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# Plane waves reversibility

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This work simulates the reversibility of plane waves in different ways. We start making theoretical classical reversing of a plane wave in two different ways exchanging $t$ by $-t$ as first step. In one case, we additionally flip the temporal orientation of the magnetic field. In the other case, we flip the electric field. Therefore, we can compare two classical approaches to time-reversed electromagnetism on plane waves.
On the other hand, we obtain two different mechanically reversed plane electromagnetic waves out of the frame of the electromagnetics reversibility theory. A theoretical experiment makes these effects, where, an infinite plane current generates two plane waves in opposite directions. After this, the waves are made to return in two different ways: (1) by retro reflecting and (2) by moving back the wave. Finally, the returning plane waves insides over a conductor plane in order to induce plane currents in the conductor.
The goal is to complete reversibility cycles, including the charges movement. The returning waves and the induced currents will be compared themselves in all the cases. Charges movements are also included in the discussion in order to have an additional felling of waves reversibility and physical insight of time-reversed waves. It is used a plane waves theoretical experiment created by Feynman as a starting point [1].

*Keywords:* Electromagnetic reversibility; plane waves.

## 1. Introduction

The simplest wave reversibility case in waves is the geometrical optics case coming from the Fermat principle. Where, it was only take into account the traveling time and not the direction of wave propagation in its derivation [2].

Theoretical electromagnetic reversibility it is performed changing $(t)$ by $(-t)$ and considering some additional condition as time flipping of one of their fields, electric or magnetic, [3] or exchanging the roles between the electric and magnetic fields themselves [4].

In this work, we discuss electromagnetic reversibility using electromagnetic waves taking into account charge movements. We start with the generation of electromagnetic plane waves with an infinite plane current $J_\infty^+$ as a source, then continue with the propagation of waves, then its reversion and finally its interaction with an infinite, and conductor plane with conductivity $\sigma$ giving an infinite plane current: $J_\infty^-$. Figure 1 schematize this process.

The plane wave generation is performed using the theoretical experiment of Feynman as is detailed in Sec. 2. We obtain reversed waves in two different ways: (i) using the theoretical electromagnetic reversibility, see Sec. 3.1, and (ii) mechanically reversing waves as detailed in Sec. 3.2.

| Generating current | | Outgoing waves | | Reversing waves | | Induced current |
|---|---|---|---|---|---|---|
| $J_\infty^+(t)$ | → | E / B ~ $J_+$ (x, t) | → | E / B ~ $J_-$ (x, t) | → | $J_\infty^-(t)$ |

FIGURE 1. Reversibility cycle: (i) Generating current (ii) Outgoing plane waves (iii) Reversing waves (iv) Induced current.

## 2. Generation of plane waves

We adapt the Feynman "experiment" [1], where an infinite sheet of current $J(t)$ generates two electromagnetic plane waves, one going in $+x$ direction and other going in $-x$ direction, as it is observed in Fig. 2. This case corresponds to a constant current $J_0$, and a constant travelling field case. In this case, we consider the sheet is made of a conductor material in order to detect the returning waves.

The idea is to leave this wave to propagate for a $\tau$ time, after this time we make the wave to return as shown in Fig. 3, inverting its propagation direction. We expect, the returning wave generates an "inverted" current in the conductor sheet, if we compare this to the initial current that generated the initial electromagnetic wave of Fig. 2. This will be an ideal simile of a radar [5].

Now, we will consider a variable current in order to get variable fields and see what will be the effect on the conduc-

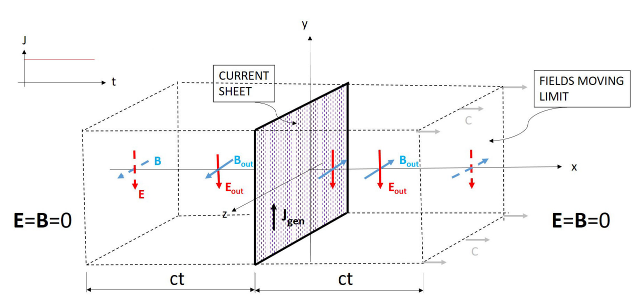


FIGURE 2. Generation of an electromagnetic wave. In red and vertical the electric fields and in blue and horizontal the magnetic field at $t$ time.

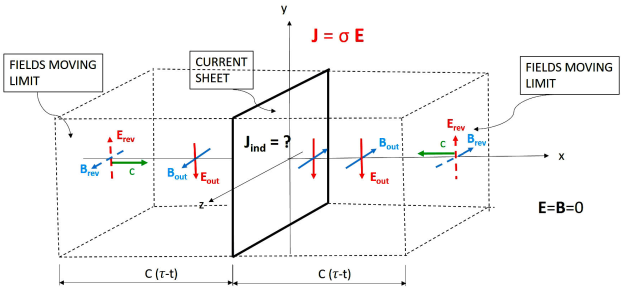


FIGURE 3. Electromagnetic wave returning toward the conductive sheet. In red and vertical the electric fields and in blue and horizontal the magnetic field after a propagation time.

