Stroboscopic observation of a random walker

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The behavior of mobile agents has received recently wide attention. There is a number of researches, ranging from albatrosses to human beings. Special emphasis has been taken in the statistical distribution of the distances that such agents cover. In some cases due to the lack of accurate data about the mobility of the agents, very ingenious and original experiments have been designed. In these experiments, loosely speaking, is intended to infer the real motion of the agents from the observed position of them at several consecutive positions with observation times taken from a known or assumed distribution. The aim of this paper is to show that, at least for a Gaussian random walker, the distribution of the distances between observed positions is conditioned by the distribution of the observation times. A wide range of numerical experiments are presented to sustain this claim.

Descriptores: Random walker; Levy flights; Gaussian walker.

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1. Introduction

In recent times, mainly due to the existence and development of internet and electronic devices such as global positioning systems and mobile phones, some properties of the motion of living beings have been unveiled. Ranging from Albatrosses [1-2], Pigeons [3], Monkeys [4], Jackals [5] to human beings [6-9], very ingenious studies have been done using data collected with electronic devices or other original forms of tracking. In these experiments, the position of the living beings is observed at consecutive time intervals. The duration of these time intervals in some experiments is constant and in others is taken from a statistical distribution calculated from the same data. Although is hard to believe that the motion of these living beings is completely random, a question is posed about, how accurate is the description of the statistical properties of real motion obtained from these observed positions.

The aim of this paper is to show that, at least for a Gaussian random walker, which is an extreme kind of motion, the distribution of the distances between observed positions is conditioned by the distribution of the observation times.

The paper is organized as follows. In Sec. 2 we describe the numerical experiments we have carried out. In Sec. 3 we analyze the data obtained from these experiments. Section 4 is devoted to conclusions.

2. Description of the numerical experiments

We study 2-dimensional random walkers. Their trajectories are a collection of independent and identical distributed pairs $(\theta_i, s_i\ )\ i = 1..., n$, where θ_i is a random variable uniformly distributed in the interval $(0, 2\pi]$ and s_i is a Gaussian random variable *e.g.*, its probability density function is:

$$p(s) = \frac{1}{\sqrt{2\pi\sigma}} e^{-1/2(s-\mu/\sigma)^2}. \tag{1}$$

Along each experiment, we keep μ and σ constants. The variable θ_i represents the angle with respect to the positive horizontal line of the i^{th} step and s_i the length of this step. In Fig. 1 a short trajectory is visualized. We assume the time to be discrete and during each unit time interval δt one and only one step of the walk is executed.

The trajectories are observed at some time intervals of length $n_k \delta t$. The length n_k of these time intervals are random variables with a distribution similar to those reported in the literature, basically Levy distributions and Levy distributions with exponential cut off. For the definitions, see Sec. 3.

We briefly describe the core of all our simulations. In Fig. 2a, the real trajectory of the random walker (blue line) and the observed trajectory (red line) are shown. Figure 2b) shows with enhanced details the portion of the trajectory inside the box of Fig. 1a). In both figures, red dots represent the positions where observations of the real trajectory were done. In this case the length of the time intervals between consecutives observation was drawn from a Levy distribution with exponential cut off. In Fig. 2c) is shown the distribution of length steps in real trajectory (blue segments) and in Fig. 2d)

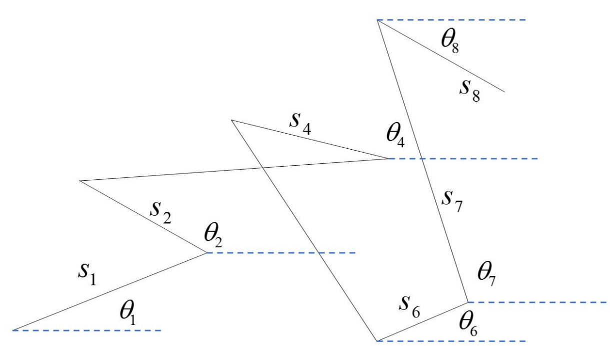

FIGURE 1. A 2-dimensional random walker. The angles θ_i are random variables which are uniformly distributed in the interval $(0, 2\pi]$. They are measured from the positive horizontal line. The values s_i are the lengths of the steps. They are taken form a Gaussian distribution.

