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# Light hyperon polarization and CP test at BESIII

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Hyperon is an ideal probe to study the QCD in the region of non-perturbative to perturbative. Using $1.31 \times 10^9$ $J/\psi$ and $448.1 \times 10^6$ $\psi(3686)$ events, the hyperon anti-hyperon pair production are well studied. The hyperon polarization and its decay parameter have been measured in high precision through their quantum entangle system. By comparing the decay parameters in hyperon and anti-hyperon decays, the CP conservation observables has been measured which is important to test the Standard Model and search for New Physics.

*Keywords:* Hyperon; polarization; CP.

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## 1. Introduction

It is assumed that the Big Bang produced exactly the same amount of matter and antimatter, but cosmological observations show that the universe is full of matter. It means that there must be a mechanism of preference for matter over antimatter in the evolution of the universe. In 1967, Sakharov gave a necessary condition for this mechanism: there must be an interaction that breaks the conservation of baryon numbers. And importantly, there must be CP violation [1]. Fortunately, CP violation has been observed in K meson decay by Cronin and Fitch [2]. In 1963, Cabbibo proposed that the eigenstates of quarks participating in weak interaction are different from their mass eigenstates [3]. It is proposed that there is a matrix transformation between the two eigenstates of d quark and s quark, which can be expressed by a Cabbibo angle. In 1970, when GIM mechanism was proposed to explain flavor-changing neutral currents are suppressed, c quark was introduced [4]. When Kobayashi and Maskawa studied the CP violation mechanism in 1973, they extended this mixing mechanism (KM mechanism) [5], and even proposed three generations of quarks before the discovery of c quarks. The explanation of CP violation by KM mechanism is the most successful by now. In experiments, the searching for CP violation process has been going on. In 2001, BaBar Collaboration and Belle Collaboration announced that CP violation (indirect CP violation) was found in B meson decay [6, 7]. Direct CP violation was observed during the rare decay of B meson [8, 9]. In 2019, the LHCb Collaboration announced that CP violation was first found in the charm decays [10]. The above experimental results are consistent with the prediction of KM mechanism in the Standard Model. However, this is not enough to explain the material dominance in the universe [11]. Therefore, people focus on searching for new physics (theoretically and experimentally), hoping that these new physics can not only explain the existing experimental results, but also contain more new sources of CP violation. Exploring possible CP violation in hyperon decay is part of the search for new physics.

The spin 1/2 hyperons could be used to study the symmetry through their two-body weak decay. Historically, it was used to study Parity conservation [12]. The current research focuses on the search of CP violation in hyperon decay. The polarization of 1/2 spin hyperon can be obtained by the angular distribution of final state particles in its two body decay. In the process of hyperon decay to nucleon (proton or neutron) and $\pi$ meson, its decay parameters can be described by a parity conserving (P-wave) and a parity violation (S-wave) amplitudes:

$$\alpha = \frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2}, \quad \beta = \frac{2\mathrm{Im}(S^*P)}{|S|^2+|P|^2},$$
$$\gamma = \sqrt{1-\alpha^2}\cos\phi, \tag{1}$$

where the parameter $\alpha$ can be described as the angular distribution asymmetry of the final state particle (nucleon):

$$dN/d\Omega = \frac{1}{4\pi}(1+\alpha\mathbf{P}\cdot\hat{\mathbf{p}}). \tag{2}$$

Then by comparing the decay parameters of hyperon and anti-hyperon, CP conservation could be tested.

