Praxeological analysis of how indonesian students learn vectors in physics: A systemic and epistemic perspective

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Vectors are fundamental in physics education, yet students frequently struggle to integrate geometric and algebraic representations. This study investigates the praxeological structures of vector content in the Physics for Senior High School/Islamic Senior High School Grade XI textbook under the Indonesian Merdeka Curriculum using the Anthropological Theory of the Didactic (ATD). ATD conceptualizes knowledge as tasks (T), techniques (τ), technologies (ϑ), and theories (Θ). Data were obtained through a detailed analysis of the vector chapter (pp. 1-26) and examined from the systemic and epistemic perspectives. The findings reveal that technical components (T/τ) dominate, while epistemic justification (ϑ/Θ) is weak. Topics such as dot and cross products are briefly mentioned but lack formal development, and connections between tasks remain fragmented. Most activities fall under Lower-Order Thinking Skills (LOTS), though all show potential for extension into Higher-Order Thinking Skills (HOTS) through proof-based, exploratory, and multirepresentational tasks. These results indicate the need to strengthen conceptual explanations, restructure content sequencing, and design coherent praxeologies to support deeper understanding. This research contributes theoretically by extending ATD applications into physics education and practically by offering insights for curriculum developers, textbook authors, and educators to create vector learning that balances procedural fluency and conceptual depth while fostering HOTS.

Keywords: Praxeological analysis; physics vectors; anthropological theory of the didactic; merdeka curriculum; LOTS-HOTS.

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1. Introduction

The concept of vectors occupies a central position in physics education as a fundamental tool for explaining various physical phenomena such as motion, force, and field interactions [1-3]. Historically, this concept evolved from geometric representations in the form of arrows indicating magnitude and direction to more formal and abstract formulations within linear spaces [4]. At the introductory level, vectors are typically presented as quantities possessing both magnitude and direction, visually represented in two- or three-dimensional space [5, 6]. As the complexity of the subject matter increases, vector representations transition toward symbolic or matrix forms, requiring deeper conceptual understanding and the ability to connect visual representations with formal reasoning.

The transition from concrete to abstract representations often constitutes a significant source of difficulty for students [7, 8]. Numerous studies indicate that learners struggle to coordinate the geometric and algebraic aspects of vectors [9, 10]. For instance, although students may correctly perform vector addition using components, they often fail to explain or interpret these operations within relevant physical contexts [11,12]. Such difficulties suggest that many students develop fragmented knowledge, confined to procedural techniques without an adequate epistemic framework to connect with conceptual meaning.

In the Indonesian context, the teaching of vectors typically begins in grade 10 within the topics of motion and force [13]. The content is delivered through textbooks and instructional materials aligned with the national curriculum [14]. Widely used physics textbooks often present problems that emphasize procedures and formulas, with limited attention to epistemic justification or the theoretical frameworks underlying the applied techniques [15, 16]. This raises a critical question: to what extent does students' engagement with textbook-based problem solving genuinely foster the development of coherent conceptual knowledge, rather than merely the mastery of isolated techniques?

To address this issue, the present study adopts the theoretical lens of the Anthropological Theory of the Didactic (ATD), developed by Chevallard (2006) [17] and further elaborated by Bosch and Gascón (2014) [18], particularly through the concept of praxeology. Within the ATD framework, scientific or mathematical knowledge is understood as the outcome of human activity, organized into a praxeological structure that systematically combines tasks (T), techniques (τ), technologies (θ), and theories (Θ). The praxis block (T/τ)

refers to the pairing of tasks and their techniques, while the logos block (θ/Θ) concerns the explanations and theoretical frameworks underpinning them. Applying this model to the teaching of vectors enables an analysis not only of what students are asked to perform, but also of how and to what extent epistemic justification and meaning are embedded in their learning materials [19].

Although the ATD approach has been widely applied in mathematics education [20–22], its use in physics education remains relatively limited, particularly in the Indonesian context. Consequently, there exists a significant research gap in understanding how praxeological structures both explicit and implicit shape the learning of vector concepts. This study seeks to address this gap by analyzing the types of praxeologies offered to students in vector learning, with particular attention to the distinctions between point, local, and regional praxeologies as proposed by Bosch and Gascón (2014) [18].

This study presents three main contributions. First, it extends the application of the ATD framework to the field of physics education, specifically to vector learning, which has received limited attention in previous didactic studies. Second, it offers an in-depth analysis of textbook-based learning tasks using the praxeological framework ($T/\tau/\theta/\Theta$), thereby enabling a mapping not only of problem-solving techniques but also of the presence or absence of theoretical justification and meaning. Third, it provides a contextual contribution to science education research in Indonesia by evaluating how the curriculum, textbooks, and classroom practices shape students' access to conceptual knowledge in physics.

The aim of this study is to investigate the praxeological structures available to Indonesian high school students in learning vector concepts, particularly through an analysis of the tasks, techniques, technologies, and theories embedded in textbooks and classroom practices. This research seeks to identify whether students are merely provided access to fragmented point praxeologies or whether opportunities exist for developing more coherent local and regional praxeologies grounded in scientific theory.

