Covalent hadronic molecules via QCD sum rules

Rui-Rui Dong, Hua-Xing Chen†, Niu Su, Hong-Zhou Xi and Yi-Xin Yan

School of Physics, Southeast University, Nanjing 210094, China.

*e-mail: hxchen@seu.edu.cn

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After carefully examining Feynman diagrams corresponding to the $D\Sigma_c$ hadronic molecule state, we propose a possible binding mechanism induced by shared light quarks. We systematically study its corresponding light-quark-exchange interaction using the method of QCD sum rules, and the obtained results suggest that there can be the $D\Sigma_c^+$ covalent molecule of $I = 1/2$. Our QCD sum rule analyses further indicate a model-independent hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that obey the Pauli principle.

Keywords: Hadronic molecule; covalent bond; QCD sum rules.

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1. Introduction

After the discovery of the $X(3872)$ by Belle in 2003 [1], a lot of charmonium-like $XYZ$ and hidden-charm $P_c/P_{cs}$ states were discovered in the past twenty years [2]. They are good candidates of multiquark states, and their experimental and theoretical studies have significantly improved our understanding of the non-perturbative behaviors of the strong interaction at the low energy region [3-11]. Some of them can be explained as hadronic molecules composed of two conventional hadrons [12-14], e.g., the $P_c(4312)$ [15] was interpreted as the $D\Sigma_c$ hadronic molecular state in Refs. [16-20] bound by the one-meson-exchange interaction, as depicted in Fig. 1a).

In this paper we shall systematically investigate Feynman diagrams corresponding to the $D\Sigma_c$ hadronic molecule. We shall use the QCD sum rule method to study the light-quark-exchange interaction, based on which we shall further propose a model-independent hypothesis: “the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that obey the Pauli principle”.

2. Correlation function

In this section we study correlation functions of the $D^-\Sigma_c^+$, $D^0\Sigma_c^+$, $I = 1/2$ $D\Sigma_c$, and $I = 3/2$ $D\Sigma_c$ hadronic molecules. Firstly, we study the $D^-$ meson and the $\Sigma_c^+$ baryon, whose interpolating currents are

$$J^{D^-}(x) = \bar{c}_a(x)\gamma_5 d_a(x),$$
$$J^{\Sigma_c^+}(x) = \frac{1}{\sqrt{2}}\epsilon^{abc}u^T_{\bar{c}}(x)\bar{C}\gamma_{\mu}u_b(x)\gamma_{\mu}\gamma_5c_c(x),$$

where $a \cdots c$ are color indices, $\bar{C} = i\gamma_0\gamma_5$ is the charge-conjugation operator, and the coefficient $1/\sqrt{2}$ is an isospin factor. Then we write down their two-point correlation functions in the coordinate space:

$$\Pi^{D^+}(x) = \langle 0|\bar{T}[J^{D^-}(x)\gamma_{\mu}J^{D^-}(0)]|0\rangle = -\text{Tr} \left[iS^o_{\bar{q}}(x)\gamma_{\mu}S^o_{\bar{q}}^{\dagger}(0)\gamma_5\right],$$
$$\Pi^{\Sigma_c^+}(x) = \langle 0|\bar{T}[J^{\Sigma_c^+}(x)\gamma_5J^{\Sigma_c^+}(0)]|0\rangle = \epsilon^{abc}\epsilon^{a'b'c'}\text{Tr} \left[S^o_{\bar{q}}(x)\gamma_{\mu}S^{o'a'}(x)\gamma_{\mu}\bar{C}(iS^o_{\bar{q}}x)T\bar{C}\gamma_{\mu}\right] \times \gamma_{\mu}\gamma_5,$$

where $iS^o_{\bar{q}}(x) = iS^o_{\bar{q}}(x) = iS^o_{\bar{q}}^{\dagger}(x)$ is the propagator of the light up/down quark and $iS^o_{\bar{q}}^{\dagger}(x)$ is the propagator of the heavy charm quark.

