massive ropes in equilibrium, are available.

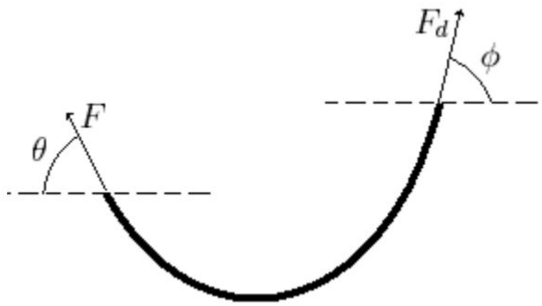

FIGURE 3. Massive rope in static-equilibrium.

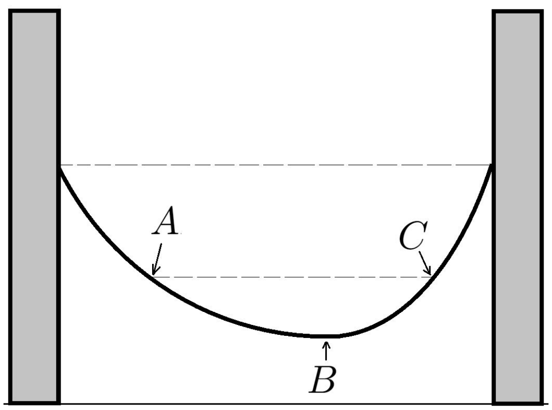

FIGURE 4. Comparison of three tension-force values.

3.4. Problems suggested

Below is shown a set of generic problems, which test understanding related to massive ropes in static equilibrium. The three rules could be very useful to deal with them. Professors could fit this set into various contexts.

3.4.1. *Massive rope*

A rope hangs under equilibrium conditions (Fig. 3). The force magnitude on the left end is F and the angles that each extreme forms with respect to the horizontal are θ and ϕ, respectively. Find the force value F_d on the right end, the rope mass and the tension magnitude on the lowest point of the rope (the minimum).

3.4.2. *Comparison of tension-force values*

In Fig. 4, let us consider T_A, T_B and T_C as magnitudes of tension-force at points A, B and C respectively. It is asked what the higher and lower tension values are.

3.4.3. *Is the tension-force value constant?*

In Fig. 5 it is shown the same rope in three situations (I, II and III). In what situation (or situations) should the tension-force value along the rope be considered constant?

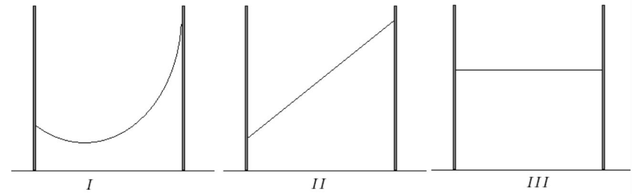

FIGURE 5. The same rope in three situations.

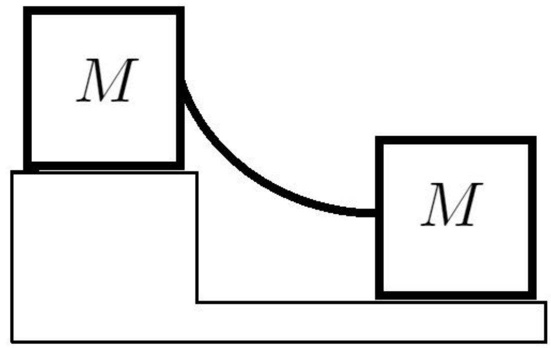

FIGURE 6. Two blocks holding a massive rope in equilibrium.

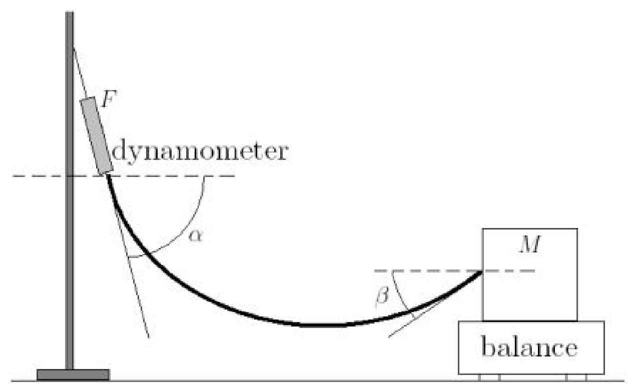

FIGURE 7. Massive rope in equilibrium hanging from a dynamometer and a block.

3.4.4. *Two blocks and massive rope*

Each block illustrated in the figure has mass M and supports one end of a massive rope. The whole system is in equilibrium. Are the values of friction forces f_l and f_r such as $f_l < f_r$, $f_l > f_r$, $f_l = f_r$ or are they both zero?