By combining the ATD theoretical framework with an empirical analysis of physics learning materials in Indonesia, this study aspires to make a theoretical contribution to the understanding of didactical transposition in science education, as well as a practical contribution to the design of more conceptually oriented instruction. The findings are expected to provide valuable insights for curriculum developers, textbook authors, and educators in structuring vector learning that not only emphasizes procedural skills but also fosters deep and meaningful conceptual understanding among students.

2. Research method

2.1. Research design

This study aims to provide a comprehensive analysis of potential learning obstacles encountered in high school physics textbooks, particularly in the understanding of vector concepts. These obstacles serve as the basis for examining the systemic and epistemic aspects of the Vector chapter in the Physics for Senior High School/Islamic Senior High School Grade XI textbook used in the Merdeka Curriculum. The study integrates the ATD framework with the Didactical Design Research (DDR) framework developed by [23, 24, 26].

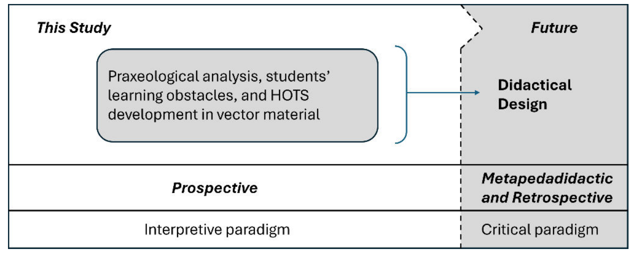


FIGURE 1. The Position of the Study within the DDR Framework (Adapted form Sahara *et al.* (2025) [28]).

DDR is grounded in two main paradigms: interpretive and critical. The interpretive paradigm emphasizes the analysis of subjective experiences and the meanings ascribed to them, whereas the critical paradigm seeks to promote change through the development of didactical designs. By adopting a transcendental approach, the results of the institutional analysis in this study are positioned as the epistemological foundation for the formulation of didactical designs. This approach aligns with the reflections of [27], who underscore the necessity of a dialectical relationship between research, design, and teaching practices.

In examining the phenomenon of textbooks, this study is situated within the interpretive paradigm to uncover the meaning of knowledge constructed by the noosphere and transposed into knowledge to be taught [18]. This position then serves as a prospective basis for employing the critical paradigm in designing new instructional models with testable hypotheses, oriented toward the development of new knowledge for learners.

2.2. Subjects and data collection

The primary data for this study were obtained from the Physics for Senior High School/Islamic Senior High School Grade XI textbook officially published by the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia. This book has been approved as a primary learning resource for senior high schools under the imple-

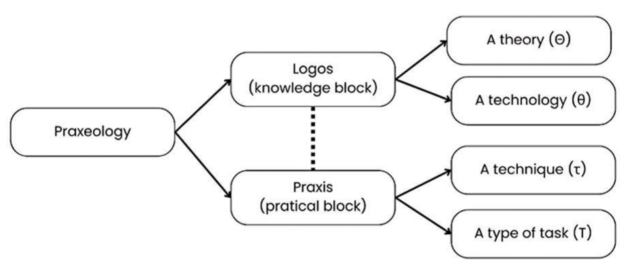


FIGURE 2. Four components of praxeology (Adapted from Chevallard (2006) [17]).

TABLE I. Praxis Block $\mathbf{T_1}$.

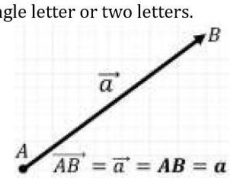
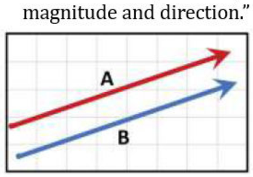
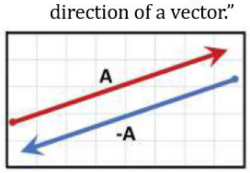
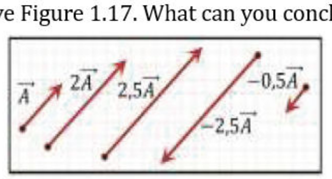
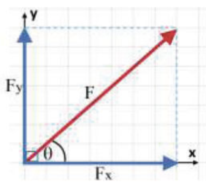
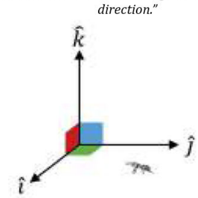
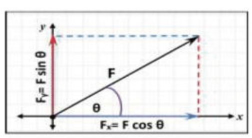
No	First Task Type (Foundational Representation of Vectors (T_1))	Technique (τ)
1	$\mathbf{T_{11}}$: Identifying the concept of vectors.	τ_{11}: Students are expected to determine the direction of a bus traveling from the terminal to the airport and subsequently construct its trajectory as a vector route.
2	$\mathbf{T_{12}}$: Representing vectors in symbolic notation. Vector notation is written as $\overrightarrow{AB}$, $\boldsymbol{AB}$, $\vec{a}$, or $\boldsymbol{a}$. Vector notation may use either a single letter or two letters.  **Figure 1.7** Vector and Its Notation Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) The magnitude or length of vector $\overrightarrow{AB}$ is written as $\lvert\overrightarrow{AB}\rvert$. It can be equal to 0 and is always non-negative.	τ_{12}: Students are expected to denote a vector symbolically when two distinct points are given.
3	$\mathbf{T_{13}}$: Constructing graphical representations of vectors. **Activity 1.2** 1. Draw the vector from Figure 1.9. Use a protractor to determine the angle formed by the tension vector, and a ruler to determine the length of the vector. 2. If 1 cm represents 10 N, determine the magnitude of each force vector in Figure 1.10. 3. Draw and label the following vectors: a. Length 8 cm and direction 150° b. Length 6 cm and direction 330°	τ_{13}: Students are expected to produce graphical representations of vectors derived from physical phenomena.
4	$\mathbf{T_{14}}$: Recognizing fundamental properties of vectors. "Two vectors are said to be equal if they have the same magnitude and direction."  **Figure 1.12** Two Equal Vectors Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) "A negative vector has the same magnitude but the opposite direction of a vector."  **Figure 1.14** A Vector and Its Negative Vector Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022)	τ_{14}: Students are expected to identify two vectors of equal magnitude, while recognizing that their orientations may either coincide or be opposite.
5	$\mathbf{T_{15}}$: Performing scalar multiplication of vectors. Observe Figure 1.17. What can you conclude?  **Figure 1.17** Various vectors resulting from scalar multiplication Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022)	τ_{15}: Students are expected to construct and interpret the multiplication of a vector by a real scalar.

