

Fermionic DIS from a deformed string/gauge correspondence model

E. Folco Capossoli^a, M. A. Martín Contreras^b, D. Li^c, A. Vega^b, and H. Boschi-Filho^d

^a*Departamento de Física / Mestrado Profissional em Práticas de Educação Básica (MPPEB),
Colégio Pedro II, 20.921-903 - Rio de Janeiro-RJ - Brazil.*

**e-mail: eduardo_capossoli@cp2.g12.br*

^b*Instituto de Física y Astronomía, Universidad de Valparaíso, A. Gran Bretaña 1111, Valparaíso, Chile.*

^c*Department of Physics and Siyuan Laboratory, Jinan University, Guangzhou 510632, China.*

^d*Instituto de Física, Universidade Federal do Rio de Janeiro, 21.941-972 - Rio de Janeiro-RJ - Brazil.*

Received 15 December 2021; accepted 20 April 2022

From a deformed AdS_5 space, we used the string/gauge duality to study the deep inelastic scattering for unpolarized fermions with spin 1/2, considering the large Bjorken x parameter regime. Here, we also took into account an anomalous dimension of an operator which represents fermions at the boundary. From this analysis, we compute the corresponding structure functions, which are dependent on x and on the photon virtuality q . The results achieved are in agreement with the experimental data.

Keywords: Deformed string/gauge correspondence; fermions; deep inelastic scattering; structure functions.

DOI: <https://doi.org/10.31349/SuplRevMexFis.3.0308076>

1. Introduction

Deep Inelastic Scattering (DIS) is an important tool in order to promote the fully understanding of the hadrons structure and its composition. As DIS belongs to the realm of strong interactions the appropriate theory to deal with it is the $SU(3)$ gauge field theory called as QCD. From the beginning of the QCD itself some important works, for instance in Refs. [1, 2], relate that the insertion of an anomalous contribution γ can change the canonical dimension Δ_{can} of an operator \mathcal{O} , such as, $[\mathcal{O}] = \Delta_{\text{can}} + \gamma$ and relate this anomalous dimension with the DIS.

In this work, we will explore the fact the perturbative techniques applied to QCD do not work well at low energy regimes, and then, we will study DIS from an alternative perspective known as anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence. This correspondence relates superstring theory in a 10-dimensional curved spacetime to a conformal $\mathcal{N} = 4$ super Yang-Mills theory (SYM) with symmetry group $SU(N)$, for $N \rightarrow \infty$, living in 3+1 dimensional Minkowski spacetime. For an important review, one can see Ref. [3].

This correspondence becomes fully applicable after breaking the conformal symmetry. By doing this, one can build a phenomenological holographic approach known as AdS/QCD. In this field, one has many important works dealing with DIS and bringing new understandings and results. This can be seen for instance in Refs. [4–12]. This set of references includes the studies of DIS for scalar, mesonic and baryonic particles. The important Ref. [4] dealt with the holographic DIS within hardwall model for scalars and fermions considering large, small and exponentially small Bjorken parameter x scenarios. Vector particle DIS was studied, for instance in Ref. [9], whilst in Refs. [6–8, 10] studied

DIS for baryons.

Here, throughout AdS/CFT duality, using a deformed background holographic model we will calculate the proton structure functions, in the large x regime, $F_1 = F_1(x, q^2)$ and $F_2 = F_2(x, q^2)$. These structure functions are written as a function of the photon virtuality q^2 , and the Bjorken parameter x .

In particular, our AdS/QCD model with a deformed AdS_5 background breaks the conformal symmetry producing a mass gap for the fermionic fields. Deformed holographic models were used for several purposes within AdS/QCD context, as can be seen in Refs. [13–20].

This work is organized as follows: in Sec. 2 we describe the DIS phenomenology. In Sec. 3 we present our deformed AdS space model where we compute the electromagnetic and fermionic fields. In Sec. 4 we calculate the DIS interaction action and derive the corresponding structure functions $F_1(q^2, x)$ and $F_2(q^2, x)$. In Sec. 6 we present our numerical results for the structure functions and compare them with available experimental data. In Sec. 6 we present our conclusion.

2. Deep Inelastic Scattering phenomenology

This kind of scattering processes occurs in high-energy regimes and it is achieved by colliding a lepton (ℓ) and a proton (p), through the exchange of a virtual photon, which is represented by $\ell p \rightarrow \ell X$. The many fragments produced are represented by X .

