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Electromagnetic Pion form factor in a deformed background

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This work discusses the electromagnetic (EM) pion form factor (πFF) in a deformed AdS geometry. We consider the conformal dimension of the hadron bulk field defined by the scaling dimension of the $q\,\bar{q}$ operator instead of the twist. We also compute the pion EM radius and compare it with the experimental data, finding a relative error of $2\,\%$.

Keywords: Deformed string/gauge correspondence; AdS/QCD; pion form factor.

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1. Introduction

Electromagnetic pion form factor (πFF) is one of the most valuable QCD quantities related to the transition from the non-perturbative to the perturbative regime. From theoretical studies such as [1], it is expected the emergence of a *Sudakov suppression* in the πFF for intermediate q^2 regions. In this work, we want to explore this phenomenology from holographic grounds.

In general, the primary motivation behind the AdS/QCD idea is how we place confinement. In pure AdS space, bulk fields are conformal. Free bulk fields have normalizable modes with continuous eigenspectra. Thus, they are not a good choice to describe hadrons. To circumvent this issue, we can add geometrical deformations to the background. In this top/down scenario, the equivalence with low energy QCD appears when we consider equally the conformal boundary theory and QCD coupling constants at some fixed point. However, this prescription is restrictive. For instance, these models do not have fundamental degrees of freedom ab initio. They should be added by geometrical deformations, such as another stack of D-branes, whose Chan-Paton charge corresponds with the number of flavors of theory [2].

Another possibility to address the emergence of bounded (hadronic) states in holography is directly transforming the continuous bulk eigenspectrum into a discrete one. This idea is achieved by slightly breaking the conformal invariance in the bulk. We can do this in two forms: by placing a dilaton (the so-called softwall [3] model) or deforming the geometry (the hardwall [4–7] and the deformed background models [8, 9]). Consequently, we will obtain a dual radial Regge trajectory, which can be linear if the dilaton or deformation are chosen to be quadratic in the holographic coordinate. We will follow the latter path.

This work is organized as follows: in Sec. 2, we describe the deformed background model for pions and photon. In Sec. 3, we summarize the holographic πFF calculation in terms of bulk interacting fields. Finally, in Sec. 4 we present our conclusions.

2. Deformed Background Model

Let us consider a general five-dimensional AdS background defined by

$$dS^2 = \frac{R^2}{z^2}\, e^{2h(z)} \left[dz^2 + \eta_{\mu\,\nu}\, dx^\mu\, dx^\nu\right], \tag{1}$$

where R defines the AdS curvature radius, and the Greek indices label four-dimensional spacetime indices. The geometrical deformation $h(z)$ sets confinement.

Hadrons are defined using the bulk fields defined by the action

$$I_{\text{Hadron}} = \int d^5x\, e^{-\Phi(z)}\, \mathcal{L}_{\text{Hadron}}, \tag{2}$$

where $\Phi(z)$ is a dilaton (which can be statically or dynamically generated) which also induces confinement. The lagrangian $\mathcal{L}_{\text{Hadron}}$ carries all the relevant information of how bulk fields will mimic hadrons at the boundary.

From the metric (1) and the action (2), we can write bulk equations of motion. These equations are transformed into a Schrodinger-like eigenvalue problem $-\psi'' + V(z)\psi = M_n^2\,\psi$, with the $V(z)$ defined as holographic potential, and $\psi(z)$ the normalizable part of the bulk field dual to hadrons.

In general, the holographic potential for integer spin hadrons has the form

$$V(z) = \frac{1}{4} B'(z)^2 - \frac{1}{2} B''(z) + \frac{M_5^2 R^2}{z^2} e^{2h(z)}, \tag{3}$$

where we have introduced $B(z) = \Phi(z) + \beta\, [\log(R/z) + h(z)]$, and $M_5^2 R^2$ defines the bulk field mass, which characterizes the hadronic identity in terms of its conformal dimension. The information initially captured in the bulk action (2) is now translated into the $B(z)$ function. The parameter β counts the hadronic spin in $3+1$ dimensions. For scalar hadrons we have $\beta = -3$, and for vectors we have $\beta = -1$. The eigenspectrum M_n^2 obtained from this holographic potential defines the so-called *holographic Regge trajectories.*

The geometric background model introduced in Refs. [9, 10] emerges when we fix $\Phi(z) = 0$ and $h(z) = (1/2)\, k\, z^2$. Notice that the parameter k is flavor dependent, *i.e*., each particle in the model has its own geometric background.