After putting the $D^-$ meson and the $\Sigma_c^+$ baryon at the same location, we construct a composite current corresponding to the $D^-\Sigma_c^+$ molecule,

$$J^{D^-\Sigma_c^+}(x) = J^{D^-}(x) \times J^{\Sigma_c^+}(x) = [\bar{c}_d(x)\gamma_5d_d(x)] \times \frac{1}{\sqrt{2}}\epsilon^{abc}u^T_{\bar{c}}(x)\gamma_5u_b(x)\gamma_{\mu}\gamma_5c_c(x).$$

Note that one needs to explicitly use the non-local current in order to exactly describe the $D^-\Sigma_c^+$ molecule, but this can
not be done yet within the present QCD sum rule framework. We similarly construct the composite currents corresponding to the $D^0\Sigma^+_c$, $I = 1/2$ $D\Sigma_c$, and $I = 3/2$ $D\Sigma_c$ molecules,

\[ J^{D^0\Sigma^+_c}(x) = \left[ \bar{c}_d(x) \gamma_5 u_d(x) \right] \times \left[ e^{abc} u^*_a(x) \Sigma^{c\mu} d_b(x) \gamma_{\mu}\gamma_5 c_c(x) \right], \]

then we expand its correlation function at the hadron level as

\[ \Pi(q^2) = \frac{f^2}{M^2} + \cdots \]

\[ \approx \frac{f^2}{M^2} - \frac{2M_0 f^2}{M^2 - q^2} \Delta M + \cdots, \]  

where the former term is contributed by non-correlated $\bar{D}$ and $\Sigma_c$, and the latter term is contributed by their interactions.

We perform the Borel transformation to Eq. (17) at the hadron level and Eqs. (11-13) at the quark-gluon level, and obtain:

\[ \Pi_0(M^2_B, s_0) = f^2 e^{-M^2_B/s_0} \int e^{-s/M^2_B} \rho_0(s) ds, \]

\[ \Pi_Q(M^2_B, s_0) = \frac{-2M_0 f^2}{M^2} \Delta M e^{-M^2_B/M^2_B} \]

\[ \approx \int e^{-s/M^2_B} \rho_Q(s) ds. \]

We use them to further obtain

\[ \frac{\Pi_Q(M^2_B, s_0)}{\Pi_0(M^2_B, s_0)} = \frac{2M_0}{M^2} \Delta M \int e^{-s/M^2_B} \rho_Q(s) ds \]

\[ = \frac{s_0}{s_0} - \frac{\int e^{-s/M^2_B} \rho_0(s) ds}{\int e^{-s/M^2_B} \rho_0(s) ds}. \]  

Since we are using local currents in QCD sum rule analyses, the parameter $\Delta M$ is actually not the binding energy. It relates to some potential $V(r)$ between $\bar{D}$ and $\Sigma_c$, satisfying

\[ V(r = 0) = \Delta M, \]

\[ V(r \to \infty) \to 0. \]  

We study the light-quark-exchange interaction $\Pi_Q(x)$ and calculate its contributions to the $D^-\Sigma^+_c$, $\bar{D}^0\Sigma^+_c$, $I =
1/2 $\bar{D}\Sigma_c$, and $I = 3/2 \bar{D}\Sigma_c$ hadronic molecules to be:

\[
\Delta M^{D^-\Sigma_c^+} = 0 ,
\]

\[
\Delta M^{D^0\Sigma_c^+} = 95 \pm 8 \text{ MeV} ,
\]

\[
\Delta M^{D_{\Sigma_c^0}^I}_{I=1/2} = -95 \pm 8 \text{ MeV} ,
\]

\[
\Delta M^{D_{\Sigma_c^0}^I}_{I=3/2} = 190 \pm 16 \text{ MeV} .
\]

Therefore, our QCD sum rule results suggest that there can be the $I = 1/2 \bar{D}\Sigma_c$ hadronic molecule, but the $D^-\Sigma_c^+, \bar{D}^0\Sigma_c^+$, and $I = 3/2 \bar{D}\Sigma_c$ hadronic molecules do not exist.