In the context of the scattering theory, DIS is ruled by some parameters, such as the photon virtuality q and the Bjorken variable x , given by:

$$x = -\frac{q^2}{2P \cdot q}, \quad (1)$$

where P is the initial proton momentum, defined as $P^2 = -M^2$.

The DIS scattering amplitude can be written as:

$$i \mathcal{M}_{l p \rightarrow l X} \propto \int d^4 y e^{i q \cdot y} \langle X | J^\mu(y) | P \rangle, \quad (2)$$

where the electromagnetic current associated to quark is represented by $J_\mu(x)$.

By using the optical theorem one can write the transition amplitude $W^{\mu\nu}$ or *the forward Compton scattering amplitude* as a function of the forward matrix element of two proton currents averaged over the spin, so that:

$$W^{\mu\nu} = \frac{i}{4\pi} \sum_s \int d^4 y e^{i q \cdot y} \langle P, s | T \{ J^\mu(y) J^\nu(0) \} | P, s \rangle, \quad (3)$$

where $|P, s\rangle$ are the normalizable proton states considering it with spin s . For a review one can see Ref. [21].

By taking the imaginary part of the transition amplitude, written in a Fourier space, and taking into account the following condition $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$, the hadronic tensor has a decomposition given by:

$$W^{\mu\nu} = F_1 \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \frac{2x}{q^2} F_2 \left(P^\mu + \frac{q^\mu}{2x} \right) \left(P^\nu + \frac{q^\nu}{2x} \right). \quad (4)$$

The scalar functions $F_{1,2} \equiv F_{1,2}(x, q^2)$ are the DIS Structure Functions that we are going to compute holographically. Besides for our purpose, we are going to consider only unpolarized leptons and target protons.

3. The deformed string/gauge correspondence model and the DIS

The deformed string/gauge model, used in this work, can be considered as a variation of the original AdS/CFT correspondence. In the deformed version, we have introduced an exponential factor e^{kz^2} in the AdS₅ metric. This deformation suggests that we should take into account an anomalous dimension in this approach.

In our model, the bulk field takes the following form:

$$S = \int d^5 x \sqrt{-g} \mathcal{L}, \quad (5)$$

where \mathcal{L} is the Lagrangean density, and g is the determinant of the metric g_{mn} of the deformed AdS₅ space, given by:

$$ds^2 = g_{mn} dx^m dx^n = e^{2A(z)} (dz^2 + \eta_{\mu\nu} dy^\mu dy^\nu). \quad (6)$$

Note that we set the AdS radius $R = 1$.

$$A(z) = -\log z + \frac{k}{2} z^2, \quad (7)$$

with z the holographic coordinate and where the constant k has dimension of GeV² which is associated with a QCD mass

scale. Here, we take m, n, \dots to refer to the AdS₅ space. We also separate into μ, ν, \dots for the Minkowski spacetime and the holographic z coordinate. The metric $\eta_{\mu\nu}$ has signature $(-, +, +, +)$ and will describe the gauge theory (boundary) of the deformed AdS space.

It is worthwhile to mention that the deformed string/gauge model presented here is different from the original softwall model (SWM) [22]. Here, in the deformed model the conformal factor is introduced in the AdS₅ metric instead in the action as in the SWM. As a consequence of this, the deformed model brakes fermion's conformal invariance producing discrete spectrum for fermions whilst SWM does not.

The deformed string/gauge correspondence model is based on Refs. [13]. The same deformation in the AdS₅ was applied to compute the spectrum of several particles, with various spins, also including spin 1/2 fermions [18]. This fermionic computation is closely related to the present discussion of DIS with fermionic target.

The holographic description of DIS used in this work is inspired by Ref. [4] considering large x regime or in the supergravity approximation. From such a description, the AdS/CFT duality connects the matrix element of DIS presented in Eq. (4) with the supergravity interaction action in AdS space, S_{int} , so that:

$$\eta_\mu \langle P + q, s_X | J^\mu(0) | P, s \rangle = S_{\text{int}}, \quad (8)$$

here, we are considering that the fermionic particle was scattered off by a virtual photon with polarization η_μ . On the other hand, the interaction action reads:

$$S_{\text{int}} = g_V \int dz d^4 y \sqrt{-g} \phi^\mu \bar{\Psi}_X \Gamma_\mu \Psi_i, \quad (9)$$

whith g_V a coupling constant which depends on the electric charge of the fermion and Γ_μ are Dirac gamma matrices placed in curved space. ϕ^μ is the massless electromagnetic gauge boson field and the spinors Ψ_i and Ψ_X are the initial and final states for the fermion.