TABLE II. Praxis Block $\mathbf{T_2}$.

No	Second Task Type (Decomposition of Vectors in Cartesian Systems (T_2))	Technique (τ)
1	T_{21}: Expressing vector components in the Cartesian plane. **1. Vector Components** A two-dimensional vector can be resolved into two perpendicular vectors. The decomposition of a vector results in two components, namely along the x-axis (horizontal) and the y-axis (vertical). Figure 1.18 shows a force **F** projected onto the x-axis and y-axis, producing **Fx** and **Fy**.  **Figure 1.18** Various vectors resulting from scalar multiplication Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022)	τ_{21}: Students are expected to express a vector in the Cartesian plane as a linear combination of the mutually perpendicular unit vectors, namely $F_x = \hat{\imath}$ and $F_y = \hat{\jmath}$.
2	T_{22}: Expressing vector components in the Cartesian space. The notations ***i*** and ***j*** represent unit vectors in the horizontal and vertical directions. For a three-dimensional vector, the following applies: $\boldsymbol{A} = A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$. Here, ***i***, ***j***, and ***k*** are unit vectors in the directions of the x-axis, y-axis, and z-axis, respectively. *"A unit vector is a dimensionless vector with a magnitude of one that points in a specific direction."*  **Figure 1.20** Unit vectors in the Cartesian coordinate system Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022)	τ_{22}: Students are expected to express a vector in the Cartesian space as a linear combination of the mutually perpendicular unit vectors, namely $F_x = \hat{\imath}$, $F_y = \hat{\jmath}$ and $F_z = \hat{k}$
3	T_{23}: Decomposing vectors based on trigonometric principles. This problem can be solved using trigonometric rules, as shown in Figure 1.22.  **Figure 1.22** Decomposition of a vector using trigonometric rules Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) The component of vector **F** along the x-axis is F_x, with a magnitude of: $F_x = F\cos\theta$ The component of vector **F** along the y-axis is F_y, with a magnitude of: $F_x = F\sin\theta$ where θ is the angle formed between vector ***F*** and the positive x-axis.	τ_{23}: Students are expected to determine $F_x F\cos\theta$ and $F_y = F\sin\theta$ where θ denotes the angleformed by vector $\vec{F}$ with the X-axis, measured counterclockwise.

plementation of the Merdeka Curriculum and is accessible through the website of the National Library of Indonesia. The material specifically analyzed is Chapter 1: Vectors, covering pages 1-26. The analysis focused on the four main components of praxeology: tasks (T), techniques (τ), technology (θ), and theory (Θ). In this study, a task is understood as a unit of learning activity that enables students to actively engage in the construction of knowledge. Not all problems or activities in the textbook were analyzed; only those designed to support conceptual understanding (rather than assessment) were examined. The didactical design K was constructed based on the types of tasks T_i such that $K = \sum T_i$, where i ranges from 1 to n. Each task type T_i consists of a set of tasks $t_{(i,j)}$ such that $T_i = \sum t_{i,j)}$, where j ranges from 1 to mi. In this study, several conceptual tasks, practice exercises, and exploratory activities were identified in the vector chapter and subsequently classified into three categories of tasks according to their characteristics.

TABLE III. Praxis Block $\mathbf{T_3}$.

