Solution of the Schrödinger equation for a particle in a uniform force field via the solution for a free particle

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We show that the solutions of the Schrödinger equation for a free particle are related in a simple manner with the solutions of the Schrödinger equation for a particle in a uniform force field. Making use of this relation we readily obtain the so-called Airy wave packets and the propagator for a particle in a uniform force field.

Keywords: Schrödinger equation; free particle; particle in a uniform force field; Airy wave packets.

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1. Introduction

By means of transformations mixing the space coordinates and the time we can see that the Schrödinger equation with a given potential is locally equivalent to the Schrödinger equation with another seemingly unrelated potential, in such a way that the corresponding solutions are related by means of this coordinate transformation, without the need of integral transforms. Essentially, the simple substitution of the original space-time coordinates by the new ones leads from the solutions of one of these Schrödinger equations to the solutions of the other. Among other things, this relation allows us to express the propagator of one of these equations as the propagator of the other multiplied by a simple factor, and to establish relations between the one-variable functions appearing in the solution of the Schrödinger equation by separation of variables.

For example, in Ref. [1] it was shown that the Schrödinger equation for a harmonic oscillator is the Schrödinger equation for a free particle written in an appropriate coordinate system and, in a similar manner, in Ref. [2] the problem of a free particle in two dimensions has been shown to be equivalent to the problem of a charged particle in a uniform magnetic field.

There is a vast amount of work on the application of point transformations in the time dependent Schrödinger equation, directly or in combination with Darboux transformations and supersymmetric techniques (see, *e.g.*, Refs. [3–6] and the references cited therein). In the example considered here the coordinate transformations are employed in a direct manner, involving only the standard elementary formalism of quantum mechanics.

In this paper we show that the Schrödinger equation for a free particle is equivalent to the Schrödinger equation for a particle in a uniform force field. The Schrödinger equation for a particle in a uniform force field can be solved making use of the standard technique of separation of variables (see, *e.g.*, Refs. [7, 8]) and the reason why this problem is not considered in the elementary textbooks on quantum mechanics is that the corresponding time-independent Schrödinger equation is Airy's equation (see Eq. (10), below), which is not commonly studied.

In Sec. 2 we show that by means of a simple change of coordinates the Schrödinger equation for a free particle can be converted into the Schrödinger equation for a particle in a uniform force field and we find the corresponding relation between the solutions of these equations. This relation is employed to construct the solutions of each of these equations corresponding to the separable solutions of the other and in this way we obtain the so-called Airy wave packets, which are nonspreading solutions of the Schrödinger equation for a free particle [9]. The relation between solutions of the two Schrödinger equations is also employed to express the propagator of one these equations in terms of the propagator of the other.

2. Equivalent Schrödinger equations

We shall consider the effect of the coordinate transformation

$$x' = x - \tfrac{1}{2}at^2, \qquad t' = t, \tag{1}$$

where a is a constant, on the Schrödinger equation for a free particle

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_{\text{free}}}{\partial x^2} = \mathrm{i}\hbar\frac{\partial\Psi_{\text{free}}}{\partial t}. \tag{2}$$

Making use of the chain rule one finds that, in terms of the primed coordinates, Eq. (2) is equivalent to

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_{\rm free}}{\partial x'^2} = {\rm i}\hbar\frac{\partial\Psi_{\rm free}}{\partial t'} - {\rm i}\hbar a t'\frac{\partial\Psi_{\rm free}}{\partial x'}. \qquad (3)$$

In order to relate this equation with the Schrödinger equation for a particle in a potential V,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x'^2} + V\Psi = {\rm i}\hbar\frac{\partial\Psi}{\partial t'}, \qquad (4)$$

we propose a relation of the form

$$\Psi_{\rm free} = (\exp {\rm i}F/\hbar)\,\Psi, \qquad (5)$$

where F is some function of x' and t' (or, equivalently, of x and t).

Substituting (5) into Eq. (3) one finds that the terms with the first derivative of Ψ with respect to x' disappear if and only if $\partial F/\partial x' = mat'$, that is, $F = mat'x' + f(t')$, where $f(t')$ is a function of t' only. Then, Ψ satisfies Eq. (4) with

$$V = max' - \frac{1}{2}ma^2t'^2 + \frac{{\rm d}f}{{\rm d}t'}.$$

This potential corresponds classically to a constant force $-\partial V/\partial x' = -ma$, regardless of the form of f. In order to simplify the identifications in what follows we take $f(t') = \frac{1}{6}ma^2t'^3$, so that $V = max'$.