4. Covalent hadronic molecule

In the previous section we apply the QCD sum rule method to study the binding mechanism induced by shared light quarks. This mechanism is similar to the covalent bond in chemical molecules induced by shared electrons, so we call such hadronic molecules “covalent hadronic molecules”.

Recalling that the two shared electrons must spin in opposite directions in order to form a chemical covalent bond, our QCD sum rule analyses indicate a similar model-independent hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that obey the Pauli principle. In this section we qualitatively study this hypothesis, whose logical chain is quite straightforward, as depicted in Fig. 1c). We assume the two light quarks $q_A$ inside $Y$ and $q_B$ inside $Z$ are totally antisymmetric, so that they obey the Pauli principle to be capable of being shared. By doing this, wave-functions of $Y$ and $Z$ overlap with each other, so that they are attracted to possibly form the covalent hadronic molecule $X = [YZ]$.

We apply the above hypothesis to study the $\bar{D}\Sigma_c$ hadronic molecular state as follows. The two shared light quarks have the same color and so the symmetric color structure, and we further assume their orbital structure to be $S$-wave and so also symmetric. Consequently, we only need to consider their spin and flavor structures.

\[\bar{D}^0[c_1 u_2] - \Sigma_c^+ [u_3 d_4 c_5] \text{ covalent molecule}\]

We exchange $u_2$ inside $\bar{D}^0$ and $u_3$ inside $\Sigma_c^+$: $c_1$ and $u_2$ inside $\bar{D}^0$ spin in opposite directions, $\bar{c}_1$ and $u_3$ also spin in opposite directions in order to form another $\bar{D}^0$, so $u_2$ and $u_3$ spin in the same direction with the symmetric spin structure. The flavor structure of $u_2$ and $u_3$ is also symmetric, so they turn out to be totally symmetric:

\[
\begin{array}{cccccc}
\text{color} & \text{flavor} & \text{spin} & \text{orbital} & \text{total} \\
\hline
u_2 \leftrightarrow u_3 & S & S & S & S & S
\end{array}
\]

Accordingly, our hypothesis suggests that the $\bar{D}^0\Sigma_c^+$ covalent molecule does not exist. This is consistent with the previous QCD sum rule analysis.

\[\bar{D}[c_1 q_2] - \Sigma_c[q_3 q_4 c_5] \text{ covalent molecule (} q = u/d \text{)}\]

After including the isospin symmetry, the exchange can take place between up and down quarks. We exchange $q_3$ inside $\bar{D}$ and $q_1$ inside $\Sigma_c$: they have the symmetric spin structure, so they can be totally antisymmetric as long as their flavor structure is also antisymmetric:

\[
\begin{array}{cccccccc}
\text{color} & \text{flavor} & \text{spin} & \text{orbital} & \text{total} \\
\hline
q_2 \leftrightarrow q_3 & S & A & S & S & A
\end{array}
\]

Accordingly, our hypothesis suggests that there can be the $I = 1/2 \bar{D}\Sigma_c$ covalent molecule, but not the $I = 3/2$ one. This is also consistent with the previous QCD sum rule analysis.

5. Summary and Discussions

In this paper we carefully examine Feynman diagrams corresponding to the $\bar{D}\Sigma_c$ hadronic molecular state. We use $\Pi^D(x)$ and $\Pi^{\Sigma_c}(x)$ to denote correlation functions of $\bar{D}$ and $\Sigma_c$ in the coordinate space, and calculate correlation functions of the $D^-\Sigma_c^+, \bar{D}^0\Sigma_c^+, I = 1/2 \bar{D}\Sigma_c$, and $I = 3/2 \bar{D}\Sigma_c$ hadronic molecules. We find that they can be systematically separated into:

