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A compact formula for the quantum fluctuations of energy

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A formula to calculate the quantum fluctuations of energy in small subsystems of a hot and relativistic gas is derived. We find an increase in fluctuations for subsystems of small sizes, but agrees with the energy fluctuations in the canonical ensemble if the size is large enough. Not only one can use our expression to find the limit of the concepts of energy density or fluid element in connection to relativistic heavy-ion collisions, but also in other areas of physics where one studies matter with high temperature and velocity.

Keywords: Quantum fluctuations; energy density; relativistic heavy-ion collisions.

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1. Introduction

Quantum fluctuations of physical observables, which commonly arise from quantum nature of the system [1], play a crucial role because they give out information about phase transitions [1–15], and large scale structure formation [16–22] as well as phenomena which are dissipative in nature [23–40]. This work studies the quantum fluctuations of energy in small systems (in other words, subsystems) of a hot and relativistic gas [41–45]. These fluctuations increase for small size of the subsystem, but agrees with the thermal fluctuations in the canonical ensemble if the size of the subsystem is large enough. Our results can be very helpful in determining the size of the subsystem (for specific particle mass and temperature) for which quantum energy fluctuations reach classical limit and may be ignored in such studies. We are specifically interested in the hot matter system which is believed to be produced during the relativistic heavy-ion collisions, and relativistic hydrodynamics has became a widely accepted theory to describe the physics of heavy-ion collisions [46–51]. Within the theory of relativistic hydrodynamics, we need to use the concepts of energy density and pressure in order to characterize the fluid cells locally, which give rise the question that how the definition of energy density of such a small system can be defined properly having the size of about 1 fm. In this article, we calculate and derive a formula to calculate the quantum fluctuations and then use this expression for some particular physical situations in the context of relativistic heavy-ion collisions. We use the metric convention as $g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$. Bold font is used to denote three-vectors and the scalar product of both four- and three-vectors is represented by a dot, *i.e.*, $a^\mu b_\mu = a\cdot b = a^0 b^0 - \boldsymbol{a}\cdot\boldsymbol{b}$.

2. Basic definitions

We assume a subsystem S_a of the larger thermodynamic system S_V which contains spinless bosonic particles of mass m. This system S_V is defined by the canonical ensemble and specified by the temperature T, where $\beta = 1/T$. Note that, the volume V of S_V is larger than the volume of S_a, and V is large enough for doing integrals over the momentum of the particles. We characterize our bosonic system by a quantum scalar field which is in thermal equilibrium as [52]

$$\phi(t,\boldsymbol{x}) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_{\boldsymbol{k}}}} \left(a_k^\dagger e^{-ik\cdot x} + a_{\boldsymbol{k}}^\dagger e^{ik\cdot x} \right), \quad (1)$$

with $a_{\boldsymbol{k}}$ and $a_{\boldsymbol{k}}^\dagger$ being the annihilation and creation operators, respectively, and $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$ the particle's energy. For the calculation of thermal averaging we need to know the following expectation values of the products of two and four creation and/or annihilation operators [53, 54]. Other combinations of creation and annihilation operators can be obtained easily from the following expression

$$\langle a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}'} \rangle = \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}') f(\omega_{\boldsymbol{k}}), \quad (2)$$

$$\langle a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}'}^\dagger a_{\boldsymbol{p}} a_{\boldsymbol{p}'} \rangle = \Big(\delta^{(3)}(\boldsymbol{k} - \boldsymbol{p})\, \delta^{(3)}(\boldsymbol{k}' - \boldsymbol{p}') + \delta^{(3)}(\boldsymbol{k} - \boldsymbol{p}')\, \delta^{(3)}(\boldsymbol{k}' - \boldsymbol{p}) \Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}), \quad (3)$$

where $f(\omega_{\boldsymbol{k}})$ is the Bose–Einstein distribution function, $f(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta\, \omega_{\boldsymbol{k}}] - 1)$.

Now we define [52] the operator $\mathcal{H}_a$ which characterizes the energy density of a *finite* size subsystem S_a. It is is placed at the origin of the coordinate system, where $\mathcal{H}(x) \equiv (\dot{\phi}^2 + (\nabla\phi)^2 + m^2\phi^2)/2$ is the Hamiltonian density for the free real scalar field

$$\mathcal{H}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\, \mathcal{H}(x)\, \exp\left(-\frac{\boldsymbol{x}^2}{a^2} \right). \quad (4)$$

We used above the Gaussian profile in order to remove boundary effects which might come from the sharp boundaries.