Thus, the wavefunction

$$\Psi = \exp\left[-\frac{\rm i}{\hbar}\left(mat'x' + \tfrac{1}{6}ma^2t'^3\right)\right]\Psi_{\rm free} \qquad (6)$$

satisfies the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x'^2} + max'\Psi = {\rm i}\hbar\frac{\partial\Psi}{\partial t'}, \qquad (7)$$

for a particle in a uniform force field, if and only if $\Psi_{\rm free}$ satisfies Eq. (2), with the coordinates related through (1) (see the examples below). (The relation (6) is considered in Ref. [10] but only in the case were $\Psi_{\rm free}$ is the free gaussian wave packet, without noticing that this relation is applicable for any solution.)

Since the wavefunctions Ψ and $\Psi_{\rm free}$ differ only by a phase factor and (for a fixed value of t) ${\rm d}x' = {\rm d}x$, Ψ is normalized if and only if $\Psi_{\rm free}$ is normalized.

It must be stressed that we are considering the coordinate transformation (1) just as a relation between different ways of labeling the space-time points. This issue is relevant because, as is well known, under changes of reference frame (Galilean transformations, for instance), the wavefunctions do not behave as scalar fields in the space-time (see, *e.g.*, Ref. [11] and the references cited therein). The coordinate transformations considered here and in Refs. [1, 2] need not be interpretable as changes of reference frame.

In other words, a given space-time point can be labeled by the coordinates (x, t) and by the coordinates (x', t'); the value of the wavefunction $\Psi_{\rm free}$ depends only on the point of the space-time where it is evaluated, though its expression depends on the coordinates employed. This is similar to what we do in electrostatics: given a charge distribution, the electrostatic potential, φ, at a given point of the space has a well defined value (assuming, for instance, that the potential is zero at infinity) but its expression depends on the coordinates being employed to label the points of the space. For example, the potential of a point dipole at the origin, parallel to the z-axis, is given by $\varphi = [p/4\pi\varepsilon_0 z](x^2+y^2+z^2)^{-3/2}$, in terms of the Cartesian coordinates and, equivalently, by $\varphi = [p/4\pi\varepsilon_0]r^{-2}\cos\theta$, in spherical coordinates, and we use the same symbol, φ, to represent the electrostatic potential, thinking that it is a single function with various expressions.

We close this section by pointing out that the potential V is already determined by the coordinate transformation (1); this coordinate transformation relates the solution of the equations of motion, in classical mechanics, of a free particle with that of a particle in a uniform force field corresponding to the potential $V = max'$.

3. Relating solutions

Both partial differential equations (2) and (7) admit separable solutions, with the big difference that the separable solutions of Eq. (2) involve elementary functions:

$$\Psi^{(k)}_{\rm free}(x,t) = \frac{1}{\sqrt{2\pi}}\,{\rm e}^{{\rm i}kx}{\rm e}^{-{\rm i}\hbar k^2 t/2m}, \qquad (8)$$

where k is an arbitrary real number, whereas the separable solutions of Eq. (7) involve the Airy functions. Indeed, substituting

$$\Psi^{(E)}(x',t') = \psi^{(E)}(x')\,{\rm e}^{-{\rm i}Et'/\hbar} \qquad (9)$$

into Eq. (7) one obtains the time-independent equation

$$-\frac{\hbar^2}{2m}\frac{{\rm d}^2\psi^{(E)}}{{\rm d}x'^2} + max'\psi^{(E)} = E\psi^{(E)},$$

which, with the standard change of variable

$$y \equiv \left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x' - \frac{E}{ma}\right),$$

is transformed into

$$\frac{{\rm d}^2\psi^{(E)}}{{\rm d}y^2} - y\psi^{(E)} = 0, \qquad (10)$$

which is the Airy equation (see, *e.g.*, Refs. [7, 8, 12]).

The solutions of the Airy equation are linear combinations of the so-called Airy functions, Ai and Bi, but the function ${\rm Bi}(y)$ diverges when $y \to \infty$. The Airy function of the first kind, Ai, can be defined by [7, 8, 12]

$${\rm Ai}(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left({\rm i}\frac{s^3}{3} + {\rm i}sy\right){\rm d}s. \qquad (11)$$

As we shall show below, this integral representation of the solution of the Airy equation is obtained in a natural way starting from the solution of the Schrödinger equation for a free particle [see Eq. (17)].