2.1. Pions in the deformed background model

Pions are defined by a bulk massive scalar field X obeying the following action

$$I_\pi = \int d^5x \sqrt{-g_\pi}\, [g_\pi^{mn} \partial_m X \partial^n X + M_5^2 X^2], \tag{4}$$

where the bulk mass M_5^2 is defined in terms of the scalar conformal dimension Δ as follows

$$M_5^2\, R^2 = \Delta(\Delta - 4). \tag{5}$$

For hadrons, Δ is identified with the dimension of the operator creating them at the boundary. In the case of mesons, they are created by the operator $q\,\bar{q}$, whose dimension is three. Therefore, in this particular case, the bulk mass is $M_5^2\, R^2 = -3$.

From the action (4) we obtain the following equation of motion for the X field:

$$\partial_z \left[e^{-B_\pi(z)}\, \partial_z X \right] + M_n^2\, e^{-B_\pi(z)}\, X + \frac{3\, R^2}{z^2}\, e^{\frac{1}{2} k_\pi^2\, z^2} X = 0, \tag{6}$$

where we have imposed the on-shell condition $-q^2 = M_n^2$ for the pion, and also we have defined $B_\pi(z) = -3\, \log(R/z) - (3/2) k_\pi\, z^2$.

Following the standard bottom-up AdS/QCD prescription [3, 11–13], after performing the transformation $X(z) = e^{\frac{1}{2} B_\pi(z)}\, \psi_{\pi,n}(z)$ we can write the following holographic potential (3) for the pion normalizable bulk modes $\psi_{\pi,n}(z)$ as

$$V_\pi(z) = \frac{15}{4\, z^2} - \frac{3}{z^2}\, e^{-k_\pi\, z^2} + \frac{k_\pi^2\, z^2}{4} + k_\pi. \tag{7}$$

The ground state of this potential is identified with the pion. Thus, using the pion mass, we can fit the value of deformation slope as $k_\pi = -0.0425^2$ GeV^2.

2.2. Virtual photons in the deformed background model

Virtual photons emerge from the non-normalizable part of an abelian massless vector bulk field $\phi_m(z, q)$, defined by the action

$$I_\gamma = -\frac{1}{\alpha_\gamma^2} \int d^5x \sqrt{-g_\gamma}\, \frac{1}{4} F^{mn} F_{mn}\,, \tag{8}$$

where α_γ is a coupling constant setting units in the action, and $F_{mn} = 2\, \partial_{[m}\, \phi_{n]}$ is field strength, and ϕ_μ is a bulk abelian vector field.

At the conformal boundary, the vector field should behave as transverse wave, *i.e.*, $\phi_\mu(x, z \to 0) = \eta_\mu\, e^{-i\, q \cdot x}$. This condition imposes that $\phi_z = 0$. Thus, the bulk vector field will be written as

$$\phi_\mu(z, q) = \eta_\mu\, e^{-i\, q \cdot x}\, \mathcal{B}(z, q), \tag{9}$$

where $\mathcal{B}(z, q)$ is the so-called *bulk-to-boundary propagator*. From the vector field action we can write the equations of motion for $\mathcal{B}(z, q)$ as follows

$$\partial_z \left[e^{-B_\gamma(z,q)}\, \partial_z\, \mathcal{B}(z, q) \right] - q^2\, e^{-B_\gamma(z)}\, \mathcal{B}(z, q) = 0, \tag{10}$$

where $B_\gamma(z) = -\log(R/z) - (1/2)\, k_\gamma\, z^2$. The equation for the bulk-to-boundary propagator has the following solution

$$\mathcal{B}(z, q) = -\frac{1}{2}\, k_\gamma\, z^2\, \Gamma\left[1 - \frac{q^2}{2k_\gamma}\right] \times \mathcal{U}\left(1 - \frac{q^2}{2k_\gamma};\, 2;\, -\frac{k_\gamma\, z^2}{2}\right), \tag{11}$$

where $\mathcal{U}(a, b, z)$ is the Tricomi function, and k_γ is the energy scale associated with the virtual photon kinematics.

Now that we have described the holographic ingredients, we can move towards the holographic calculation of the pion form factor.