3.4.5. *Example of context problem*

The lecture of the balance is equal to the normal value N over its surface divided by gravity g, it means N/g. Dynamometer lecture F, mass of the block M, gravity of the place g, and angles α and β are known. To determine the lecture of the balance.

3.5. Teachnig experience

From work developed on ten groups of first-year engineering students from three universities (UGC,PUJ, and UPTC) in two cities (Bogotá and Tunja). Each group consists of an average of 25 students. They attend three classes per week, each lasting two hours: two theoretical classes and one lab session. This report focuses on the experiences gained during the three theoretical classes.

As a preamble to the teaching experience, it includes understanding what weigh, contact and intermolecular forces are, how a force diagram should be done, and how to get a scalar system of equations from an equilibrium force situation. After, these requirements are achieved, the students are ready to face the first challenge illustrated in Sec. 3.2. After that, it is necessary to spend six hours of class per group.

In this experience, the professor begins by presenting the first challenge (3.2.) to the students. They should from a team of three or four members based on their own criteria. Each team that achieves some advance should show it to the other teams. This methodology encourages each other to work towards a common goal. It is a cooperative way of approaching the task, it is not a competition. First of all, they should make a first force diagram to the first mass on the left (Fig. 2).

The students quickly find a way to get T_{1x}. Unlike the vertical component of the tension T_{1y} demands additional thinking. At this point, the student asks for masses of the beads, hoping for some symmetric property in search of simplicity. However, the masses are unknown and are not necessarily equal. After a while, usually at least one student, out of thirty, reflects that a force is a vector, two-dimensional in this case, so that T_{1y} can be obtained by triangle geometry. Throughout this activity, the professor acts as a moderator, but at the end summarizes the achievements using Table I.

TABLE I. Results of calculations.

$i-mass$	T_{ix}	T_{1y}	T_i
1	$T_0\cos\theta_0$	$T_0\cos\theta_0\tan\theta_1$	$T_0\cos\theta_0/\cos\theta_1$

Now the second mass ($i=2$) now is considered. The students draw the corresponding diagram of force and write the system of equations from it. At this point, this procedure seems a bit boring; however, it turns out easy to solve using the results obtained previously, and the students can complete the next row of the table. The feeling about procedure described by the adjective “boring” could be considered negative, but it turns out to be a tool. Then, Table II becomes.

TABLE II. Results of calculations.

$i-mass$	T_{ix}	T_{1y}	T_i
1	$T_0\cos\theta_0$	$T_0\cos\theta_0\tan\theta_1$	$T_0\cos\theta_0/\cos\theta_1$
2	$T_0\cos\theta_0$	$T_0\cos\theta_0\tan\theta_2$	$T_0\cos\theta_0/\cos\theta_2$

At this point, it is asking the students for the next tension force and so on. Of course, the procedure is really tedious so they look for a shortcut, which highlights from the table of results. Then they realize about the three rules for obtaining the tension force and their Cartesian components.

Next, at a new class, the professor ask the students how look the same system of beads, Fig. 2, is the number of masses is increasing. They comment that the bead system look like a heavy rope if the number of beads is increased. Then, the three rules need for an interpretation of the continuum of mass (the rope). In that way, the line between two neighboring masses becomes a tangent line to the rope and the corresponding $\tan\theta$ turns out the slope, namely, the first derivative of the function describes the shape of the string. This now, they are ready to move to the exercise illustrated in 3.4.1. There, the student questioned how to understand the angle at the minimum of the string. Finally, through collaborative methodology they find the complete solution after thirty to forty minutes (on average). Students understand how they can use these three rules analytically to analyze and conclude, this is one of the objectives achieved here.

To exercises written on Secs. 3.4.2. and 3.4.3. frequently necessary that insist to the students support their answers from the three rules. These exercises lead students to deal with preconceived ideas about tension force (same rope same tension, for instance), here the students discover when the tension force can considered constant and when it cannot.

The exercise illustrated in Sec. 3.4.4. is done as homework. When students return to class, some (30%) comment that there is no data to address it, and others (20%) do not find a way to solve it. The key here is to remember the angle strategy from exercise 3.4.2. and encourage them to start the analysis from a force diagram. At lest 10% of the students get the correct answer. Later, this exercise is discussed in class and the professor highlights diagram force, the scalar system of equilibrium equations and the three rules.