3.1. From plane waves to Airy functions

As is well known, the general solution of Eq. (2) can be written in the form (see, *e.g.*, Ref. [13])

$$\Psi_{\rm free}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, {\rm e}^{{\rm i}(kx - \frac{\hbar k^2}{2m} t)}\, {\rm d}k, \tag{12}$$

where $\phi(k)$ is a possibly complex-valued function. Hence, according to Eqs. (6) and (1), the general solution of the Schrödinger equation for a particle in a uniform force field must be given by

$$\begin{aligned} \Psi(x',t') &= \exp\left[-\frac{\rm i}{\hbar}\big(mat'x' + \tfrac{1}{6}ma^2t'^3\big)\right] \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, \exp\left[{\rm i}\left(kx' + \frac{1}{2}kat'^2 - \frac{\hbar k^2}{2m}t'\right)\right] {\rm d}k \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, \exp\left\{{\rm i}\left[\left(k - \frac{mat'}{\hbar}\right)x' - \frac{\hbar k^2}{2m}t' + \frac{ka}{2}t'^2 - \frac{ma^2}{6\hbar}t'^3\right]\right\} {\rm d}k \end{aligned} \tag{13}$$

or, replacing the integration variable k by $s \equiv \left(\frac{\hbar^2}{2m^2a}\right)^{1/3}\left(k - \frac{ma}{\hbar}t'\right)$,

$$\begin{aligned} \Psi(x',t') = \frac{1}{\sqrt{2\pi}} \left(\frac{2m^2a}{\hbar^2}\right)^{1/3} \int_{-\infty}^{\infty} \exp\left[{\rm i}\frac{s^3}{3} + {\rm i}s\left(\frac{2m^2a}{\hbar^2}\right)^{1/3} x'\right] \\ \times\, \phi\left(\frac{ma}{\hbar}t' + \left(\frac{2m^2a}{\hbar^2}\right)^{1/3} s\right) \exp\left\{-\frac{\rm i}{3}\left[s + \frac{ma}{\hbar}\left(\frac{\hbar^2}{2m^2a}\right)^{1/3} t'\right]^3\right\} {\rm d}s. \end{aligned} \tag{14}$$

As usual, one expects that the general solution of Eq. (7) be a superposition of the separable solutions (9). We shall verify that this is indeed the case and, in the process, we shall obtain the relevant solutions of the Airy equation.

In order to express the right-hand side of Eq. (14) as a superposition of the separable solutions of Eq. (7), the last two factors of (14) will be written as the Fourier transform of some function χ:

$$\phi(k)\exp\left[-\frac{\rm i}{3}\left(\frac{\hbar^2}{2m^2a}\right)k^3\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \chi(w)\, {\rm e}^{-{\rm i}kw} {\rm d}w. \tag{15}$$

(Thus, χ can be defined by

$$\chi(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) \exp\left[-\frac{\rm i}{3}\left(\frac{\hbar^2}{2m^2a}\right)k^3\right] {\rm e}^{{\rm i}kw} {\rm d}k, \tag{16}$$

in terms of ϕ.) Substituting (15) into (14) we obtain

$$\Psi(x',t') = \frac{1}{2\pi}\left(\frac{2m^2a}{\hbar^2}\right)^{1/3} \int_{-\infty}^{\infty} \chi(w) \int_{-\infty}^{\infty} \exp\left[{\rm i}\frac{s^3}{3} + {\rm i}s\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}(x'-w)\right] {\rm d}s\, \exp\left(-\frac{{\rm i}ma}{\hbar}wt'\right) {\rm d}w, \tag{17}$$

which is a superposition of products of the form (9). This allows us to identify w with E/ma, where E is the energy of the stationary states of the particle in the uniform force field, and the required solutions of the Airy equation.

Comparing the last expression with (11) we have

$$\Psi(x',t') = \frac{1}{ma}\left(\frac{2m^2a}{\hbar^2}\right)^{1/3} \int_{-\infty}^{\infty} \chi(E/ma)\, {\rm Ai}\left(\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x' - \frac{E}{ma}\right)\right) \exp\left(-\frac{{\rm i}E}{\hbar}t'\right) {\rm d}E. \tag{18}$$

According to the standard postulates of quantum mechanics, $|\chi(E/ma)|^2$ is, essentially, the probability density of measuring the energy E if the particle is in the state (18).