3. Holographic πFF calculation

In holography, form factors are defined via interaction terms in the bulk action (2). These interaction terms may arise in two possible forms. One is from high-order expansions in the bulk lagrangian, encoded in the group covariant derivatives due to the inner group structure associated with the bulk fields [14]. The other comes from phenomenological interaction terms written *ab initio* inspired by expected hadronic properties, such as sum rules or OPE's [15].

In this scenario, the electromagnetic form factor comes from the minimal coupling between scalar bulk field normalizable Schrodinger modes (dual to the incoming and outgoing pions) and a vector bulk field non-normalizable part $\mathcal{B}(z, q^2)$ (dual to the virtual photon). Therefore we have [16]

$$F_\pi(q^2) = \int dz\, \psi_{\pi,1}(z)\, \mathcal{B}(z, q^2)\, \psi_{\pi,1}(z), \tag{12}$$

where $\psi_{\pi,1}(z)$ stands for the pion ground state eigenfunction.

In our particular case, since we have a background for each particle, we have set (*or choose*) the frame where we will calculate the interaction term (12). In this sense, we propose the *pion geometric background* as the most natural scenario, since in this frame, we have defined the pion Schrodinger-like modes.

This explicit form for the πFF was introduced in the context of light-front (LF) holography [15], where Δ is not associated with the dimension of the operator creating hadrons. Instead, Δ is connected with the *twist*, which carries the particle content information. Thus, in the case of mesons, $\Delta = \tau = 2$. With this choice, the πFF has the correct large q^2 behavior, expected from QCD sum rules [29]. In the next paragraphs, we will explore the case with $\Delta = 3$, outside the LF holography context.

3.1. k_γ fixed case

As usual in AdS/QCD, we will take the conformal dimension for the scalar bulk field as $\Delta = 3$. In Fig. 1 we summarize our results comparing with experimental and theoretical (holographic and non-holographic) available data. To test the consistency with the sum rules, we will examine the Brodsky-Lepage counting rule calculated in this model:

$$F_\pi\left(q^2\right)\big|_{q^2\to\infty} = \frac{1+\gamma_e(\Delta-1)\Delta+\dots}{8\,\Delta^2(\Delta-1)} \times\left(\frac{1}{q^2}\right)^{\Delta-1} \propto \frac{1}{q^4}. \tag{13}$$

Notice that when we consider the LF case, *i.e.* $\Delta = 2$ [15], we recover the expected behavior $1/q^2$ for the πFF. However, in the non-LF case, our geometric deformed background model with $\Delta = 3$ and using Eq. (12), we observed that the πFF captures the low q^2 (below 1 GeV^2) behavior (see Fig. 1), nevertheless it does not satisfy the expected counting rule. The πFF in this situation is highly suppressed for high q^2 values also.

The low q^2 behavior is tested by calculating the pion electromagnetic radius, defined as

$$\langle r_\pi^2\rangle = -6\,\left.\frac{dF_\pi(q^2)}{dq^2}\right|_{q^2=0}. \tag{14}$$

In this situation with $\Delta = 3$ and $k_\gamma = -3.8$ GeV^2, we obtain $r_\pi = 0.458$ fm, with an error around 30% compared with experimental data [30].

3.2. k_γ running with q case

A possible form to circumvent this issue with the counting rule appears when we assume the photon slope k_γ to be a function of the transferred momentum q. The parameter k_γ does not set confinement, and it is related with the kinematic scale of the virtual photon. Thus, we will fix $k_\gamma(q) = q\,\kappa_\gamma$, where κ_γ has energy units. If we compute the πFF at large q^2, following the procedure described in Ref. [15],

$$F_\pi\left(q^2\right)\big|_{q^2\to\infty} = \frac{32\,k_\gamma^2(q)}{\left[q^2+4\,|k_\gamma(q)|\right]\left[q^2+8\,|k_\gamma(q)|\right]} \propto \frac{1}{q^2}, \tag{15}$$

we will fulfill the Brodsky-Lepage rule. The results in this situation are depicted in Fig. 2. Notice that we also have a softened Sudakov suppression, as expected from lattice analysis. The plots also show that this approach captures the low q^2 phenomenology. When we compute the pion electromagnetic radius with $\kappa_\gamma = -2.8$ GeV, we obtain $r_\pi = 0.671$, having $2\,\%$ of error compared with experimental data [30].