The last exercise 3.4.5. leads to a context. The students put the learned strategies into action. In the first experience, difficulty is not longer the tension in a massive rope, but rather it is related to knowing what a scale is measuring?. A discussion should be held on this matter to establish that normal on the scale divided by gravity value is the value reported by it. Due to, the main goal is to understand how to address problems with massive rope,it becomes necessary to add to the text the statement “The lecture of the balance is equal to the normal value (N) over its surface divided by gravity (g)”. Then, some problems arise related to how to solve the system of equations. In the end, at least 70% of the students find the answer, working in teams.

Finally, comparison between the evaluation of several topics and subject massive rope showed that the level of the difficulty of this topic is similar to that of the parabolic movement.

This work focuses on an analytic vision. However, a sensitive and experimental experience is recommended and it will be an opportunity for future work.

4. Pedagogical opportunities

It is not common in the teaching of the so-called "hard sciences" to have a didactic approach outside the framework of a lecture, where the oratory skills of the teacher determine the apprehension of content that is very complex to bring into experience. In this sense, an active pedagogy that focuses its efforts on the construction of knowledge through cooperative work can become an invaluable tool in highly theoretical contexts. According to Johnson, Johnson, and Holubec [19], "collaborative learning is the use of small groups in which students work together to maximize their own learning and that of others", making it possible for the learning curve to strengthen, as the heterogeneous interaction of individuals promotes thinking and, especially, social behavior that benefits academic activities in the classroom. Some research in the area corroborates that the use of these methodologies increases students' perception when facing problem-solving, enhancing their decision-making ability and communication skills [20]. It is important to mention that these types of proposals promote meaningful learning, whose essence 'lies in the fact that symbolically expressed ideas are related in a non-arbitrary and substantial way (not literally) to what the student already knows [16].

In our experience working with collaborative groups, the role of the teacher is fundamental when proposing each of the activities that allow students to establish procedural strategies to answer the questions posed by the program, while being able to transfer this knowledge to real-life application contexts. In this environment, the teacher designs the work materials in advance, posing meaningful questions that promote analytical reflection, leading students to inquire about their prior knowledge [21], as well as possible ways to solve the problems posed. Here, the teacher is a facilitator of the experience, guiding students through the paths they themselves propose; organizing, suggesting, questioning, and providing tools that enable the mobilization of thinking where each student "promotes the learning of others by verbally explaining how to solve problems, analyzing the nature of the concepts being learned, teaching what one knows to peers, and connecting present learning with past learning" [19]. Clearly, the learning outcomes observed at the end of the course exponentially elevate the learning curve, allowing the teacher to go beyond suggesting that students review bibliographies that, for students at this level, are difficult to understand.

Undoubtedly, the application of cooperative learning in the classroom promotes a paradigm shift that favors individual achievement, very common in the sciences, towards a "model based on teamwork and high performance" [19], providing students with a vision of collective achievement, which is needed in educational and research spaces. The purpose of this methodology is for more and more students to successfully face the resolution of complex problems and for the knowledge they acquire to not only be permanent but also become part of their engrained responses to the challenges of their existence. Additionally, an important finding is that cooperative learning balances the cognitive differences among students, which originate from the educational and social gaps they come from [22], providing an added value that promotes equity and social justice.

Furthermore, the teacher can configure this didactic-proposal according to various pedagogical needs. For instance, experience refers to the experiences lived by the subject, mediated through the senses, which transform into actions or sensory-motor patterns that become incorporated into the soma and determine their behavior [23], could lead to an experimental perspective in the laboratory (next work).

5. Conclusions

A didactic-proposal to teach massive-rope in equilibrium developed from well-established theoretical foundations is shown. In this, a set of three rules valid for uniform and non-uniform massive-rope under static equilibrium conditions are addressed with basic physics.

The teaching experience with first-year engineering students shows that they deduced the rules and solved different problems from them. Also, this shows that the students' effort to understand the tension in massive-rope is similar with parabolic motion.

The first approaching to test the didactic-proposal was developed into collaborative learning framework. However, it can be configured to suit pedagogical needs. For example, from experimental and sensorial view.

1. S. Flores *et al.*, Students' difficulties with tension in massless strings. Part I., *Rev. Mex. Fis.* **55** (2009) 21.
2. S. Flores *et al*., Students' difficulties with tension in massless strings. Part II, *Rev. Mex. Fis.* 55 (2009) 118.
3. K. R. Symon, Mechanics, 3rd ed. (Addison-Wesley Publishing Company, USA, 1971)
4. A. Higdon *et al*., Engineering Mechanics. Statics and Dynamics. (Prentice Hall, 1976).
5. R. Feynman, R. Leighton, and M. Sands, Mechanics, radiation and heat. (Addison Wesley Iberoamericana, 1987).
6. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics (Dover Publications Inc., NY, 1963).
7. F. W. Sears, M. W. Zemansky, and H. D. Young, Física Universitaria, 6th ed. (Addison-Wesley Iberoamericana, 1998)
8. P. A. Tipler, Physics for scientists and engineers (W. H. Freeman and Company, 1999).
9. R. A. Serway and J. W. Jewett, Physics for scientists and engineers, 6th ed. (Thomson-Brooks/Cole, 2004).