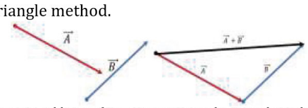
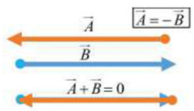
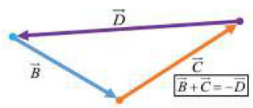
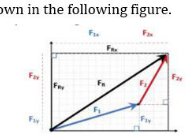
No	Third Task Type (Vector Operations and Resultants (T_3))	Technique (τ)
1	T_{31}: Performing vector addition and subtraction using the graphical method. Observe Figure 1.24, which shows the addition of two vectors using the triangle method.  **Figure 1.24** Addition of Two Vectors Using the Triangle Method Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022)	τ_{31}: Students are expected to perform vector addition and subtraction through the graphical method.
2	T_{32}: Understanding the concept of a zero resultant vector. 1. Two vectors of equal magnitude but opposite direction The sum of the two results in a zero vector.  **Figure 1.29** Addition of Two Vectors Resulting in a Zero Vector Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) 2. The addition of vector B, vector D, and vector C results in a zero vector. The sum of vectors B and C is the negative of vector D.  **Figure 1.30** Addition of Three Vectors Resulting in a Zero Vector Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) "A zero vector is a vector whose initial point and terminal point coincide. The zero vector has zero length and no definite direction."	τ_{32}: Students are expected to understand the concept of the additive inverse of a vector within the operation of vector addition.
3	T_{33}: Performing vector addition and subtraction using the analytical method. The analytical solution of vector addition can be carried out by adding components, as shown in the following figure.  **Figure 1.31** Addition of Vector Components Source: Alvius Tinambunan / Ministry of Education, Culture, Research, and Technology (2022) The magnitude of each vector component along the x-axis and the y-axis is given by: $F_x = F\cos\alpha$ and $F_y = F\sin\alpha$ The resultant vector along the x-axis and y-axis is: $F_{Rx} = \sum F_x = F_{1x} + F_{2x}$ $F_{Ry} = \sum F_y = F_{1y} + F_{2y}$ The magnitude and direction of the resultant vector are: $F_x = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ and $\tan\alpha = \frac{F_y}{F_x}$ If there are more than two vectors, then the resultant vector along the x-axis and y-axis is: $F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} + \ldots$ $F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} + \ldots$	τ_{33}: Students are expected to perform vector addition and subtraction through the analytical method.
4	T_{34}: Determining the resultant vector by applying the law of cosines. Refer again to Figure 1.26. The magnitude of the resultant of two vectors F_1 and F_2 forming an angle α can be calculated using the following equation: $F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$, where: F_R= magnitude of the resultant of the two vectors, F_1 = magnitude of the first vector F_2 =magnitude of the second vector, and α =the angle between the two vectors.	τ_{34}: Students are expected to determine the. resultant of two vectors forming an angle between them by applying the law of cosines

TABLE IV. Logos Block $\mathbf{T_1}$.

No	$\mathbf{T_1}$	τ_1	Technology (θ)	Theory (Θ)
1	$\mathbf{T_{11}}$	τ_{11}	θ_{11}:Constructing a trajectory by representing a directed line segment from the bus terminal to the airport.	
2	$\mathbf{T_{12}}$	τ_{12}	θ_{12}:Defining a vector by identifying its initial and terminal points, thereby establishing both its orientation (direction) and magnitude, which are then denoted symbolically.	Θ_1: Introducing the concept of a directed line segment, which constitutes the foundational idea for students to understand vectors.
3	$\mathbf{T_{13}}$	τ_{13}	θ_{13}:Representing a vector by identifying a physical force as its origin.	
4	$\mathbf{T_{14}}$	τ_{14}	θ_{14}:Identifying two vectors based on their properties: if two vectors have the same magnitude and the same direction, they are considered equal; if they have the same magnitude but opposite directions, they are considered opposites.	Θ_2: Establishing the theoretical properties of vectors, particularly those concerning the equality of two vectors.
5	$\mathbf{T_{15}}$	τ_{15}	θ_{15}: Scalar multiplication of a vector is determined by multiplying the scalar with the magnitude of the vector, while its direction is defined by whether the scalar is a positive or a negative real number.	Θ_3: Establishing the theoretical properties of vectors with respect to scalar multiplication, vector addition and subtraction, as well as dot product and cross product operations.

2.3. Data analysis

The data analysis was carried out in two stages. The first stage focused on the practical aspects of praxeology, namely examining the types of tasks (T) and techniques (τ) employed in the textbook. The second stage involved a deeper analysis of the theoretical and epistemological aspects, specifically the technologies (θ) and theories (Θ) underpinning the construction of those tasks. Each praxeological component was elaborated within the framework of vector concepts, encompassing representations, operations, and their applications in physics contexts. As illustrated in Fig. 2, the praxeological framework interrelates the four components (T,τ,θ,Θ) to provide a systemic model for analyzing knowledge organization.

Within the framework of the Anthropological Theory of Didactics (ATD), praxeology serves as the fundamental unit for analyzing human activity in learning contexts. Chevallard and Bosch (2020) [29] emphasized that praxeology enables the evaluation of whether an activity constitutes a form of knowledge. Thus, praxeology provides a theoretical foundation for assessing whether the didactical design embedded in vector textbooks is epistemic (knowledge-based) and systemic (structured as part of the discipline of physics).

3. Results

3.1. Praxis blocks

The praxeological analysis of the Vector chapter in the Physics for Senior High School/Islamic Senior High School Grade XI textbook of the Merdeka Curriculum reveals that the material is structured into three praxis blocks, which connect tasks (T) with techniques (τ), and three logos blocks, which encompass technologies (θ) and theories (Θ) as epistemic justifications [17, 18].