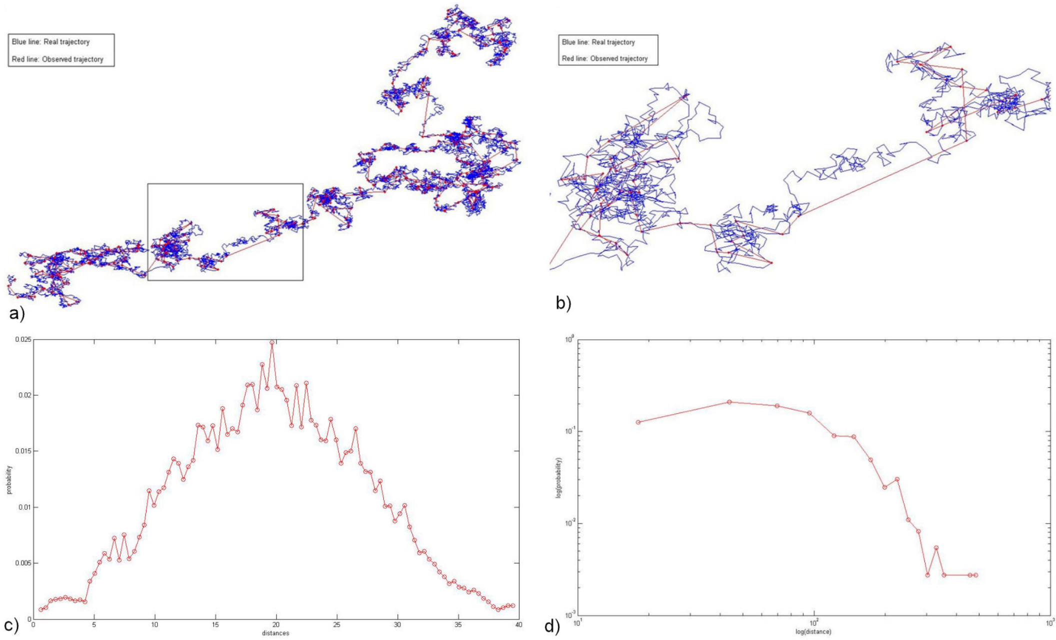

FIGURE 2. a) The trajectory of the Gaussian random walker (blue line) and the observed trajectory (red line). The red dots represent the position where observations of real trajectory were done. b) Section of the trajectories in the box of Fig. 2a with enhanced details. The red dots represent the position where observations of real trajectory were done. c) The probability density function of the steps in real trajectory. d) The probability density function of the steps in the observed trajectory.

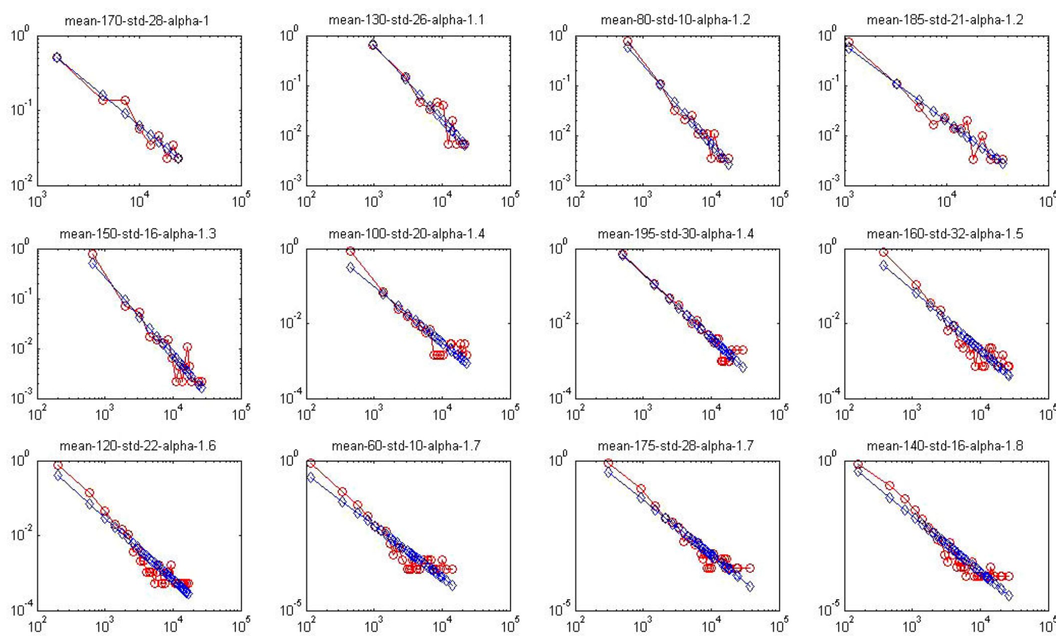

FIGURE 3. A random sample of 12 numerical experiments developed taken the probability density function of the time intervals between observations as a Levy distribution. The red lines represent the distribution of length steps and the blue line the fitted Levy distribution. See text for details.

is shown the distribution of length steps (red segments) in the observed trajectory.