3.2. Two special cases

As shown above, given a solution of the Schrödinger equation for a free particle, $\Psi_{\rm free}$, the wavefunction Ψ given by (6) is a solution of the Schrödinger equation (7) for a particle in a uniform force field. Thus, if we take $\Psi_{\rm free}$ given by (8) (characterized by a particular value of k), we obtain the solution of (7) given by

$$\Psi^{(k)}(x',t') = \frac{1}{\sqrt{2\pi}} \exp\left\{{\rm i}\left[\left(k - \frac{mat'}{\hbar}\right)x' - \frac{\hbar k^2}{2m}t' + \frac{ka}{2}t'^2 - \frac{ma^2}{6\hbar}t'^3\right]\right\} \tag{19}$$

[*cf.* Eq. (13)]. This is the solution of the Schrödinger equation for a particle in a uniform force field (7) with the initial condition $\Psi^{(k)}(x',0) = [1/\sqrt{2\pi}]\,\mathrm{e}^{\mathrm{i}kx'}$.

The separable solution (8) is obtained from the general solution (12) taking $\phi(k') = \delta(k-k')$, and substituting this function into Eq. (16) we obtain

$$\chi(w) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\mathrm{i}}{3}\left(\frac{\hbar^2}{2m^2a}\right)k^3 + \mathrm{i}kw\right] \tag{20}$$

(remember that k has a fixed value here). Since $|\chi(w)| = 1/\sqrt{2\pi}$, in the state (19) of the particle in a uniform force field, all values of the energy are equally probable. (Note that the substitution of (20) into Eq. (18) must reproduce (19).)

Similarly, we can take the separable solution of the Schrödinger equation (7) given by

$$\Psi^{(E)}(x',t') = \mathrm{Ai}\left(\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x' - \frac{E}{ma}\right)\right)\exp\left(-\frac{\mathrm{i}E}{\hbar}t'\right) \tag{21}$$

[characterized by a particular value of E, *cf.* Eq. (18)], which gives rise to the solution of the Schrödinger equation for a free particle

$$\Psi^{(E)}_{\text{free}}(x,t) = \exp\left[\frac{\mathrm{i}}{\hbar}\left(matx - \tfrac{1}{3}ma^2t^3\right)\right]\mathrm{Ai}\left(\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x - \frac{1}{2}at^2 - \frac{E}{ma}\right)\right)\exp\left(-\frac{\mathrm{i}E}{\hbar}t\right). \tag{22}$$

This is the solution of the Schrödinger equation for a free particle (2) with the initial condition

$$\Psi^{(E)}_{\text{free}}(x,0) = \mathrm{Ai}\left(\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x - \frac{E}{ma}\right)\right).$$

The solution (22) has been previously given in Ref. [9] (see also Ref. [14] and the references cited therein). It is distinguished by the fact that it is a nonspreading wave packet which moves with constant acceleration a in spite of the fact that is a solution of the Schrödinger equation for a free particle. In fact, this solution satisfies

$$|\Psi^{(E)}_{\text{free}}(x,t)| = \left|\mathrm{Ai}\left(\left(\frac{2m^2a}{\hbar^2}\right)^{1/3}\left(x - \frac{1}{2}at^2 - \frac{E}{ma}\right)\right)\right|.$$

This solution is known as the Airy wave packet. (A discussion about the properties of this wave packet can be found in Refs. [9, 14].)

The separable solution (21), which is the state of a particle in a uniform force field with energy E, is a special case of the general solution (18) with

$$\chi(E'/ma) = ma\left(\frac{\hbar^2}{2m^2a}\right)^{1/3}\delta\big((E'-E)/ma\big)$$

(now E has a fixed value), and the corresponding solution of the Schrödinger equation for a free particle is given by (12) with $\phi(k)$ given by [see Eq. (15)]

$$\phi(k) = \frac{1}{\sqrt{2\pi}}\left(\frac{\hbar^2}{2m^2a}\right)^{1/3}\exp\left[\frac{\mathrm{i}}{3}\left(\frac{\hbar^2}{2m^2a}\right)k^3 - \frac{\mathrm{i}kE}{ma}\right].$$

Substituting this function into Eq. (12) one obtains the Airy wave packet (22). Since $|\phi(k)|$ is a constant, in the Airy wave packet all values of k are equally probable.