The first praxis block, summarized in Table I, centers on fundamental vector concepts. The tasks include determining the direction and magnitude of a vector, denoting a vector given two points, drawing vectors based on physical phenomena, identifying similarities or differences in vector directions, and illustrating the result of scalar multiplication of a vector. From the perspective of ATD, this structure represents a praxeology in which each task stands independently without explicit connections to more complex tasks, thereby limiting the coherence among concepts [30].

The second praxis block, presented in Table II, extends the focus to vector representations in Cartesian planes and spaces, as well as vector decomposition using trigonometric rules. Although there is potential to establish a local praxeology namely, interconnections among tasks through techniques within the presentation of vector material this potential remains underutilized, as the relationship between geometric and algebraic representations of vectors is not sufficiently reinforced with conceptual explanations.

The third praxis block, presented in Table III, encompasses vector operations such as addition and subtraction using graphical and analytical methods, determination of inverse vectors, and the calculation of resultants through the cosine rule. The variety of techniques demonstrates completeness in terms of technical praxis; however, the approach remains largely algorithmic and procedural. The absence of formal discussions on the dot product and cross product indicates a tendency toward an epistemology of use rather than an epistemology of understanding, which is essential for building stable conceptual comprehension.

3.2. Logo blocks

The logos blocks in this analysis consist of two main components: technology (θ) and theory (Θ). Technology (θ) refers to the tools or methods used to justify strategies (τ), while theory (Θ) represents conclusions in the form of conceptual knowledge that can generalize the entire praxeological process. On the logos side, Table IV illustrates that the technologies provided are limited to intuitive explanations concerning directed line segments, vector properties, and scalar multiplication. These explanations are not connected to a formal axiomatic framework. Table V justifies that vectors can be represented as linear combinations of standard unit vectors and trigonometric components, yet it does not explicitly link this representation to vector space theory. Table VI contains procedural explanations of vector addition, subtraction, and the determination of resultants; however, the dot product and cross product are merely mentioned without formal elaboration, thereby constraining the development of structural understanding [31].

When viewed as a whole, the integration between the praxis and logos blocks across the six tables reveals an imbalance: the praxis blocks dominate, while the logos blocks remain comparatively weak. According to the ATD literature, this condition suggests that the resulting learning is primarily technical-procedural in nature and risks generating learning obstacles [18] when students attempt to generalize knowledge to new contexts. The limited connections among tasks also hinder the transition from point praxeology to local or regional praxeology, which is essential for establishing systematic coherence across various vector concepts [29].

4. Discussion

The praxeological analysis referring to Tables I-III (praxis blocks) and Tables IV-VI (logos blocks) reveals a significant imbalance between the components of technical practice (T, τ) and epistemic justification (θ, Θ). Within the framework of the Anthropological Theory of the Didactic (ATD), each praxeology consists of four elements: tasks (T), techniques (τ), technologies (θ) as justifications for techniques, and theories (Θ) as general conceptual frameworks [29, 32]. This imbalance can be examined through both the systemic perspective and the epistemic perspective, which emphasize the institutional and epistemological relations of knowledge organization [33, 34]. If such a condition persists, it may give rise to learning obstacles of both epistemological and didactical nature, as identified in various praxeological studies in the contexts of mathematics and applied sciences [35, 36].

4.1. Systemic perspective

From the systemic perspective, the sequencing of material in Tables I-III illustrates a didactic transposition that prioritizes practical instructional strategies over alignment with the axiomatic structure of vector concepts. For instance, in $[T_{15}, \tau_{15}, \theta_{15}, \Theta_3]$, scalar multiplication is introduced prior to $[T_{31}, \tau_{31}, \theta_{31}, \Theta_3]$ and $[T_{33}, \tau_{33}, \theta_{33}, \Theta_3]$, which represent vector addition and subtraction, even though axiomatically τ_+ and τ_- form the foundational basis of vector algebra before τ_{scalar}. This sequence indicates the occurrence of praxeology drift, namely a shift in the structure (T,τ,θ,Θ) away from disciplinary order as a result of adaptation to short-term pedagogical goals [37, 38].

Furthermore, in $[T_{34}, \tau_{34}, \theta_{34}, \Theta_6]$, which outlines the cosine rule, the expected connections to the dot product $[T_\bullet, \tau_\bullet, \theta_\bullet, \Theta_3]$ and the cross product $[T_\times, \tau_\times, \theta_\times, \Theta_3]$ typically integrated within a regional praxeology of vectors, are

TABLE V. Logos Block $\mathbf{T_2}$.