3.1. Calculating the electromagnetic field

In this section we will present the massless gauge field in the deformed AdS space. It will represent the virtual photon at UV, responsible the electromagnetic interaction, needed in DIS.

The action for a five dimensional massless gauge field ϕ^n is written as:

$$S = - \int d^5 x \sqrt{-g} \frac{1}{4} F^{mn} F_{mn}, \quad (10)$$

where $F^{mn} = \partial^m \phi^n - \partial^n \phi^m$ and the following equations of motion, given by $\partial_m [\sqrt{-g} F^{mn}] = 0$. By Considering the gauge fixing as follows:

$$\partial_\mu \phi^\mu + e^{-A} \partial_z (e^A \phi_z) = 0, \quad (11)$$

where $A = A(z)$ is given by Eq. (7), one gets after some algebra

$$\phi_\mu(z, q) = -\frac{\eta_\mu e^{iq \cdot y}}{2} B(z, q), \quad (12)$$

where $\Gamma[a]$ is the Gamma function and $B(z, q) \equiv \mathcal{U}(1 - q^2/2k; 2; -kz^2/2)$ is the Tricomi hypergeometric function.

3.2. Calculating the fermionic states

The action for the fermionic fields in the deformed AdS space is given by [28]:

$$S = \int d^5x \sqrt{g} \bar{\Psi} (\not{D} - m_5) \Psi, \quad (13)$$

From this action one can derive the corresponding equation of motion given by $(\not{D} - m_5)\Psi = 0$, where \not{D} is defined as:

$$\not{D} \equiv e^{-A(z)} \gamma^5 \partial_5 + e^{-A(z)} \gamma^\mu \partial_\mu + 2A'(z) \gamma^5, \quad (14)$$

where $\partial_5 \equiv \partial_z$, and m_5 is the baryon bulk mass. Now, one can decompose the spinor Ψ into right- and left-handed chiral components, and applying some math, one has

$$-\psi''_{R/L}(z) + [V_{R/L}(z)] \psi_{R/L}(z) = M_n^2 \psi_{R/L}^n(z). \quad (15)$$

Note that the above equation represents a Schrödinger equation which considers right and left sectors. Also note that M_n is the boundary baryon mass for each mode $\psi_{R/L}^n$ and the corresponding potentials is written as:

$$V_{R/L}(z) = m_5^2 e^{2A(z)} \pm m_5 e^{A(z)} A'(z). \quad (16)$$

By using the solutions provided by Eq. (15) one can read the final spinor state Ψ_X and the initial spinor state Ψ_i . These are linear combinations of the chiral solutions $\psi_{R/L}$. Besides s_i and s_X are the spin of the initial and final states, respectively.

Regarding the bulk mass, one can relate it to the conformal dimension Δ_{can} of a boundary operator \mathcal{O} . In pure AdS₅ space, one has:

$$|m_5^{\text{AdS}}| = \Delta_{\text{can}} - 2. \quad (17)$$

Within the holographic context the concept of the anomalous dimension has appeared in some studies, for instance, Refs. [23–27]. Taking into account those cases, the effective conformal dimension is $\Delta_{\text{eff}} = \Delta_{\text{can}} + \gamma$, and then, one can write $|m_5| = \Delta_{\text{can}} + \gamma - 2$, where γ is the anomalous contribution.

In this work we will resort to the anomalous dimension to calculate the structure functions in fermionic DIS. Besides, we will follow [29] where one can see that if we are dealing with the Bjorken variable $x \rightarrow 1$ (large x regime) the parton distribution functions (PDFs) seem to indicate that the proton can be considered as a single particle (disregarding the internal constituents). This is applied in Ref. [10], with $\Delta_{\text{can}} = 3/2$.