$$A = \frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}. \tag{3}$$

The Standard Model predicts that the hyperon CP violation happens at the level of $10^{-5}$ to $10^{-6}$ [13]. In the Eq. (2), we notice that $\alpha$ (decay parameter) and P (polarization) appear in the form of product in the angular momentum distribution, which means we can not obtain the decay parameters from the angular distribution of a single final particle. However, in the BESIII experiment, the hyperon and anti-hyperon pairs are obtained by the decay of the intermediate resonance states ($J/\psi$ and $\psi(3686)$) produced by the annihilation of electron and positron. This provides a unique environment for the study of the generation and decay of hyperons. This quantum entanglement system provides a controllable and accurate CP conservation test. The process of hyperon and anti-hyperon generated by electron and positron annihilation through $J/\psi$ (or $\psi(3686)$) can be described by $\psi$ like electromagnetic shape factor [14]. Its form is equivalent to the electromagnetic form factor of hyperons [15–17]. These two form factors can be written as two real parameters $\alpha_\psi$ and $\Delta\Phi$, which represent the asymmetry of angular distribution and the phase angle between the two form factors. The observable $\Delta\Phi$ is related to the spin polarization of hyperon and anti-hyperon. If $\Delta\Phi$ is not equal to 0, the polarization of the hyperon will be perpendicular to its generation plane. These variables can be measured in BESIII experiment. We can write the differential cross section formula of hyperon anti-hyperon pairs generated through $J/\psi$(or $\psi(3686)$), then they decay to nucleons and $\pi$ mesons (and their anti-particles). The differential cross-section is given as $d\sigma \propto \mathcal{W}(\boldsymbol{\xi})d\boldsymbol{\xi}$, where $\mathcal{W}(\boldsymbol{\xi})$ is

$$\begin{aligned}\mathcal{W}(\boldsymbol{\xi}) &= \mathcal{T}_0(\boldsymbol{\xi}) + \alpha_\psi \mathcal{T}_5(\boldsymbol{\xi}) \\ &+ \alpha\bar{\alpha}\left(\mathcal{T}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)\mathcal{T}_2(\boldsymbol{\xi}) + \alpha_\psi \mathcal{T}_6(\boldsymbol{\xi})\right) \\ &+ \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)\left(\alpha\mathcal{T}_3(\boldsymbol{\xi}) + \bar{\alpha}\mathcal{T}_4(\boldsymbol{\xi})\right).\end{aligned}$$

and $\mathcal{T}_i, (i = 0, 1...6)$ are angular functions dependent on $\boldsymbol{\xi}$, where $\boldsymbol{\xi}$ represents polar angles of hyperons and polar angles and azimuthal angles of nucleons [14].

With the data collected in 2009 and 2012 at BESIII, the observation of hyperon polarizations opens a new window for testing CP conservation in baryon sector, as it allows for simultaneous production and detection of hyperon and anti-hyperon pair two-body weak decay. The CP symmetry tests are performed in the process of $\Lambda$, $\Sigma$ and $\Xi$ pair production. Furthermore in the $\Xi$ decay, it is possible to perform three independent CP tests and determine the strong phase and weak phase difference.

## 2. $J/\psi \to \Lambda\bar{\Lambda}$

Using $1.31 \times 10^9$ $J/\psi$ events collected with BESIII detector, the process of $J/\psi \to \Lambda\bar{\Lambda}$ is studied [18]. $\Lambda$ is reconstructed with $p\pi^-$. $\bar{\Lambda}$ is reconstructed with two modes: $\bar{p}\pi^+$ and $\bar{n}\pi^0$. In the real data, there are 420,593 and 47,009 signal events are selected for the decay final states $p\pi^-\bar{p}\pi^+$ and $p\pi^-\bar{n}\pi^0$ respectively. With the selected signal events, the angular observables $\boldsymbol{\xi}$ are fully reconstructed. In the differential cross section formula, the free parameters $\alpha_\psi$, $\Delta\Phi$, $\alpha_-$, $\alpha_+$ and $\bar{\alpha}_0$ are extracted by a simultaneous unbinned maximum likelihood fit. The $\Delta\Phi$ is measured to be $(42.4 \pm 0.6 \pm 0.5)^\circ$ which means there are a clear polarization existence in the $J/\psi \to \Lambda\bar{\Lambda}$. In Fig. 1, to show the polarization, the moment $\mu(\cos\theta_\Lambda) = (m/N)\sum_i^{N(\cos\theta_\Lambda)}(\sin\theta_p^i \sin\phi_p^i - \sin\theta_{\bar{p}}^i \sin\phi_{\bar{p}}^i)$ related to the polarization is calculated in $m$ bins of $\cos\theta_\Lambda$. Therefore, we could determine the decay asymmetry parameters $\alpha_-$, $\alpha_+$ and $\bar{\alpha}_0$ separately. The CP violation test $A_{\mathrm{CP}} = (\alpha_- + \alpha_+)/(\alpha_- - \alpha_+)$ is measured to