No	$\mathbf{T_2}$	τ_2	Technology (θ)	Theory (Θ)
1	$\mathbf{T_{21}}$	τ_{21}	θ_{21}:Understanding that any vector in the Cartesian plane, $\vec{F} = \langle a, b\rangle$, can be expressed as $\vec{F} = \langle a, b\rangle = a\vec{F}_x + b\vec{F}_y$, where $\vec{F}_x = \langle 1, 0\rangle = \hat{\imath}$ and $\vec{F}_y = \langle 0, 1\rangle = \hat{\jmath}$.	Θ_4: Formalizing the representation of a vector in the Cartesian plane or Cartesian space as a linear combination of standard unit vectors.
2	$\mathbf{T_{22}}$	τ_{22}	θ_{22}: Understanding that any vector in the Cartesian space, $\vec{F} = \langle a, b, c\rangle$, can be expressed as $\vec{F} = \langle a, b, c\rangle = a\vec{F}_x + b\vec{F}_y + c\vec{F}_z$, where $\vec{F}_x = \langle 1, 0, 0\rangle = \hat{\imath}$, $\vec{F}_y = \langle 0, 1, 0\rangle = \hat{\jmath}$, and $\vec{F}_z = \langle 0, 0, 1\rangle = \hat{k}$.	Θ_4: Formalizing the representation of a vector in the Cartesian plane or Cartesian space as a linear combination of standard unit vectors.
3	$\mathbf{T_{23}}$	τ_{23}	θ_{23}:Understanding and computing $F_x = F\cos\theta$ and $F_y = F\sin\theta$, where θ denotes the angle formed by vector $\vec{F}$ with the X-axis, measured counterclockwise.	Θ_5: Establishing the determination of vector components F_x and F_y through trigonometric

TABLE VI. Logos Block $\mathbf{T_3}$.

No	$\mathbf{T_3}$	τ_3	Technology (θ)	Theory (Θ)
1	$\mathbf{T_{31}}$	τ_{31}	θ_{31}:Vector addition and subtraction can be performed by connecting the terminal point of vector $\vec{A}$ to the initial point of vector $\vec{B}$; the resultant is then defined as a new vector whose initial point coincides with that of $\vec{A}$ and whose terminal point coincides with that of $\vec{B}$.	
2	$\mathbf{T_{32}}$	τ_{32}	θ_{32}:The additive inverse of a vector $\vec{u}$ is obtained by determining another vector, say $\vec{v}$, such that $\vec{u}+\vec{v}=\vec{0}$, where $\vec{0}$ denotes the zero vector.	Θ_3: Establishing the properties of vectors, including scalar multiplication, vector addition and subtraction, as well as dot product cross product operations.
3	$\mathbf{T_{33}}$	τ_{33}	θ_{33}:Vector addition and subtraction of $\vec{F_1}$ and $\vec{F_2}$ can be achieved analytically by adding or subtracting their respective components along the x-axis and y-axis.	
4	$\mathbf{T_{34}}$	τ_{34}	θ_{34}:The resultant of two vectors forming an included angle can be determined by applying the law of cosines from trigonometric principles.	Θ_6: Formalizing the determination of a resultant vector within the framework of vector operations.

are not realized. Both are only mentioned at the end of Table VI with a statement promising further elaboration in Chapter 3, yet no continuation is provided in that chapter. This phenomenon generates a didactic gap that disrupts the chain of inter-praxeological connections, thereby impeding the formation of a regional praxeology that unifies various vector operations under a coherent theoretical framework of Θ_{vector} [39, 40].

On the other hand, the mapping results in the HOTS-LOTS-Development Potential Table (Table VII) show that although the majority of (T,τ,θ,Θ) remain within the LOTS category, almost all possess the potential to be recontextualized in order to align with the axiomatic structure while simultaneously facilitating the development of HOTS. Thus, the opportunities for improvement are not only pedagogical but also structural, namely restoring systemic coherence between the sequencing of concepts, problem-solving techniques, technological explanations, and theoretical coverage [41, 42].

4.2. Epistemic perspective

From the epistemic perspective, Tables IV-VI show that the available logos blocks provide only intuitive justifications ($\theta_{\text{intuitive}}$) and procedural justifications ($\theta_{\text{procedural}}$) without mathematical definitions, property proofs, or extended applications that link (τ) to (Θ). For example, in $[T_{11}, \tau_{11}, \theta_{11}, \Theta_1]$ through $[T_{15}, \tau_{15}, \theta_{15}, \Theta_3]$, the explanations ($\theta$) are merely descriptive outlining how to draw or denote vectors without an axiomatic foundation that connects them to vector space theory. A similar condition appears in $[T_{21}, \tau_{21}, \theta_{21}, \Theta_4]$ and $[T_{22}, \tau_{22}, \theta_{22}, \Theta_4]$ which present vectors as linear combinations of unit vectors. Although (θ) states the relation $\vec{F} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, there is no formal discussion of orthonormal bases or vector space dimensions, leaving the connection between praxis and logos weak.

The absence of formal definitions becomes even more apparent in $[T_{34}, \tau_{34}, \theta_{34}, \Theta_6]$ which introduces the cosine rule to determine vector resultants but does not connect it to the concept of the dot product $[T_\bullet, \tau_\bullet, \theta_\bullet, \Theta_3]$ or the cross product $[T_\times, \tau_\times, \theta_\times, \Theta_3]$. In fact, ($\tau_\bullet$) and ($\tau_\times$) are merely mentioned without sufficient elaboration of (τ), (θ), or (Θ). These findings indicate the dominance of praxis blocks detached from logos blocks [43], where techniques (τ) are treated as practical rules within limited contexts rather than as components of a conceptual system (Θ) that can be generalized. Without such connections, students risk mastering procedures without developing structural understanding [44, 45], which is essential for transferring knowledge to new situations or across domains.