4. The DIS interaction action

Here, we will compute the DIS interaction action by using Eqs. (8) and (9), together with the final and the initial spinor states Ψ_X and Ψ_i , respectively, so that:

$$S_{\text{int}} = \frac{gV}{2} (2\pi)^4 \delta^4(P_X - P - q) \eta^\mu \times \left[\bar{u}_{s_X} \gamma_\mu \hat{P}_R u_{s_i} \mathcal{I}_L + \bar{u}_{s_X} \gamma_\mu \hat{P}_L u_{s_i} \mathcal{I}_R \right], \quad (18)$$

where the $\mathcal{I}_{R/L}$ are defined in terms of the solutions of the chiral fermions and the solution of the field B , so that:

$$\mathcal{I}_{R/L} = \int dz B(z, q) \psi_{R/L}^X(z, P_X) \psi_{R/L}^i(z, P). \quad (19)$$

As we interested in a spin independent scenario, and by summing over spin, one can write the structure functions, such as:

$$F_1(q^2, x) = \frac{g_{\text{eff}}^2}{4} \left[M_0 \sqrt{M_0^2 + q^2} \left(\frac{1-x}{x} \right) \mathcal{I}_L \mathcal{I}_R + (\mathcal{I}_L^2 + \mathcal{I}_R^2) \left(\frac{q^2}{4x} + \frac{M_0^2}{2} \right) \right] \frac{1}{M_X^2}, \quad (20)$$

$$F_2(q^2, x) = \frac{g_{\text{eff}}^2}{8} \frac{q^2}{x} (\mathcal{I}_L^2 + \mathcal{I}_R^2) \frac{1}{M_X^2}, \quad (21)$$

where $M_X \equiv M_X(q^2, x)$ is mass of the effective final hadron related to the mass of the initial hadron:

$$M_X^2(q^2, x) = M_0^2 + q^2 \left(\frac{1-x}{x} \right). \quad (22)$$

5. Numerical results for the structure functions

Here, we will summarize our numerical results related to the structure functions $F_1(x, q^2)$ and $F_2(x, q^2)$, and considering some values of the Bjorken parameter, such as, $x = 0.65, 0.75, 0.85$. Besides, we compare our results with the available experimental data, as presented in Ref. [30], from SLAC and in Ref. [31], from BCDMS.

In order to get our results, we solved numerically Eqs. (15) and (21). In the figure below one can see the main results achieved in this work. This figure shows the structure function $F_2(x, q^2)$ as a function of q^2 considering $x = 0.65$ with $m_5 = 0.505$ GeV, $k = 0.374$ GeV², $g_{\text{eff}}^2 = 1.83$ and $\gamma = 0.005$; $x = 0.75$ with $m_5 = 0.565$ GeV, $k = 0.340$ GeV², $g_{\text{eff}}^2 = 1.65$ and $\gamma = 0.065$, and $x = 0.85$ with $m_5 = 0.878$ GeV, $k = 0.196$ GeV², $g_{\text{eff}}^2 = 1.83$ and $\gamma = 0.378$.

Note that these values of m_5 , k , g_{eff}^2 and γ for each value of x assure that the computed proton mass, from Eq. (15), is $m_p = 0.938$ GeV.

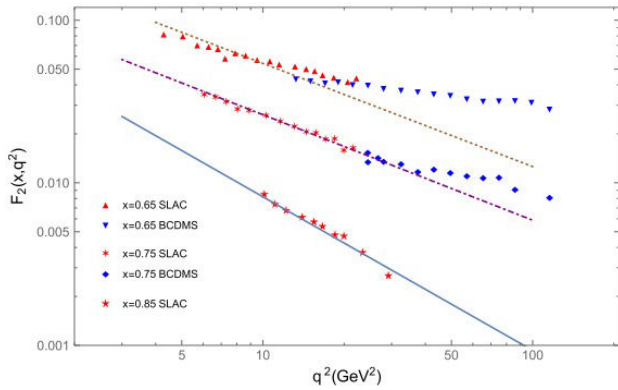


FIGURE 1. Here we are presenting our results for the structure function $F_2(x, q^2)$ in terms of q^2 . Also in this figure, we have considered the cases for $x = 0.65$, $x = 0.75$ and $x = 0.85$ and its corresponding theoretical fitting represented by the dotted, dot-dashed, and solid lines, respectively. Notice that we also brought in this figure the experimental data from SLAC [30] and BCDMS [31].

6. Conclusions

In this section we will present our last comments on our holographic description for the DIS structure functions $F_1(x, q^2)$ and $F_2(x, q^2)$ for baryons. The present work is based on a deformed AdS metric and takes into account an anomalous

contribution to the canonical conformal dimension of a CFT operator.

The results for $F_2(x, q^2)$ when compared with ones coming from experimental data are in agreement. In particular, for large x (or $x = 0.85$) regime.

The complete calculations done in this work, including all details, can be seen in Ref. [32].