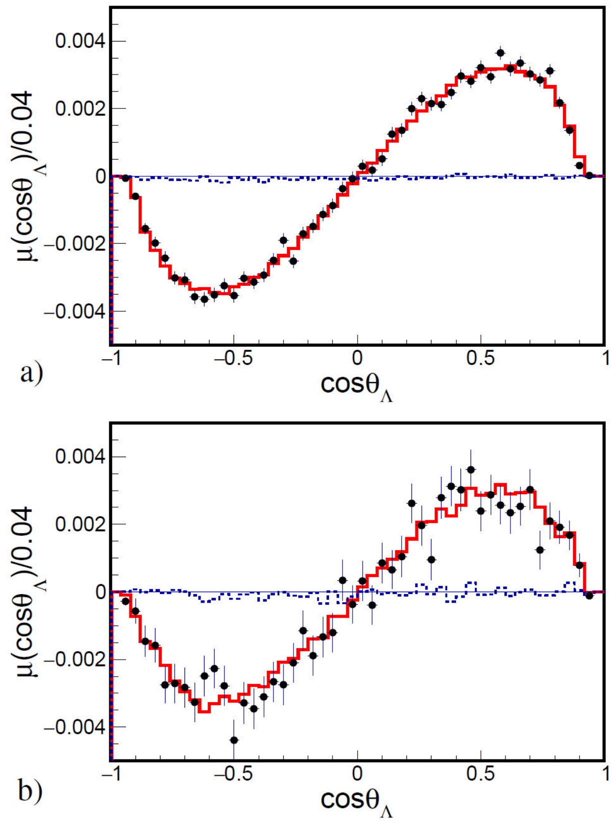


FIGURE 1. Moments $\mu(\cos\theta_\Lambda)$ for acceptance uncorrected data as a function of $\cos\theta_\Lambda$ for a) $p\pi^-\bar{p}\pi^+$ and b) $p\pi^-\bar{n}\pi^0$ data sets. The points with error bars are the data, and the solid-line histogram is the global fit result. The dashed histogram shows the no polarization scenario.

be $-0.006 \pm 0.012 \pm 0.007$ which is consistent with SM prediction. The obtained value of $\alpha_-$ is $0.750 \pm 0.009 \pm 0.004$, which is $17 \pm 3\%$ higher than the world average at that time. As a fundamental parameter, this measurement has been confirmed by analysis the CLAS data [19].

## 3. $J/\psi$ and $\psi(3686) \to \Sigma^+\bar{\Sigma}^-$

The polarizations are studied not only in the $J/\psi$ decays, but also in $\psi(3686)$ decays. With $1310.6 \times 10^6$ $J/\psi$ and $448.1 \times 10^6$ $\psi(3686)$ events, the combined fit is performed for the process of $J/\psi$ and $\psi(3686) \to \Sigma^+\bar{\Sigma}^-$ [20]. We selected the final states with proton, anti-proton and $2\pi^0$ to reconstruct the signal events. To further suppress the background events, the four momentum kinematic constraints are applied. The peaking background level is below 0.1%, and other background are 5% for $J/\psi$ decay and 1% for $\psi(3686)$ decay which could be well estimated with sideband method. An unbinned maximum likelihood fit is performed in the five angular dimensions $\boldsymbol{\xi}$, simultaneously fitting both the $J/\psi \to \Sigma^+\bar{\Sigma}^-$ and $\psi' \to \Sigma^+\bar{\Sigma}^-$ data in order to determine the parameters $\boldsymbol{\Omega} = \{\alpha_{J/\psi}, \alpha_{\psi'}, \Delta\Phi_{J/\psi}, \Delta\Phi_{\psi'}, \alpha_0, \bar{\alpha}_0\}$. The numerical fit results are summarized in Table I. We could