4.3. Potential learning obstacles

The imbalance between praxis and logos blocks may result in learning that is predominantly technical-procedural and can generate learning obstacles, particularly when the logos components are underdeveloped so that techniques are not accompanied by justifications or conceptual explanations. From a curriculum reform perspective, Bosch *et al.* (2023) [46] associate this phenomenon with weak internal curriculum coherence: when the alignment among goals, content, pedagogy, and assessment is lacking, students become more vulnerable to gaps in conceptual understanding, even if they can perform tasks procedurally. This condition reveals several potential learning obstacles identified through praxeological analysis and categorized by Brousseau, namely ontogenetic, didactical, and epistemological obstacles.

TABLE VII. Mapping of LOTS, HOTS, and development potentials of vector tasks based on praxeological analysis.

T	Task Description	τ	θ	Θ	Category	Rationale	Development Potential
T_{11}	Determining the concept of vectors (Activity 1.1)	τ_{11}	θ_{11}	Θ_1	LOTS	Only involves identifying magnitude and direction; no alternative reasoning required	May develop into HOTS if students are asked to compare trajectories by considering distance vs. time or additional variables
T_{12}	Vector notation from two given points	τ_{12}	θ_{12}	Θ_1	LOTS	Direct substitution; no verification or physical context	May develop into HOTS if students are asked to verify notation through measurement or unit conversion
T_{13}	Constructing vectors from physical phenomena (Activity 1.2)	τ_{13}	θ_{13}	Θ_1	LOTS	Limited to drawing from given data; lacks conceptual modeling	May develop into HOTS if students are required to model the phenomenon independently into vector representation
T_{14}	Identifying vector properties (Activity 1.3)	τ_{14}	θ_{14}	Θ_2	LOTS	Deterministic answers; no justification or proof involved	May develop into HOTS if tasks require formal proof or generalization of vector properties.
T_{15}	Scalar multiplication of a vector (Figure 1.17)	τ_{15}	θ_{15}	Θ_3	LOTS	Procedure-based; no analysis of physical implications	May develop into HOTS if students analyze the impact of scalar multiplication in specific physical systems
T_{21}	Vector components in the Cartesian plane (Figures 1.19–1.21)	τ_{21}	θ_{21}	Θ_4	LOTS	Direct translation from diagrams; no derivation process	May develop into HOTS if students are asked to derive the relation from the concept of orthonormal bases
T_{22}	Vector components in Cartesian space	τ_{22}	θ_{22}	Θ_4	LOTS	Similar to T_{21}; lacks application beyond Cartesian systems	May develop into HOTS if students compare orthogonal and non-orthogonal coordinate systems
T_{23}	Vector decomposition using trigonometry (Figures 1.22–1.23)	τ_{23}	θ_{23}	Θ_5	LOTS	Fully provided data; requires only formula substitution	May develop into HOTS if students must estimate angles from incomplete or experimental data.
T_{31}	Vector addition/ subtraction using the graphical method (Activity 1.4)	τ_{31}	θ_{31}	Θ_3	LOTS	Involves a single method; lacks strategy comparison	May develop into HOTS if students compare accuracy across triangle, parallelogram, and polygon methods
T_{32}	Zero resultant vector (Figure 1.30)	τ_{32}	θ_{32}	Θ_3	LOTS	Not connected to equilibrium applications	May develop into HOTS if linked to real-world systems of balanced forces
T_{33}	Vector addition/ subtraction using the analytical method (Activity 1.5)	τ_{33}	θ_{33}	Θ_3	LOTS	Does not compare results with graphical methods	May develop into HOTS if students compare both methods or construct computational models
T_{34}	Resultant vector using the law of cosines (Activity 1.6)	τ_{34}	θ_{34}	Θ_6	LOTS	No proof or generalization beyond two vectors	May develop into HOTS if students are required to prove the law of cosines or generalize it to multi-vector systems

Ontogenetic obstacles, particularly of a conceptual nature, may occur when the sequencing of (T, τ) does not follow the epistemic logic of vector algebra. For example, $[T_{15}, \tau_{15}, \theta_{15}, \Theta_3]$ on scalar multiplication is introduced before $[T_{31}, \tau_{31}, \theta_{31}, \Theta_3]$ and $[T_{33}, \tau_{33}, \theta_{33}, \Theta_3]$ on vector addition and subtraction. Such misalignment with the structure of knowledge risks producing fragmented conceptual schemes that are difficult to reconstruct at more advanced stages of learning [47].