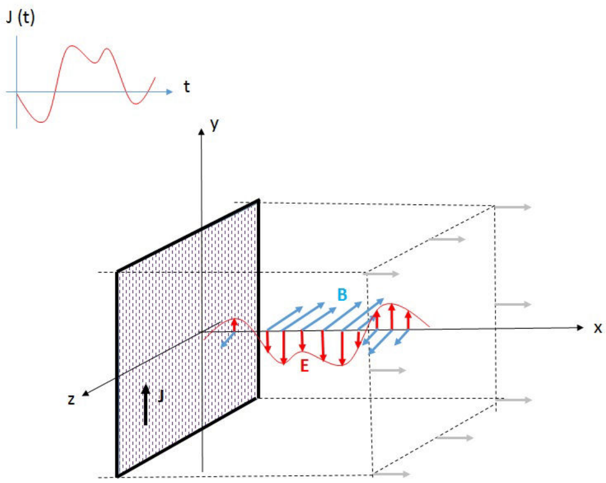


FIGURE 4. Plane electromagnetic wave "outgoing" from the conductive sheet of current $J(t)$. It is omitted the wave is outgoing in $-x$ direction. In red and vertical the electric fields and in blue and horizontal the magnetic field.

tor sheet when the wave return. Fig. 4 shows this case of variable current $J(t)$.

A current $J(t)$ will generate fields described by next equations [6]:

$$B(x,t) = \frac{J(t-x/c)}{2\varepsilon_o c^2}, \tag{1a}$$

$$\boldsymbol{B}(x,t) = \frac{J(t-x/c)}{2\varepsilon_o c^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{1b}$$

$$E(x,t) = \frac{J(t-x/c)}{2\varepsilon_o c}, \tag{1c}$$

$$\boldsymbol{E}(x,t) = -\frac{J(t-x/c)}{2\varepsilon_o c}\boldsymbol{u}_J. \tag{1d}$$

In order to perform the reversibility, we will make the wave goes back. Then we will find the resultant electric field by the two left and right going waves.

## 3. Theoretical reversing of waves

In this section we will reverse the waves using two of the most popular parameters changes for perform reversibility found in literature, called textbook reversibility.

### 3.1. Electromagnetic reversibility: $t \longleftrightarrow -t$ and $E \longleftrightarrow -E$ case

Applying electromagnetic reversibility $t \longleftrightarrow -t$ and $E \longleftrightarrow -E$, on Eq. (1b) and Eq. (1d) will get

$$\boldsymbol{B}(x,t) = \frac{J(-t-x/c)}{2\varepsilon o c^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{2a}$$

$$\boldsymbol{E}(x,t) = \frac{J(-t-x/c)}{2\varepsilon o c}\boldsymbol{u}_J. \tag{2b}$$

And, on the conductive plate ($x = 0$) we get

$$\boldsymbol{B}(0,t) = \frac{J(-t)}{2\varepsilon o c^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{2c}$$

$$\boldsymbol{E}(0,t) = \frac{J(-t)}{2\varepsilon o c}\boldsymbol{u}_J. \tag{2d}$$

Finally, the induced current, using $\boldsymbol{J}(t) = \sigma\boldsymbol{E}(0,t)$, considering the two incoming waves is

$$\boldsymbol{J}(t) = \sigma\frac{J(-t)}{\varepsilon o c}\boldsymbol{u}_J, . \tag{2e}$$

### 3.2. Electromagnetic reversibility: $t \longleftrightarrow -t$ and $B \longleftrightarrow -B$ case

Applying electromagnetic reversibility $t \longleftrightarrow -t$ and $B \longleftrightarrow -B$ on Eq. (1b) and Eq. (1d) we will get

$$\boldsymbol{B}(x,t) = -\frac{J(-t-x/c)}{2\varepsilon o c^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{3a}$$

$$\boldsymbol{E}(x,t) = -\frac{J(-t-x/c)}{2\varepsilon o c}\boldsymbol{u}_J. \tag{3b}$$

On the conductive plate (x=0):

$$\boldsymbol{B}(0,t) = -\frac{J(-t)}{2\varepsilon o c^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{3c}$$

$$\boldsymbol{E}(0,t) = -\frac{J(-t)}{2\varepsilon o c}\boldsymbol{u}_J. \tag{3d}$$

The induced current considering the two incoming waves, and using $\boldsymbol{J}(t) = \sigma\boldsymbol{E}(0,t)$ is

$$\boldsymbol{J}(t) = -\sigma\frac{J(-t)}{\varepsilon o c}\boldsymbol{u}_J. \tag{3e}$$

## 4. Mechanically reversed plane electromagnetic waves

In this Section we return the wave in two different ways: one is with the same profile, the other with inverted profile.

