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Study of heavy bottom mesons in a variational approach

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Using a variational approach we calculate the mass spectra of the bottom mesons in the framework of the phenomenological quark anti-quark potential. Decay constant and Leptonic decay widths of the bottom mesons are also calculated. The obtained results are compared with the available experimental data and other theoretical predictions.

Keywords: Potential model; non-relativistic limit; bottom mesons; decay constant.

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1. Introduction

Recently, significant experimental progress has been achieved regarding the discoveries of the structures and decay properties of heavy mesons [1–4]. Many theoretical attempts are also proposed to explain the heavy meson properties [5–9]. No experimental data of the decay constants of B mesons is Many theoretical attempts are also. However, a variety of approaches have been applied to investigate the B mesonic decays [10–13]. Bottom meson decays to leptons are of renewed interest in hadron physics that gives us good information about the internal structure of heavy meson states. Non-relativistic quark model have been quite successful in describing the heavy quark dynamics [14–16]. In this paper we present a non-relativistic formalism to study the heavy meson states containing one (or two) bottom quark(s).

The main aim of the current work is the study of the leptonic B to l mesonic decay constants and widths. In Sec. 2 we introduce our potential model and solve the two-body Schrödinger equation of the mesonic system using a simple variational method and then obtain the eigenfunctions and the corresponding energy eigenvalues of the heavy states. In Sec. 3 our calculations for bottom meson masses are listed. Leptonic decay constants, decay widths, branching ratios and also a comparison with other theoretical models are presented in Sec. 4.

2. Theoretical framework

We use a phenomenological potential model to estimate the meson properties. The interaction Hamiltonian for the quark-antiquark interaction is given by

$$H(r) = V(r) + H_S(r), \tag{1}$$

where the potential $V(r)$ consists of two parts:

$$V(r) = \lambda r - \frac{\beta}{r}. \tag{2}$$

The first term is a linear potential that confines the quarks inside the hadron and the second term is a color Coulomb-like potential due to one-gluon exchange processes. λ and β are constants and are determined in our phenomenological models. The variable r is the relative quark-antiquark coordinate. The hyperfine interaction is given by:

$$H_S = As(\vec{s}_q \cdot \vec{s}_{\bar{q}}), \tag{3}$$

where s_q is the quark spin operator and A_s is a constant. In the heavy mesons, we have at least one heavy quark and then, the iso-spin interaction terms are equal to zero.

Since the introduced potential depends on r only, the two-body Schrödinger equation for the ground-state, $l = 0$, is given by

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right]\psi(r) = -2m[E - V(r)]\psi(r), \tag{4}$$

where $\psi(r)$ is the radial wave function and m is the reduced mass

$$m = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}, \tag{5}$$

where m_q and $m_{\bar{q}}$ are the quark and antiquark masses respectively. The transformation

$$R(r) = r\psi(r), \tag{6}$$

reduces Eq. (4) to the form

$$\frac{d^2}{dr^2}R(r) + \left[2mE - 2m\lambda r + \frac{\beta}{r}\right]R(r) = 0. \tag{7}$$

The Schrödinger equation (7) has no analytical solution.

For the ground state mesons, $n = l = 0$, we use the variational method and choose the trial wave function as

$$R(p, r) = Np^{3/2} r e^{-pr}, \tag{8}$$

where p is the variational parameter and N is the normalization constant. Now the variational parameter p must be defined. Using the usual condition

$$\frac{\partial}{\partial p}\left(\frac{\langle R(p,r)|H|R(p,r)\rangle}{\langle R(p,r)|R(p,r)\rangle}\right) = 0, \tag{9}$$

we minimize the energy of the system and get the variational parameters for the heavy meson states and then the energy eigenvalues and corresponding eigenfunctions of the systems can be obtained. The values of p parameter are different for each meson. Therefore, we have different wave functions for heavy mesonic states containing different quark flavors.

TABLE I. The model parameters.

Parameter	Value
m_u	330 MeV
m_s	460 MeV
m_c	1550 MeV
m_b	4980 MeV
α	2.1 fm^{-2}
β	1.27
A_s	48 MeV

TABLE II. Comparison of our results for the bottom meson masses and experimental data [20] (in MeV).

Meson	Theory	Experiment	Meson	Theory	Experiment
B	5277	5279	B_c	6274	6274
B^*	5325	5325	B_c^*	6323	-
B_s	5369	5366	η_b	9431	9400
B_s^*	5417	5415	Υ_b	9478	9460

3. Meson masses

By using the energy eigenvalues and eigenfunctions we can calculate the bottom mesons masses. M_{meson} is obtained as a combination of the energy of the systems and the constituent quark masses [17]:

$$M_{\text{meson}} = m_q + m_{\bar{q}} + E_{q\bar{q}} + \langle H_S \rangle, \tag{10}$$

$E_{q\bar{q}}$ is the energy eigenvalue calculated in Sec. 2. The expectation value of the perturbed energy of the system can be obtained using the unperturbed wave function obtained in Ref. 8.