Acknowledgments

We would like to thank Alfonso Ballon Bayona for useful discussions. E.F.C. and M.A.M.C would like to thank the hospitality of the Jinan University where part of this work was done. E.F.C also would like to thank the hospitality of the Valparaíso University where part of this work was done. A. V. and M. A. M. C. would like to thank the financial support given by FONDECYT (Chile) under Grants No. 1180753 and No. 3180592, respectively. D.L. is supported by the National Natural Science Foundation of China (11805084), the PhD Start-up Fund of Natural Science Foundation of Guangdong Province (2018030310457) and Guangdong Pearl River Talents Plan (2017GC010480). H.B.-F. is partially supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under Grant No. 311079/2019-9.

1. H. Georgi and H. Politzer, Electroproduction scaling in an asymptotically free theory of strong interactions, *Phys. Rev. D* **9** (1974) 416, <https://doi.org/10.1103/PhysRevD.9.416>.
2. D. Gross, S. Treiman and F. A. Wilczek, Mass Corrections in Deep Inelastic Scattering, *Phys. Rev. D* **15** (1977) 2486, <https://doi.org/10.1103/PhysRevD.15.2486>.
3. O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Large N field theories, string theory and gravity, *Phys. Rept.* **323** (2000) 183, [https://doi.org/10.1016/S0370-1573\(99\)00083-6](https://doi.org/10.1016/S0370-1573(99)00083-6). [arXiv:hep-th/9905111 [hep-th]].
4. J. Polchinski and M. J. Strassler, Deep inelastic scattering and gauge / string duality, *JHEP* **0305** (2003) 012, <https://doi.org/10.1088/1126-6708/2003/05/012>.
5. C. A. Ballon Bayona, H. Boschi-Filho and N. R. F. Braga, Deep inelastic structure functions from supergravity at small x , *JHEP* **0810** (2008) 088, <https://doi.org/10.1088/1126-6708/2008/10/088>.
6. C. A. Ballon Bayona, H. Boschi-Filho and N. R. F. Braga, Deep inelastic scattering from gauge string duality in the soft wall model, *JHEP* **0803** (2008) 064, [arXiv:0711.0221 [hep-th]].
7. J. H. Gao and B. W. Xiao, Polarized Deep Inelastic and Elastic Scattering From Gauge/String Duality, *Phys. Rev. D* **80** (2009) 015025, <https://doi.org/10.1103/PhysRevD.80.015025>.
8. J. H. Gao and Z. G. Mou, Polarized Deep Inelastic Scattering Off the Neutron From Gauge/String Duality, *Phys. Rev. D* **81** (2010) 096006, <https://doi.org/10.1103/PhysRevD.81.096006>.
9. C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga and M. A. C. Torres, Deep inelastic scattering for vector mesons in holographic D4-D8 model, *JHEP* **1010** (2010) 055, [arXiv:1007.2448 [hep-th]].
10. N. R. F. Braga and A. Vega, Deep inelastic scattering of baryons in a modified soft wall model, *Eur. Phys. J. C* **72** (2012) 2236, [arXiv:1110.2548 [hep-ph]].
11. J. H. Gao and Z. G. Mou, Structure functions in deep inelastic scattering from gauge/string duality beyond single-hadron final states, *Phys. Rev. D* **90** (2014) 075018, <https://doi.org/10.1103/PhysRevD.90.075018>.
12. E. Folco Capossoli and H. Boschi-Filho, Deep Inelastic Scattering in the Exponentially Small Bjorken Parameter Regime from the Holographic Softwall Model, *Phys. Rev. D* **92** (2015) 126012, <https://doi.org/10.1103/PhysRevD.92.126012>.
13. O. Andreev and V. I. Zakharov, Heavy-quark potentials and AdS/QCD, *Phys. Rev. D* **74** (2006) 025023, <https://doi.org/10.1103/PhysRevD.74.025023>.
14. S. Afonin, Generalized Soft Wall Model, *Phys. Lett. B* **719** (2013) 399, <https://doi.org/10.1016/j.physletb.2013.01.055>.