The simulations were done for $\mu = 50, 55, 60, ..., 200$. For each value of μ we did simulations with $\sigma \in \{\mu/10, \mu/10+1, ..., \mu/5\}$, that is, ranging from the 10 percent of μ to the 20 percent. In all of our calculations, the length of the trajectories was n=250000. For each of the 58513 above described pairs of (μ, σ) we developed several simulations, changing the values of parameters of the selected time intervals distribution. In each of these simulations the distribution of the length steps in the real trajectory as well as the distribution of the length steps in the observed trajectory were calculated. Further, in each case we calculated the values of the parameters of a theoretical distribution that best fit the values of the observed distribution calculated from the data. Interestingly, we found the following result: if the length of the time intervals between consecutive observations is drawn from a Levy distribution, the corresponding empirical distribution of the space steps is also a Levy type. Similar results were found for a Levy distribution with exponential cut off.

3. Analysis of the results

In our study two types of distribution for the intervals between consecutive observations were used. Our selection is based on the ubiquity of these distributions in the most recent researches [6-9]. Obviously dollar bills and human beings do not behave as random walkers.

Let us denote by Δt the time interval between two consecutive observations of the random walker. The distribution of lengths of these time intervals is Levy type if $P(\Delta t) \sim \Delta t^{-\alpha}$ for some exponent α. In Ref. [6] is found that the distribution of rests between observed displacements of dollar bills fits this distribution with $\alpha \approx 1.6$.

We made simulations with $\alpha = 1$, 1.1, 1.2,..., 2. For each of these values of α we did the 58513 simulations described in Sec. 2. The distribution of the observed lengths was calculated in each case. The best fit occurred with the Levy distribution. In Fig. 3 is shown a random sample of these experiments. The title in each subfigure is **mean-X-std-Y-alpha-Z** where X stands for the mean μ of the steps distribution of the random walker, Y is its standard deviation σ and Z the value of α used in the Levy distribution of time interval lengths between observed positions. The procedure for fitting to a Levy distribution was done in logarithmic variables and the R^2 of each regression were also calculated. The distribution of these R^2 is shown in Fig. 4.

Let assume now that the distribution of the time intervals between observations is Levy type with an exponential cutoff, that is, $P(\Delta t) \sim \Delta t^{-\alpha} e^{-\tau \Delta t}$, where α and τ are both constants. The cutoff here is $1/\tau$. In Ref. [8] it is found that the distribution of the time interval Δt between two consecutive phone calls fits this distribution with $\alpha = 0.9$ and $\tau = 1/48$ days when time intervals Δt and the probability $P(\Delta t)$ are properly rescaled. Further, the authors claim that this is a universal property of the system. The above-mentioned distribution is obtained from the calling activity of 6×10^6 mobile phone users during one month.

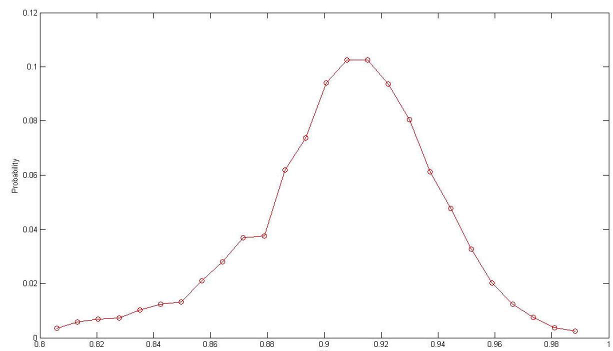

FIGURE 4. The distribution of the R^2 coefficients in the experiments shown in Fig. 3.