The model parameters used in our model are listed in Table I. The quark masses m_u, m_s and m_b are taken from our previous works [18,19] and the value of the rest potential parameters are chosen such that we get the best meson spectra. The obtained values of p parameter for B and B_s mesons are 2.68 and 3.32 respectively.

In Table II our results for the bottom meson masses are compared to the existing experimental data [20].

4. Decay constant and Branching ratios of $B^+ \to l^+\nu$ decay

The decay Constants are important parameters in the study of leptonic and non-leptonic meson decay properties and calculating the branching ratios.

In the non-relativistic limit, the decay constant can be obtained by using the following formula [11,21]

$$f_p^2 = \frac{12|\psi_p(0)|^2}{M_p}, \tag{11}$$

TABLE III. The decay constants of pseudoscalar B mesons (in GeV).

Constant	Present work	[21]	[22]
f_B	0.149	0.189	0.189
f_{Bs}	0.208	0.218	0.230
$\frac{f_{Bs}}{f_B}$	1.395	1.153	1.218

TABLE IV. The leptonic branching ratios for B meson.

Mode	Present work	[23]	[24]
$B^+ \to \tau^+\nu$	0.58×10^{-4}	1.8×10^{-4}	$1.8 \pm 0.5 \times 10^{-3}$
$B^+ \to \mu^+\nu$	2.61×10^{-7}	$< 11 \times 10^{-6}$	$< 1.0 \times 10^{-4}$
$B^+ \to e^+\nu$	6.17×10^{-12}	$< 7.7 \times 10^{-6}$	$< 1.9 \times 10^{-6}$

where M_p is the pseudoscalar meson masses as obtained in Eq. (2) and the square of the wave function at the origin is defined as

$$|\psi_p(0)|^2 = \frac{m}{2\pi\hbar^2}\left\langle \frac{dV(r)}{dr} \right\rangle. \tag{12}$$

The expectation value of $\langle dV(r)/dr \rangle$ can be obtained by using the wave function introduced in Eq. (8). In our model the obtained values for $|\psi_p(0)|^2$ are equal to 0.0097 and 0.0196 (in GeV^3) for pseudoscalar B and B_s respectively. The decay constants of pseudoscalar B mesons are listed in Table III and compared to the the other predictions [21,22].

The total leptonic decay widths for $B^+ \to l^+\nu$ can be written

$$\Gamma = \frac{G_F^2 |V_{bu}|^2 f_p^2}{8\pi} m_l^2 \left(1 - \frac{m_l^2}{M_P^2}\right)^2, \tag{13}$$

where $|V_{bu}| = 3.89 \times 10^{-3}$ is CKM matrix element and G_F is the Fermi constant.

The used lepton masses are as follows

$$m_\tau = 1.776 \text{ GeV}, \qquad m_\mu = 0.105 \text{ GeV},$$
$$m_e = 0.510 \times 10^{-3} \text{ GeV}.$$

Using Eq. (13) and the meson masses obtained in Sec. 3, one can get the leptonic decay widths.

For $B^+ \to \tau^+\nu$ decay we get $\Gamma = 2.37 \times 10^{-17} s^{-1}$. For $B^+ \to \mu^+\nu$ and $B^+ \to e^+\nu$ decays we also get $\Gamma = 1.05 \times 10^{-19} s^{-1}$ and $\Gamma = 2.49 \times 10^{-24} s^{-1}$ respectively.

We can get the branching ratio (Br) of heavy-light meson decays as follows:

$$Br = \Gamma\tau. \tag{14}$$

Using the obtained decay widths and meson mean lifetime τ we can calculate the corresponding branching ratios for B mesons. To obtain the branching ratios, we use the PDG experimental value for the mean life time of the B meson ($\tau_B = 1.630 ps$). Our results for the leptonic branching ratios for B mesons are listed in Table IV and compared to the the results of Refs. [23,24].

5. Conclusions

In this work, we have made an analysis of B meson states. We have calculated the masses of the ground-state heavy bottom mesons in the framework of a non-relativistic formalism with the lager potential as well as spin-spin interaction. The variational approach is used to solve the two-body equation of the meson systems. The spin dependent potential is considered as perturbation. Using the perturbed and unperturbed energies we could obtain the masses of bottom mesons. Finally we obtained the square of the wave function at the origin, decay constants, decay widths and branching ratios for leptonic decays of B mesons. In most cases our results and those of the mentioned references are in good agreement. Our phenomenological potential model is simple and useful to study the quark dynamics of the heavy hadron states. Future experimental studies of the B mesonic decays will be significantly helpful for the test and validation of our model.

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