Didactical obstacles arise from instructional designs that fail to integrate formal explanations (logos) with procedures. For instance, the absence of formal definitions $(\Theta_\bullet, \Theta_\times)$ for the dot product and cross product, where $\tau_\bullet$ and $\tau_\times$ are merely mentioned without sufficient elaboration of (θ) and (Θ) to convey their geometric and algebraic meanings. This condition leads students to perceive these operations merely as computational rules devoid of deeper conceptual meaning [48]. Epistemological obstacles emerge when student knowledge is limited to certain contexts, making it difficult to apply in new situations. For example, the procedural techniques of $[T_{21}, \tau_{21}, \theta_{21}, \Theta_4]$ or $[T_{23}, \tau_{23}, \theta_{23}, \Theta_5]$, such as vector decomposition based on trigonometry in orthogonal systems, are applied directly to non-orthogonal systems without understanding the conditions of validity [49]. A similar issue arises from the overemphasis on visual representation (θ_{geo}) in $[T_{11}, \tau_{11}, \theta_{11}, \Theta_1]$ and $[T_{13}, \tau_{13}, \theta_{13}, \Theta_1]$, y which, without the support of symbolic representation (θ_{alg}) restricts students' ability to transfer knowledge into more abstract contexts [50].

Moreover, the lack of reinforcement at the theoretical level $(\Theta_3, \Theta_4, \Theta_6)$ results in disconnected praxeologies that fail to develop into complete local or regional praxeologies [51]. This condition is consistent with findings that overly contextual learning without formal justification produces fragile and easily fragmented knowledge [52]. From the ATD perspective, these findings highlight a didactic transposition that has not yet successfully integrated the components $(T, \tau, \theta, \Theta)$ in a balanced manner. Improvement requires restructuring the material and pedagogical interventions that guide students from merely procedural knowledge toward conceptual understanding, thereby enabling the construction of a more coherent regional praxeology.

4.4. Analysis of LOTS Categories and the Potential for HOTS Development in Vector Material

International assessments such as PISA have shown that Indonesian students' performance in the domain of Higher Order Thinking Skills (HOTS), particularly in mathematical and scientific literacy, remains at a low level [53]. This condition highlights the need for continuous evaluation of learning resources, including textbooks, to ensure that the presented material not only trains procedural skills (Lower Order Thinking Skills / LOTS) but also facilitates the development of higher-order skills such as analysis, synthesis, evaluation, and non-routine problem solving. The praxeological analysis $[T, \tau, \theta, \Theta]$ of vector material in high school textbooks reveals that the majority of task types fall within the LOTS category. This is reflected in task characteristics that are deterministic in nature, with data and procedures fully provided, and techniques (τ) that tend to rely on single algorithmic approaches. At the technological dimension (θ), explanations are dominated by operational "how-to" descriptions without conceptual justification of the "why", whereas the theoretical dimension (Θ) emphasizes definitions and formal properties with limited cross-topic connections.

Nevertheless, the analysis also identifies that all task types hold potential for development into HOTS. For example, $[T_34, \tau_34, \theta_34, \Theta_6]$ related to the cosine rule can be extended into HOTS tasks through proof-based activities or generalizations to multi-vector cases. Similarly, $[T_23, \tau_23, \theta_23, \Theta_5]$ on vector decomposition could be enhanced by providing incomplete data, requiring students to engage in estimation or measurement. These findings are not intended as criticism of textbook authors but rather as constructive input for enriching instructional content. Recommendations for strengthening HOTS in textbooks align with the findings of Johnson & Ohtani (2025) [54], who emphasize the importance of curriculum coherence and task design that integrates conceptual understanding, procedural skills, and contextual problem solving. Furthermore, in line with the PISA framework for assessing HOTS, tasks that require indirect data interpretation, real-world modeling, and formal proof can stimulate students to operate at higher cognitive levels [55]. By leveraging the identified developmental potential, textbooks can maintain their strength in providing fundamental conceptual foundations while expanding toward activities that foster analytical, evaluative, and creative skills. This effort is consistent with broader initiatives to improve the quality of education in Indonesia in response to international assessment challenges and the demands of 21st-century learning.

5. Conclusion

The praxeological analysis of the Vector chapter in the Physics for Senior High School/Islamic Senior High School Grade XI textbook under the Merdeka Curriculum reveals a dominance of technical components (T, τ) with limited epistemic reinforcement (θ, Θ). The sequencing of content indicates both praxeology drift and a didactic gap, such as the introduction of scalar multiplication prior to vector addition/subtraction and the absence of further discussion on the dot product and cross product. From an epistemic perspective, the logos blocks are primarily intuitive and procedural, lacking formal definitions or property proofs, thereby leaving techniques disconnected from their theoretical frameworks. The HOTS analysis further shows that all $(T, \tau, \theta, \Theta)$ remain at the LOTS level, although each has the potential to be developed into HOTS through task design modifications, integration of multiple representations, and formal proofs. These conditions generate various learning obstacles that necessitate restructuring of content, strengthening of logos blocks, and the design of exploratory tasks to facilitate the transition from LOTS to HOTS. Such efforts would enable learning to move toward a coherent regional praxeology, thereby enhancing conceptual understanding and fostering students' higher-order thinking skills.

This study contributes to the development of the Anthropological Theory of the Didactic (ATD) in the context of physics education in Indonesia by demonstrating how the concept of praxeology widely applied in mathematics education research can be employed to analyze the structure of physics knowledge, particularly in the topic of vectors. This approach opens opportunities for further studies to explore didactic transposition in other areas of science while providing an empirical foundation for curriculum developers, textbook authors, and educators to design materials that are not only procedural but also enriched with conceptual justifications, aligned with the epistemology of the discipline, and oriented toward HOTS-based learning.

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