Now one can easily obtain the thermal average of $\mathcal{H}_a$ as [1]

$$\langle :\mathcal{H}_a: \rangle = \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}}\, f(\omega_{\boldsymbol{k}}) \equiv \varepsilon(T). \quad (5)$$

To obtain Eq. (5), we used normal ordering in order to remove the infinite vacuum contribution coming from the zero-point energy term. Eq. (5) is also independent of the size a reflecting the spatial uniformity of the thermodynamic system S_V. In order to calculate the quantum fluctuation of energy of the subsystem S_a we use the formula for variation and the normalized standard deviation, given respectively as

$$\sigma^2(a,m,T) = \langle : \mathcal{H}_a :: \mathcal{H}_a : \rangle - \langle : \mathcal{H}_a : \rangle^2 , \tag{6}$$

$$\sigma_n(a,m,T) = \frac{(\langle : \mathcal{H}_a :: \mathcal{H}_a : \rangle - \langle : \mathcal{H}_a : \rangle^2)^{1/2}}{\langle : \mathcal{H}_a : \rangle} . \tag{7}$$

Using Eqs. (2) and (3), we find

$$\sigma^2(a,m,T) = \int dK\, dK' f(\omega_{\boldsymbol{k}})(1+f(\omega_{\boldsymbol{k}'})) \times \Big[(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' + m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} + (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' - m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2} \Big]. \tag{8}$$

where $dK = d^3k/((2\pi)^3 2\omega_{\boldsymbol{k}})$. Here, we have neglected a temperature independent divergent term which may be related to the pure vacuum energy fluctuation [55].

Equation (8) is our main result which determine the quantum fluctuations of energy of the "Gaussian" subsystems S_a of the larger thermodynamic system S_V. Numerically we can calculate the results for any system of size a, temperature T, and mass m. These plots will be shown below.

One should also keep in mind the factor of degeneracy which is important when we study the thermodynamic properties of particles. This degeneracy factor is related to the internal degrees of freedom, for instance, spin, isospin or color charge. Therefore to take into consideration in our framework g identical particles, one needs to consider g copies of scalar field which is given by both creation and annihilation operators which commute for different species of particles. Hence, the following replacements are required for the calculation of the quantum fluctuations

$$\varepsilon \to g\varepsilon, \quad \sigma^2 \to g\sigma^2. \tag{9}$$

3. Thermodynamic limit

We believe that in the limit of large subsystem size, that is, $a \to \infty$ (but still with $a^3 \ll V$), our quantum fluctuation expression should agree with classical statistical mechanics [1]. Hence, we use the following representation of the Dirac delta function in Eq. (8)

$$\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}) = \lim_{a\to\infty} \frac{a^3}{(2\pi)^{3/2}} e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{p})^2}, \tag{10}$$

which give rise to the formula valid in the large size a limit

$$\sigma^2 \sim \frac{g}{(2\pi)^{3/2}a^3} \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}}^2\, f(\omega_{\boldsymbol{k}})(1+f(\omega_{\boldsymbol{k}})).$$

Note that we calculate the quantum fluctuation for massless particles as $\sigma^2 \sim T^5/a^3$, therefore RHS of the last equation can be written as

$$c_V = \frac{d\varepsilon}{dT} = \frac{g}{T^2} \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}}^2\, f(\omega_{\boldsymbol{k}})(1+f(\omega_{\boldsymbol{k}})), \tag{11}$$

where c_V is the specific heat at constant volume. Hence, in the large a limit [56]

$$V_a\sigma_n^2 = \frac{T^2 c_V}{\varepsilon^2} = V\frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2} \equiv V\sigma_H^2, \tag{12}$$

with $V_a = a^3(2\pi)^{3/2}$ and H being the Hamiltonian of S_V. The RHS of Eq. (12) is the normalized energy fluctuation in the thermodynamical system S_V [1]. Here, $V_a = a^3(2\pi)^{3/2}$ is the volume of S_a.