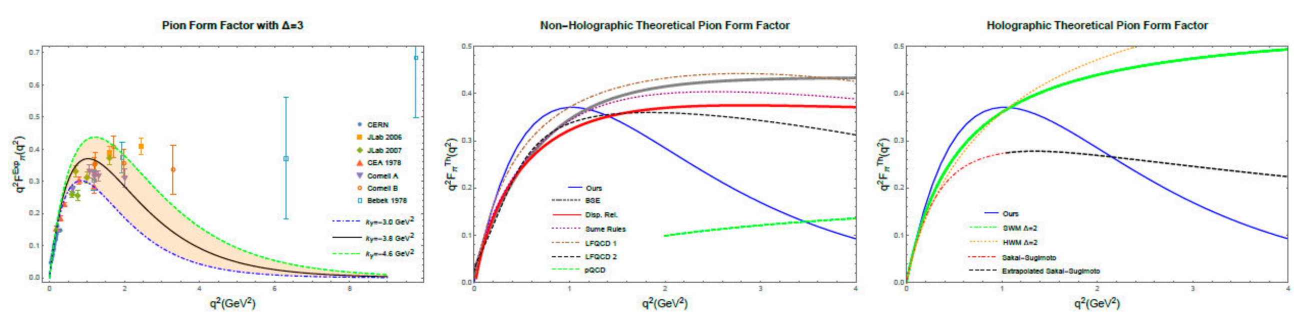

FIGURE 1. The left panel compares our results for the pion form factor with the available experimental data [17–22]. In the next panels, we have a comparison of our results with non-holographic models (center panel) such as BSE [23], perturbative QCD [25], dispersion relations [26], sum rules [27], and LFQCD [1]. In the right panel, we depict a comparison with other holographic models such as hardwall and softwall with $\Delta = 2$ [15], and Sakai-Sugimoto/extrapolated Sakai-Sugimoto [28]. In our results we have taken $k_\gamma = -3.8$ GeV^2.

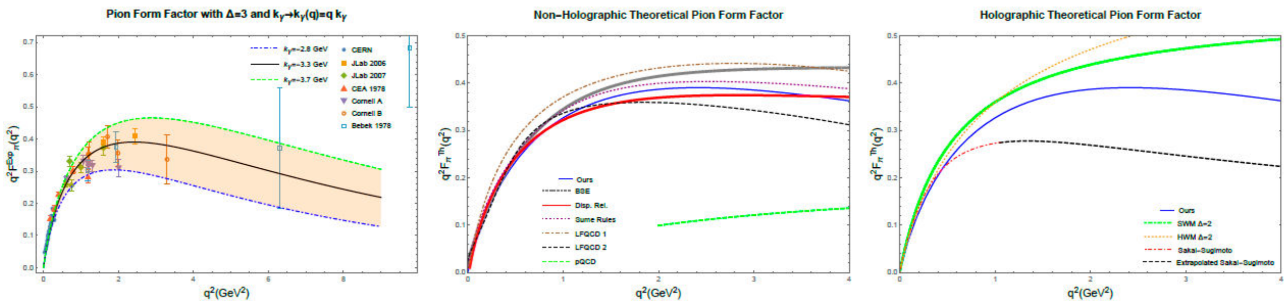

FIGURE 2. The left panel compares our results for the pion form factor with the available experimental data [17–22] using the proposed scaling for k_γ. In the next panels, we depict a comparison of our results with non-holographic models (center panel) such as BSE [23] or Light-Front BSE [24], perturbative QCD [25], dispersion relations [26], sum rules [27], and LFQCD [1]. In the right panel, we show a comparison with other holographic models such as hardwall and softwall with $\Delta = 2$ [15], and Sakai-Sugimoto/extrapolated Sakai-Sugimoto [28]. In our results, for lower panels we have taken $k_\gamma = -2.8$ GeV^2.

4. Conclusions

In this work, we have calculated the electromagnetic pion form factor holographically by considering non-LF formalism, *i.e.*, we have considered $\Delta = 3$. We show the issue with the large q^2 behavior that this prescription has and how to solve it by considering a re-scaling in the photon slope k_γ that sets the corresponding geometric background. Our calculation captures the low q^2 behavior and also exhibits a Sudakov-like suppression in the intermediate q^2 region, as it was suggested by lattice [1]. Our calculation of the pion electromagnetic radius give us a result of 0.671 fm, with a $2\,\%$ error in comparison with available experimental data.

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