TABLE I. Electric and magnetic fields of the returning waves and induced currents for different cases.

| CASE (Reference) | Electric field | Magnetic field | Induced current |
|---|---|---|---|
| $t \longleftrightarrow -t$ <br> $E \longleftrightarrow -E$ <br> (3.1) | $\boldsymbol{E} = \frac{J(-t-x/c)}{2\varepsilon oc}\boldsymbol{u}_J$ | $\boldsymbol{B} = \frac{J(-t-x/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_S$ | $\boldsymbol{J} = \sigma \frac{J(-t)}{\varepsilon oc}\boldsymbol{u}_J$ |
| $t \longleftrightarrow -t$ <br> $B \longleftrightarrow -B$ <br> (3.2) | $\boldsymbol{E} = -\frac{J(-t-x/c)}{2\varepsilon oc}\boldsymbol{u}_J$ | $\boldsymbol{B} = -\frac{J(-t-x/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_S$ | $\boldsymbol{J} = -\sigma \frac{J(-t)}{\varepsilon oc}\boldsymbol{u}_J$ |
| $x \longleftrightarrow x+ct$ <br> (4.1) | $\boldsymbol{E} = -\frac{J[(\tau-t)-x/c]}{2\varepsilon oc}\boldsymbol{u}_J$ | $\boldsymbol{B} = -\frac{J[(\tau-t)-x/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_S$ | $\boldsymbol{J} = -\sigma \frac{J(\tau-t)}{\varepsilon oc}\boldsymbol{u}_J$ |
| $x \longleftrightarrow (2L-x)/c$ <br> (4.2) | $\boldsymbol{E} = -\frac{J[t-(2L-x)/c]}{2\varepsilon oc}\boldsymbol{u}_J$ | $\boldsymbol{B} = -\frac{J[t-(2L-x)/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_S$ | $\boldsymbol{J} = -\sigma \frac{J(t-2L/c)}{\varepsilon oc}\boldsymbol{u}_J$ |

### 4.1. Moving back wave: $x \longleftrightarrow x+ct$

Considering a propagation time $\tau$, the fields will be:

$$\boldsymbol{B}(x,t) = \frac{J(\tau - x/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{4a}$$

$$\boldsymbol{E}(x,t) = -\frac{J(\tau - x/c)}{2\varepsilon oc}\boldsymbol{u}_J. \tag{4b}$$

Considering these right going waves, we can do to "walk" the fields in reverse, applying $x \longleftrightarrow x+ct$

$$\boldsymbol{B}(x,t) = \frac{J(\tau - t - x/c)}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{4c}$$

$$\boldsymbol{E}(x,t) = -\frac{J(\tau - t - x/c)}{2\varepsilon oc}\boldsymbol{u}_J. \tag{4d}$$

On the conductive plate the magnetic field will be

$$\boldsymbol{B}(0,t) = \frac{J[\tau - t]}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{4e}$$

and the electric field

$$\boldsymbol{E}(0,t) = -\frac{J[\tau - t]}{2\varepsilon oc^2}\boldsymbol{u}_J. \tag{4f}$$

If we use the relation $\boldsymbol{J}(t) = \sigma \boldsymbol{E}(0,t)$, and considering the two waves on the sheet, we obtain the current on it

$$\boldsymbol{J}(t) = -\sigma \frac{J[\tau - t]}{\varepsilon oc}\boldsymbol{u}_J \quad 0 \le t \le \tau. \tag{4g}$$

### 4.2. Reflected back wave: $x \longleftrightarrow (2L-x)/c$

Now, we make the outgoing wave described by Eqs. (1) reflects on a perfect mirror set at a distance $L$ from the current sheet. In this case, Eqs. (1) transforms into

$$\boldsymbol{B}(x,t) = \frac{J\left[t - \frac{2L-x}{c}\right]}{2\varepsilon oc^2}\boldsymbol{u}_J x \boldsymbol{u}_s, \tag{5a}$$

$$\boldsymbol{E}(x,t) = -\frac{J\left[t - \frac{2L-x}{c}\right]}{2\varepsilon oc}u_J. \tag{5b}$$

And, on the conductive plane we get

$$\boldsymbol{E}(0,t) = -\frac{J(t - 2L/c)}{2\varepsilon oc}\boldsymbol{u}_J. \tag{5c}$$

The induced current will be

$$J = -\sigma \frac{J(t - 2L/c)}{\varepsilon oc}\boldsymbol{u}_J \quad t \ge 2L/c. \tag{5d}$$

## 5. Resume

The Table I summarize the different cases to facilitate the analysis.

From results presented in Table I, it is easy to see that all reversible - returning wave equations for electric and magnetic fields have physical meaning. On the other hand, the induced current equations coming from theoretical reversibility (case 3.1 and case 3.2), do not have physical meaning while the ones coming from mechanically reversed plane waves have physical interpretation.

## 6. Conclusions

- We have presented a way to compare two theoretical approach for time reversibility in plane electromagnetic waves.
- We have presented a way to compare two mechanical approaches for motion reversion of plane electromagnetic waves.

- We have presented theoretical experiments to study the electromagnetic reversibility including currents.
- Theoretical reversibility works for plane wave reversibility but not for induced currents.
- Mechanical reversibility works well for plane wave reversibility and for induced currents.
- We expect this work helps to formulate new description of electromagnetic reversibility.


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