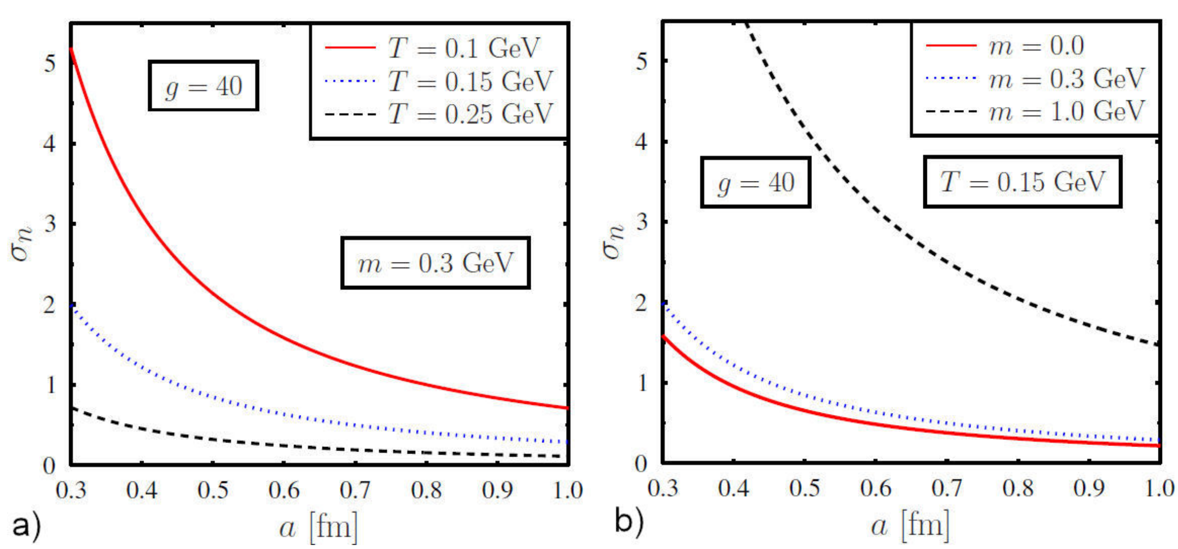

FIGURE 1. a) Variation of the normalized energy density fluctuation σ_n in the subsystem S_a with size a for various values of the temperature T but with fixed particle mass $m = 0.3$ GeV. b) Same as left figure but for fixed temperature $T = 0.15$ GeV and different particles masses.

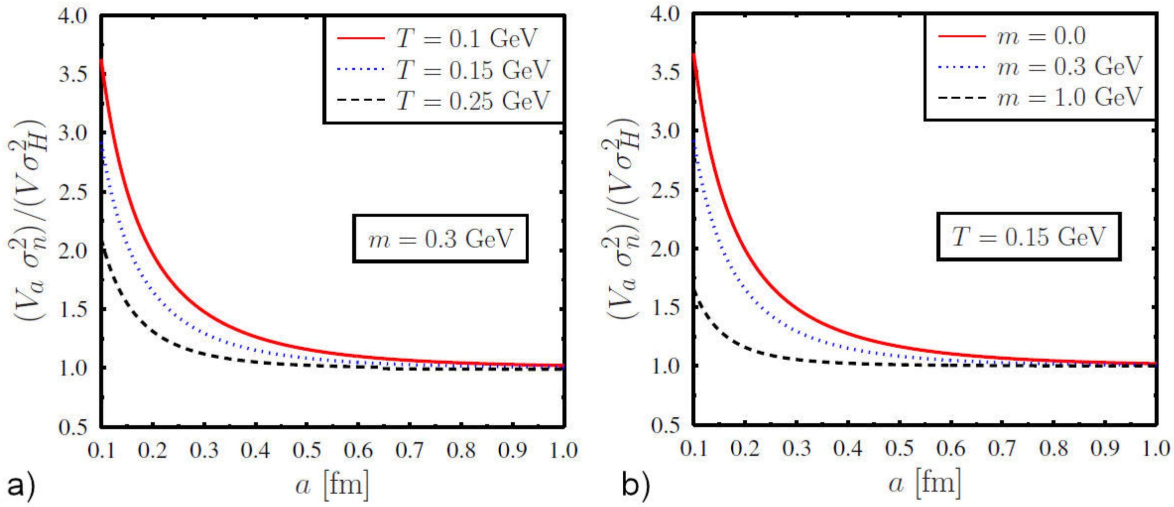

FIGURE 2. a) Variation of the normalized energy fluctuation in the subsystem S_a with the size a for various values of the temperature at $m = 0.3$ GeV. b) Same as left figure but for different values of mass at $T = 0.15$ GeV.

4. Numerical results

Since we correctly reproduced the thermodynamic limit for $a \to \infty$, we can do some numerical calculations. Figure 1 shows the variation of the normalized energy density fluctuation σ_n having the size a for the subsystem with various values of temperature and mass of the system. We use degeneracy factor to be $g = 40$ in order to be consistent with the heavy-ion experiments [57]. Figure 1 indicate that σ_n decreases with increasing the system size a as expected from the behavior of fluctuations. Since, from Eq. (8) we see that σ_n explodes in the limit $a \to 0$ hence we start our numerical calculations at $a = 0.3$ fm. Having fixed the particle mass we find a decrease in the normalized fluctuations while increasing temperature. On the other hand, at fixed T the fluctuations grow with m.

Figure 2 presents the variation of $V_a\sigma_n^2/V\sigma_H^2$ with the subsystem size a for particles with mass m obeying Bose-Einstein statistics. From Eq. (12) we see that it approaches the thermodynamic limit [58] and hence $V_a\sigma_n^2/V\sigma_H^2$ should go to unity, which is seen from Fig. 2.

5. Conclusions

In this work we have derived an expression for the quantum fluctuation of energy for the subsystems of a hot and relativistic gas which beautifully agrees with the thermodynamic fluctuation expression for the large system size. We also explained and outlined the possible consequences of our results for the description of relativistic heavy-ion systems.

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