TABLE I. Values and uncertainties of the fit parameters extracted in this work.

| Parameter | Measured value |
|---|---|
| $\alpha_{J/\psi}$ | $-0.508 \pm 0.006 \pm 0.004$ |
| $\Delta\Phi_{J/\psi}$ | $-0.270 \pm 0.012 \pm 0.009$ |
| $\alpha_{\psi'}$ | $0.682 \pm 0.03 \pm 0.011$ |
| $\Delta\Phi_{\psi'}$ | $0.379 \pm 0.07 \pm 0.014$ |
| $\alpha_0$ | $-0.998 \pm 0.037 \pm 0.009$ |
| $\bar{\alpha}_0$ | $0.990 \pm 0.037 \pm 0.011$ |

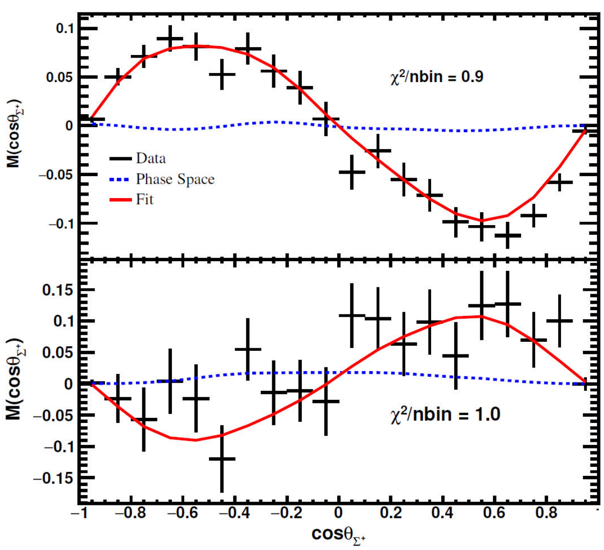


FIGURE 2. The moments $\mu(\cos\theta_{\Sigma+})$ for data that is not corrected for acceptance and reconstruction efficiency as function of $\cos\theta_{\Sigma+}$ for the decays $J/\psi \to \Sigma^+\bar{\Sigma}^-$ (top) and $\psi(3686) \to \Sigma^+\bar{\Sigma}^-$ (bottom). The black points with error bars are experimental data, the red solid lines are the fit results and the blue dashed line represents the distribution without polarization from the simulated events evenly distributed in phase space.

could find that $\alpha_{J/\psi}$ is determined to be $-0.508 \pm 0.006 \pm 0.004$ , which is negative as observations made in the decays $J/\psi \to \Sigma^0\bar{\Sigma}^0$, $J/\psi \to \Sigma(1385)^-\bar{\Sigma}(1835)^+$ and $J/\psi \to \Sigma(1385)^+\bar{\Sigma}(1835)^-$. The relative phases $\Delta\Phi_{J/\psi}$ and $\Delta\Phi_{\psi'}$ are determined simultaneously for the first time for both reactions $J/\psi \to \Sigma^+\bar{\Sigma}^-$ and $\psi' \to \Sigma^+\bar{\Sigma}^-$. Considering the $\Delta\Phi_{J/\psi}$ and $\Delta\Phi_{\psi'}$ are non-zero, both of them could be used to extract the decay asymmetry parameters $\alpha_0$ and $\bar{\alpha}_0$ simultaneously. While the value of $\alpha_0$ determined in this work is consistent with the PDG average at significantly improved precision, $\bar{\alpha}_0$ is measured for the first time. The value of $A_{\rm CP}$ is found to be consistent with CP-conservation and is in agreement with the Standard Model prediction.