3.3. Relation between propagators

In general, the solution to the (time-dependent) Schrödinger equation can be expressed in the form

$$\Psi(x_{\mathrm{f}},t_{\mathrm{f}}) = \int_{-\infty}^{\infty} K(x_{\mathrm{f}},t_{\mathrm{f}};x_{\mathrm{i}},t_{\mathrm{i}})\Psi(x_{\mathrm{i}},t_{\mathrm{i}})\,\mathrm{d}x_{\mathrm{i}}, \tag{23}$$

where $K(x_{\mathrm{f}},t_{\mathrm{f}};x_{\mathrm{i}},t_{\mathrm{i}})$ is the so-called propagator. In the case of a free particle the propagator can be obtained in a wide variety of ways with the result

$$K_{\text{free}}(x_{\mathrm{f}},t_{\mathrm{f}};x_{\mathrm{i}},t_{\mathrm{i}}) = \sqrt{\frac{m}{2\pi\mathrm{i}\hbar(t_{\mathrm{f}}-t_{\mathrm{i}})}}\exp\left[\frac{\mathrm{i}}{\hbar}\frac{m(x_{\mathrm{f}}-x_{\mathrm{i}})^2}{2(t_{\mathrm{f}}-t_{\mathrm{i}})}\right]. \tag{24}$$

Making use of the relation (6) for the initial and final points (with $x'_{\rm f}, t'_{\rm f}$ related to $x_{\rm f}, t_{\rm f}$ by means of (1) and, similarly, $x'_{\rm i}, t'_{\rm i}$ related to $x_{\rm i}, t_{\rm i}$) we have

$$\begin{aligned}\Psi(x'_{\rm f}, t'_{\rm f}) &= \exp\left[-\frac{\rm i}{\hbar}\big(mat'_{\rm f}x'_{\rm f} + \tfrac{1}{6}ma^2 t'^3_{\rm f}\big)\right]\Psi_{\rm free}(x_{\rm f}, t_{\rm f}) \\ &= \exp\left[-\frac{\rm i}{\hbar}\big(mat'_{\rm f}x'_{\rm f} + \tfrac{1}{6}ma^2 t'^3_{\rm f}\big)\right]\int_{-\infty}^{\infty} K_{\rm free}(x_{\rm f}, t_{\rm f}; x_{\rm i}, t_{\rm i})\Psi_{\rm free}(x_{\rm i}, t_{\rm i})\,{\rm d}x_{\rm i} \\ &= \exp\left[-\frac{\rm i}{\hbar}\big(mat'_{\rm f}x'_{\rm f} + \tfrac{1}{6}ma^2 t'^3_{\rm f}\big)\right]\int_{-\infty}^{\infty} K_{\rm free}(x_{\rm f}, t_{\rm f}; x_{\rm i}, t_{\rm i})\exp\left[\frac{\rm i}{\hbar}\big(mat'_{\rm i}x'_{\rm i} + \tfrac{1}{6}ma^2 t'^3_{\rm i}\big)\right]\Psi(x'_{\rm i}, t'_{\rm i})\,{\rm d}x'_{\rm i}\end{aligned}$$

which means that the propagator for the particle in a uniform force field is given by

$$\begin{aligned}K(x'_{\rm f}, t'_{\rm f}; x'_{\rm i}, t'_{\rm i}) &= K_{\rm free}(x_{\rm f}, t_{\rm f}; x_{\rm i}, t_{\rm i})\exp\left\{-\frac{\rm i}{\hbar}\Big[ma(t'_{\rm f}x'_{\rm f} - t'_{\rm i}x'_{\rm i}) + \tfrac{1}{6}ma^2(t'^3_{\rm f} - t'^3_{\rm i})\Big]\right\} \\ &= \sqrt{\frac{m}{2\pi{\rm i}\hbar(t'_{\rm f} - t'_{\rm i})}}\exp\left\{\frac{{\rm i}m}{2\hbar}\left[\frac{(x'_{\rm f} - x'_{\rm i})^2}{t'_{\rm f} - t'_{\rm i}} - a(t'_{\rm f} - t'_{\rm i})(x'_{\rm i} + x'_{\rm f}) - \frac{a^2}{12}(t'_{\rm f} - t'_{\rm i})^3\right]\right\}.\end{aligned}$$

It seems difficult to devise a simpler procedure to obtain this propagator.

4. Concluding remark

The coordinate transformation considered here belongs to a restricted class of transformations with $t' = t$, but we can consider more general transformations, mixing the space coordinates and the time (see Refs. [1, 2]). An interesting question is which problems can be related in this way with a given one.

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