\[
\Pi^{D^-\Sigma_c^+}(x) = \Pi_0^{D^+\Sigma_c}(x) + \Pi_G^{D\Sigma_c}(x) ,
\]

\[
\Pi^{\bar{D}^0\Sigma_c^+}(x) = \Pi_0^{\bar{D}^0\Sigma_c}(x) + \Pi_G^{\bar{D}^0\Sigma_c}(x) - \Pi_Q^{\bar{D}^0\Sigma_c}(x) ,
\]

\[
\Pi^{\Sigma_c^0}_{I=1/2}(x) = \Pi_0^{\Sigma_c^0}(x) + \Pi_G^{\Sigma_c^0}(x) + \Pi_Q^{\Sigma_c^0}(x) ,
\]

\[
\Pi^{\Sigma_c^0}_{I=3/2}(x) = \Pi_0^{\Sigma_c^0}(x) + \Pi_G^{\Sigma_c^0}(x) - 2\Pi_Q^{\Sigma_c^0}(x) ,
\]

where $\Pi_Q^{\Sigma_c^0}(x) = \Pi_0^{\bar{D}}(x) \times \Pi_{\Sigma_c^0}(x)$ is the leading term contributed by non-correlated $\bar{D}$ and $\Sigma_c$. $\Pi_G^{\Sigma_c^0}(x)$ describes the double-gluon-exchange interaction between them, and $\Pi_Q^{\Sigma_c^0}(x)$ describes the light-quark-exchange interaction between them.

The term $\Pi_Q$ indicates that $\bar{D}$ and $\Sigma_c$ are exchanging and so sharing two light up/down quarks. We systematically study it using the method of QCD sum rules, and calculate the mass correction $\Delta M$ induced by this term. Since we are using local currents in QCD sum rule analyses, the parameter $\Delta M$ is actually not the binding energy, but relates to some potential $V(r)$ between $\bar{D}$ and $\Sigma_c$, satisfying:

\[
V(r = 0) = \Delta M ,
\]

\[
V(r \to \infty) \to 0.
\]

We systematically calculate the light-quark-exchange interaction $\Pi_Q$, and evaluate its contributions to the $D^-\Sigma_c^{++}$, $\bar{D}^0\Sigma_c^+$, $I = 1/2 \bar{D}\Sigma_c$, and $I = 3/2 \bar{D}\Sigma_c$ hadronic
molecules:
\[\Delta M^{D^-\Sigma_c^+} = 0,\]
\[\Delta M^{\bar{D}^0\Sigma_c^+} = 95 \text{ MeV},\]
\[\Delta M_{I=1/2}^{\Sigma_c^+} = -95 \text{ MeV},\]
\[\Delta M_{I=3/2}^{\Sigma_c^+} = 190 \text{ MeV}.
\]
Therefore, our QCD sum rule results suggest that there can be the \( I = 1/2 \) \( \bar{D}\Sigma_c \) hadronic molecule, but the \( D^-\Sigma_c^+ \), \( \bar{D}^0\Sigma_c^+ \), and \( I = 3/2 \) \( D\Sigma_c \) hadronic molecules do not exist.

The binding mechanism induced by shared light quarks is similar to the covalent bond in chemical molecules induced by shared electrons, so we call such hadronic molecules “covalent hadronic molecules”. We have also investigated the other \( \bar{D}(+)\Sigma_c^+, D(+)\Lambda_c, D(+)\bar{K}^*, \) and \( D(+)\bar{D}(+) \) hadronic molecular states using the QCD sum rule method, and the obtained results indicate a model-independent hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that obey the Pauli principle. Its mechanism is similar to the chemical covalent bond, where the two shared electrons spin in opposite directions and so totally antisymmetric obeying the Pauli principle. We apply the above hypothesis to reanalysis the \( D\Sigma_c \) hadronic molecule, and the obtained results are consistent with our QCD sum rule calculations.

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