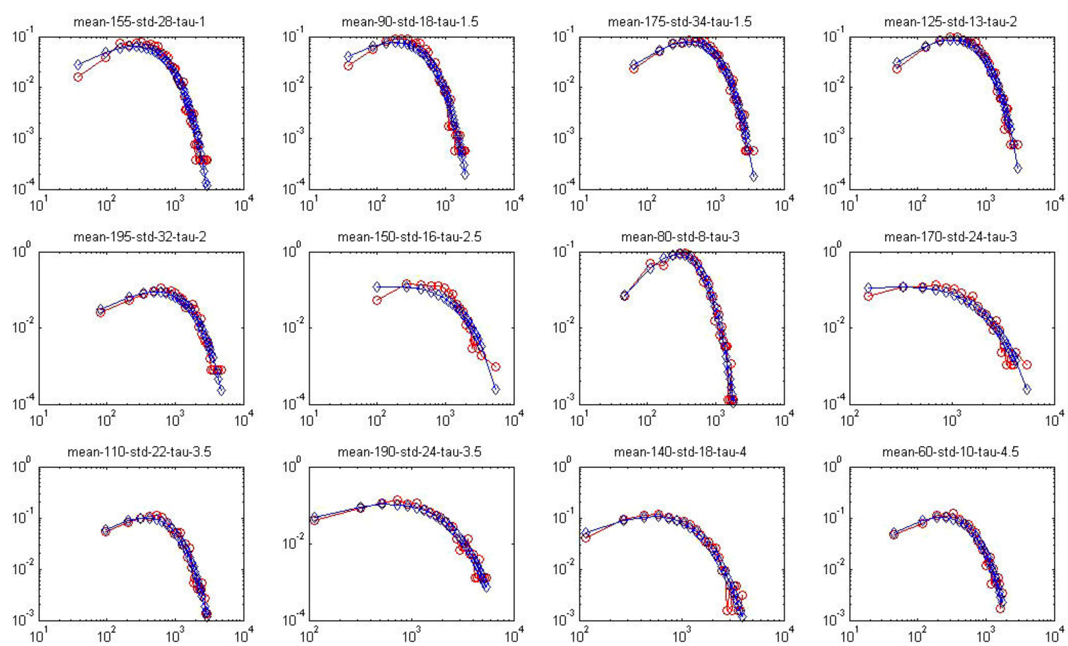

FIGURE 5. A random sample of 12 numerical experiments developed taken the probability density function of the time intervals between observations as a Levy distribution with exponential cutoff. The red lines represent the distribution of length steps and the blue line the fitted Levy distribution. See text for details.

We developed simulations with $\tau = 1$, 1.1, 1.2,..., 5. For each of these values of τ we did the 58513 simulations described in Sec. 2. The distribution of the observed lengths was calculated in each case. The best fit for the distribution of the observed lengths was obtained by a Levy distribution with an exponential cutoff. In Fig. 5 a random sample of these experiments is shown. As above; the title in each subfigure is **mean-X-std-Y-tau-Z**, where X stands for the mean μ of the steps distribution of the random walker, Y is its standard deviation σ and Z the value of τ used as exponential cutoff of the distribution of time interval lengths between observed positions. The procedure for fitting to a Levy distribution was also done in logarithmic variables and the R^2 of each regression was calculated. The distribution of the R^2 is shown in Fig. 6.

A complementary test which shows that observed trajectories do not follow a Gaussian random walk is the behavior of the mean squared displacement $\langle F^2(\Delta t)\rangle$ [4-5]. It is well known that for Gaussian random walker $\langle F^2(\Delta t) \sim \Delta t$.

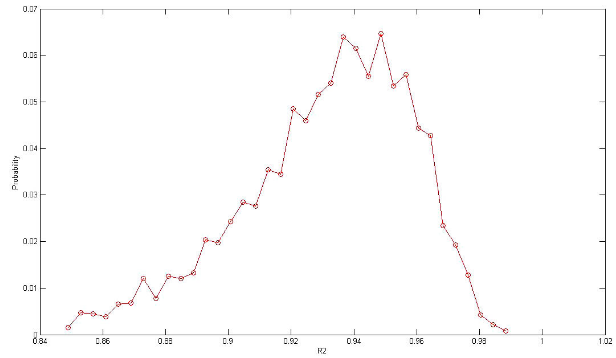

FIGURE 6. Distribution of the R^2 coefficients in the experiments shown in Fig. 5.