15. M. Rinaldi and V. Vento, Scalar and Tensor Glueballs as Gravitons, *Eur. Phys. J. A* **54** (2018) 151, <https://doi.org/10.1140/epja/i2018-12600-9>.
16. R. C. L. Bruni, E. Folco Capossoli and H. Boschi-Filho, Quark-antiquark potential from a deformed AdS/QCD, *Adv. High Energy Phys.* **2019** (2019) 1901659, <https://doi.org/10.1155/2019/1901659>.
17. S. Afonin and A. Katanaeva, Glueballs and deconfinement temperature in AdS/QCD, *Phys. Rev. D* **98** (2018) 114027, <https://doi.org/10.1103/PhysRevD.98.114027>.
18. E. Folco Capossoli, M. A. M. Contreras, D. Li, A. Vega and H. Boschi-Filho, Hadronic Spectra from Deformed AdS Backgrounds, *Chin. Phys. C* **44** (2020) 064104, <https://doi.org/10.1088/1674-1137/44/6/064104>.
19. M. A. Martín Contreras, E. Folco Capossoli, D. Li, A. Vega and H. Boschi-Filho, Pion form factor from an AdS deformed background, *Nucl. Phys. B* **977** (2022) 115726, <https://doi.org/10.1016/j.nuclphysb.2022.115726>.
20. M. A. M. Contreras, E. F. Capossoli, D. Li, A. Vega and H. Boschi-Filho, Proton and neutron form factors from deformed gravity/gauge duality, *Phys. Lett. B* **822** (2021) 136638, <https://doi.org/10.1016/j.physletb.2021.136638>.
21. A. V. Manohar, An Introduction to spin dependent deep inelastic scattering, In *Lake Louise 1992, Symmetry and spin in the standard model* 1-46 [hep-ph/9204208].
22. A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, *Phys. Rev. D* **74** (2006) 015005, [hep-ph/0602229].
23. S. S. Gubser, A. Nellore, S. S. Pufu and F. D. Rocha, ‘Thermodynamics and bulk viscosity of approximate black hole duals to finite temperature quantum chromodynamics, *Phys. Rev. Lett.* **101**, (2008) 131601, <https://doi.org/10.1103/PhysRevLett.101.131601>.
24. A. Vega and I. Schmidt, Hadrons in AdS/QCD correspondence, *Phys. Rev. D* **79** (2009) 055003, <https://doi.org/10.1103/PhysRevD.79.055003>.
25. H. Boschi-Filho, N. Braga, F. Jugeau and M. Torres, Anomalous dimensions and scalar glueball spectroscopy in AdS/QCD, *Eur. Phys. J. C* **73** (2013) 2540, <https://doi.org/10.1140/epjc/s10052-013-2540-5>.
26. E. Folco Capossoli, D. Li and H. Boschi-Filho, Dynamical corrections to the anomalous holographic soft-wall model: the pomeron and the odderon, *Eur. Phys. J. C* **76** (2016) 320, <https://doi.org/10.1140/epjc/s10052-016-4171-0>.
27. D. M. Rodrigues, E. Folco Capossoli and H. Boschi-Filho, Scalar and higher even spin glueball masses from an anomalous modified holographic model, *EPL* **122** (2018) 21001, <https://doi.org/10.1209/0295-5075/122/21001>.
28. M. Henningson and K. Sfetsos, Spinors and the AdS / CFT correspondence, *Phys. Lett. B* **431** (1998) 63, [hep-th/9803251].
29. P.A. Zyla *et al.*, (Particle Data Group), *Prog. Theor. Exp. Phys.* **2020** (2020) 083C01.
30. L. Whitlow, E. Riordan, S. Dasu, S. Rock and A. Bodek, Precise measurements of the proton and deuteron structure functions from a global analysis of the SLAC deep inelastic electron scattering cross-sections, *Phys. Lett. B* **282** (1992) 475, [https://doi.org/10.1016/0370-2693\(92\)90672-Q](https://doi.org/10.1016/0370-2693(92)90672-Q).
31. A. Benvenuti *et al.* [BCDMS], A High Statistics Measurement of the Proton Structure Functions $F(2)(x, Q^{*2})$ and R from Deep Inelastic Muon Scattering at High Q^{*2} , *Phys. Lett. B* **223** (1989) 485, [https://doi.org/10.1016/0370-2693\(89\)91637-7](https://doi.org/10.1016/0370-2693(89)91637-7).
32. E. Folco Capossoli, M. A. Martín Contreras, D. Li, A. Vega and H. Boschi-Filho, Proton structure functions from an AdS/QCD model with a deformed background, *Phys. Rev. D* **102** (2020) 086004, <https://doi.org/10.1103/PhysRevD.102.086004>.