10. R. Lane, Fisica Universitaria (International Thomson Editores, 2002).

11. A. French, Mecanica newtoniana (Editorial Reverté, 1974), p. 151.

12. D. Kleppner and R. Kolenkow, An Introduction to Mechanics (Cambridge University Press, 2014), pp. 89-90.

13. J. C. McCormac and J. K. Nelson, Structural analysis: a classical and matrix approach, 2nd ed. (Addison-Wesley Educational Publishers Inc., 1997).

14. S. P. Timoshenko and D. H. Young, Theory of Structures, 2nd ed. (McGraw-Hill, 1975).

15. E. Butkov, Mathematical Physics (Addison-Wesley Publishing Company, 1973).

16. D. P. Ausubel, J. D. Novak, and H. Hanesian, Psicología educativa: un punto de vista cognoscitivo, Segunda edición ed. (Trillas SA de CV, Mexico City, Mexico, 1983).

17. S. Gil, Experimentos de física:usando las TIC y elementos de bajo costo (Alfaomega Grupo Editor Argentino, 2014), pp. 216-221.

18. R. Hibbeler, Estática 12th ed. (Prentice Hall, 2010), pp. 336-349.

19. D. W. Johnson, R. T. Johnson, and E. J. Holubec, El aprendizaje cooperativo en el aula, 1st ed. (PAIDOS, Buenos Aires, Argentina, 1999), pp. 1-144.

20. Z. Z. Zúñiga, El aprendizaje cooperativo y su influencia en el aprendizaje del curso de Matemática Financiera en los estudiantes de la Facultad de Ciencias Empresariales de la Universidad Nacional de Educación (UNE), Revista Dilemas Contemporáneos: Educación, Política y Valores **IX** (2022) 1.

21. W. L. Velásquez Álava *et al.*, Aprendizaje colaborativo en la enseñanza de matemática, *Revista Científica Arbitrada Multidisciplinaria PENTACIENCIAS* **5** (2023) 103, `https://doi.org/10.59169/pentaciencias.v5i6.842`.

22. L. Morales-Maure *et al.*, Habilidades Cognitivas a través de la Estrategia de Aprendizaje Cooperativo y Perfeccionamiento Epistemológico en Matemática de Estudiantes de Primer Año de Universidad, Formación universitaria **11** (2018) 45, `https://dx.doi.org/10.4067/S0718-50062018000200045`.

23. M. Fukumoto, The Principle of Somatic Learning by T. Hanna, *Journal of the Philosophy of Sport and Physical Education* **35** (2013) 83, `https://doi.org/10.9772/jpspe.35.83`

24. P. Navarro, El aprendizaje cooperativo, 4th ed. (Editorial Cara Parens, 2012).

25. J. de D. Arias Silva, C. C. Roa, and f. Estupiñan Tarapuez.

26. S. Sheppard and B. Tongue, Estática: análisis y diseño de sistemas en equilibrio (Limusa Willey, 2008), pp. 572-594.

27. J. M. Goicolea Ruigómez, Cálculo de cables (Archivo Digital UPM, 2021), `https://doi.org/10.20868/UPM.book.67125`

28. A. Beléndez Vázquez, T. Beléndez Vázquez, and C. Neipp López, Estudio estático de un cable homogéneo bajo la acción de su propio peso: catenaria, *Rev. Esp. Fis.* **15** (2001) 38.

29. T. Beléndez, C. Neipp, and A. Beléndez, Cables Under Concentrated Loads: A Laboratory Project for an Engineering Mechanics Course, *Int. J. Engng.* **19** (2003) 272.

30. H. Irvine and G. Sinclair, The suspended elastic cable under the action of concentrated vertical loads, *Int. J. Solids Struct.* **12** (1976) 309, `https://doi.org/10.1016/0020-7683(76)90080-9`.

31. M. Babaz *et al.*, Unusual expression of tension of a massless cable with application to the oscillations of a mass suspended to a cable with a variable length, *J. Sound Vib.* **363** (2016) 446, `https://doi.org/10.1016/j.jsv.2015.10.020`.

32. S. A. Ali, Dynamic response of sagged cables, *Comput. Struct.* **23** (1986) 51, `https://doi.org/10.1016/0045-7949(86)90106-9`