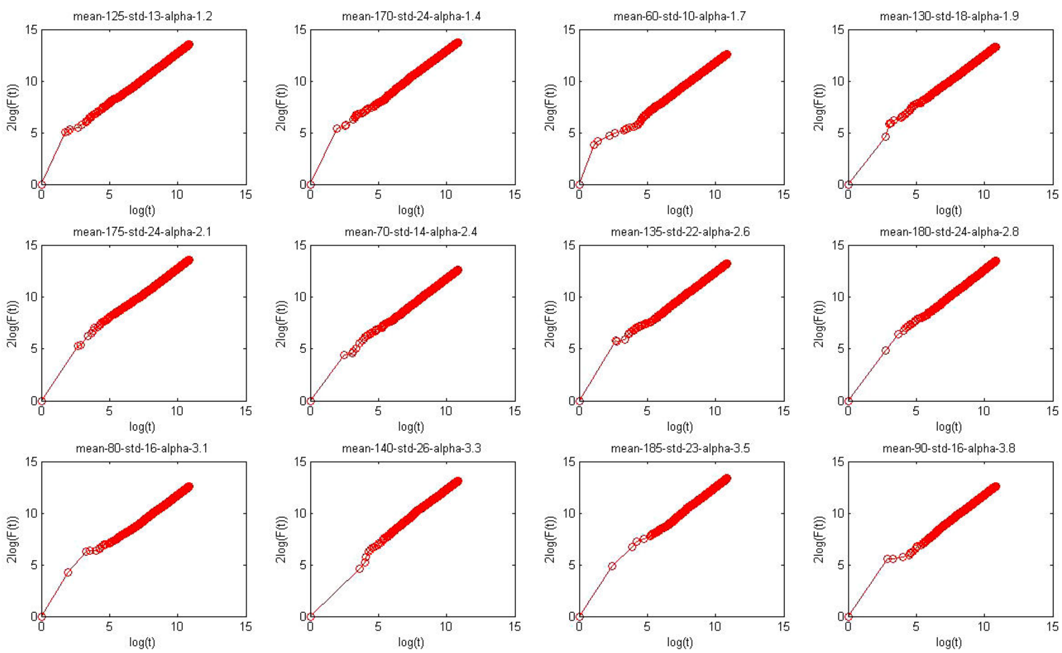

FIGURE 7. A random sample of 12 calculated $\langle F^2(\Delta t)\rangle$ in the experiments shown in Fig. 3. See text for details.

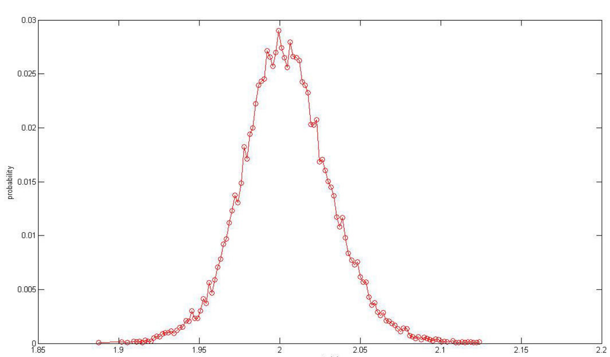

FIGURE 8. The distribution of the α exponents in the fittings $\langle F^2(\Delta t)\rangle \sim \Delta t^{\alpha}$ shown in Fig. 7.

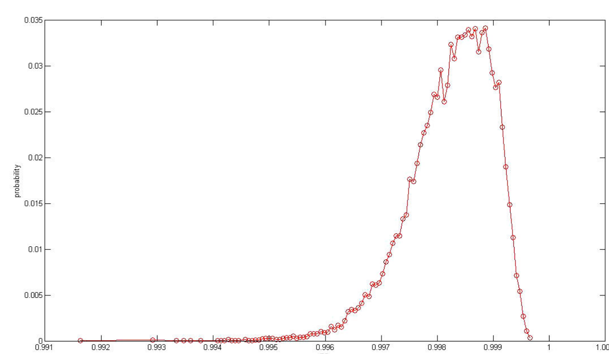

FIGURE 9. The distribution of the R^2 in the fittings shown in Fig. 7. See the text for details.

We calculated the mean squared displacement for each simulation of the first case studied, *e.g*., where the distribution of the time interval lengths between observations is Levy type. In Fig. 7 a random sample of these experiments is shown. Here the titles of the subfigures are **mean-X-std-Y-alpha-Z**, where X stands for the mean μ of the steps distribution of the random walker, Y is its standard deviation σ and Z the value of α used in the Levy type distribution of time interval lengths between observed positions. We found the following scaling $\langle F^2(\Delta t)\rangle \sim \Delta t^{\alpha}$ with $\alpha = 2 \pm 0.05$. The distribution of the exponents α can be shown in Fig. 8. The procedure for fitting $\langle F^2(\Delta t)\rangle \sim \Delta t^{\alpha}$ was also done in logarithmic variables the same scale of Fig. 7 and the R^2 of each regression was calculated. The distribution of the R^2 is shown in Fig. 9.

4. Concluding remarks

If the agents under study behave as Gaussian random walkers, our study concludes that the distribution of the length steps between observed positions is similar to the distribution of the time interval lengths between such observations, at least for the ubiquitous Levy type or Levy type with cutoff distributions. Hence an accurate recovery of the trajectories of the Gaussian random walker only could be done with a constant length interval of observation. In this case, the Central Limit Theorem assures similar parameters in the distribution.

The subordination of the observed distribution to the spatial properties of the environment where agents move have been already studied [10]. Similar results were found.

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