## 4. $J/\psi \to \Xi^-\bar{\Xi}^+$

To describe the spin 1/2 hyperon decay, there are three decay parameters are defined as in Eq. (1). And these three parameters are not independent as $\alpha^2+\beta^2+\gamma^2 = 1$. By comparing these decay parameters in hyperon and anti-hyperon decays, the CP-conservation could be tested. But in the $\Lambda$ and $\Sigma$ decays, the final state polarization of proton could not be determined by the detector. Therefore we can only determine the decay asymmetry parameter $\alpha$ in $\Lambda$ and $\Sigma$ decays. In the decay process of $\Xi^- \to \Lambda\pi^-$, we could also determine the $\alpha_\Xi$ as before. The different is that the polarization of $\Lambda$ could be determined through $\Lambda \to p\pi^-$ which allow us to get the all decay parameters: $\alpha_\Xi$, $\beta_\Xi$ and $\gamma_\Xi$. By defining the parameter $\phi_\Xi$ according to

$$\beta_\Xi = \sqrt{1-\alpha_\Xi^2}\sin\phi_\Xi, \quad \gamma_\Xi = \sqrt{1-\alpha_\Xi^2}\cos\phi_\Xi, \qquad (4)$$

the decay is completely described by two independent parameters $\alpha_\Xi$ and $\phi_\Xi$. Then two CP violation observables are defined to be

$$A_{\rm CP} = \frac{\alpha_\Xi + \bar{\alpha}_\Xi}{\alpha_\Xi - \bar{\alpha}_\Xi}, \quad \Delta\phi_{\rm CP} = \frac{\phi_\Xi + \bar{\phi}_\Xi}{2}. \qquad (5)$$

Since the decay amplitude for $\Xi^- \to \Lambda\pi^-$ consists of both a P-wave and an S-wave part, the leading-order contribution to the CP asymmetry $A_{\rm CP}$ can be written as

$$A_{\rm CP} \approx -\tan(\delta_P - \delta_S)\tan(\xi_P - \xi_S), \qquad (6)$$

where $\tan(\delta_P - \delta_S) = \beta/\alpha$ denotes the strong phase difference of the final-state interaction between the $\Lambda$ and $\pi^-$ from the $\Xi^-$ decay. An independent CP-symmetry test in $\Xi^- \to \Lambda\pi^-$ is provided by determining the value of $\Delta\phi_{\rm CP}$. At leading order, this observable is related directly to the weak-phase difference

$$(\xi_P - \xi_S)_{\rm LO} = \frac{\beta + \bar{\beta}}{\alpha - \bar{\alpha}} \approx \frac{\sqrt{1-\alpha^2}}{\alpha}\Delta\phi_{\rm CP}, \qquad (7)$$

and can be measured even if $\delta_P = \delta_S$.

With $1.31 \times 10^9$ $J/\psi$ events, the multi-dimensional angular fitting is performed to measure the decay parameters in $J/\psi \to \Xi^-\bar{\Xi}^+$ [21]. Nine kinematic observables are used to describe this decay process of $J/\psi \to \Xi^-\bar{\Xi}^+$, $\Xi^- \to \Lambda\pi^-$ and $\bar{\Xi}^+ \to \bar{\Lambda}\pi^+$. To show the fit quality of the fitting, the polarization and spin correlation plots are shown in Fig. 3. Based on the fitting results, $A_{\rm CP}$ and $\Delta\phi_\Xi$ are measured to be $(6.0 \pm 13.4 \pm 5.6) \times 10^{-3}$ and $(-4.8 \pm 13.7 \pm 2.9) \times 10^{-3}$ rad, which are consistent with CP symmetry. In addition, the strong and weak $\Xi \to \Lambda\pi$ decay amplitudes could be separated. The direct weak phase difference is measured to be $(\xi_P - \xi_S) = (1.2 \pm 3.4 \pm 0.8) \times 10^{-2}$ rad and is one of the most precise tests of CP symmetry for strange baryons. The strong phase difference is measured to be $(\delta_P - \delta_S) = (-4.0 \pm 3.3 \pm 1.7) \times 10^{-2}$ rad. Moreover, we provide an independent measurement of the $\Lambda$ decay parameter, $\alpha_\Lambda = 0.757 \pm 0.011 \pm 0.008$ which is consistent with previous BESIII measurement [18].

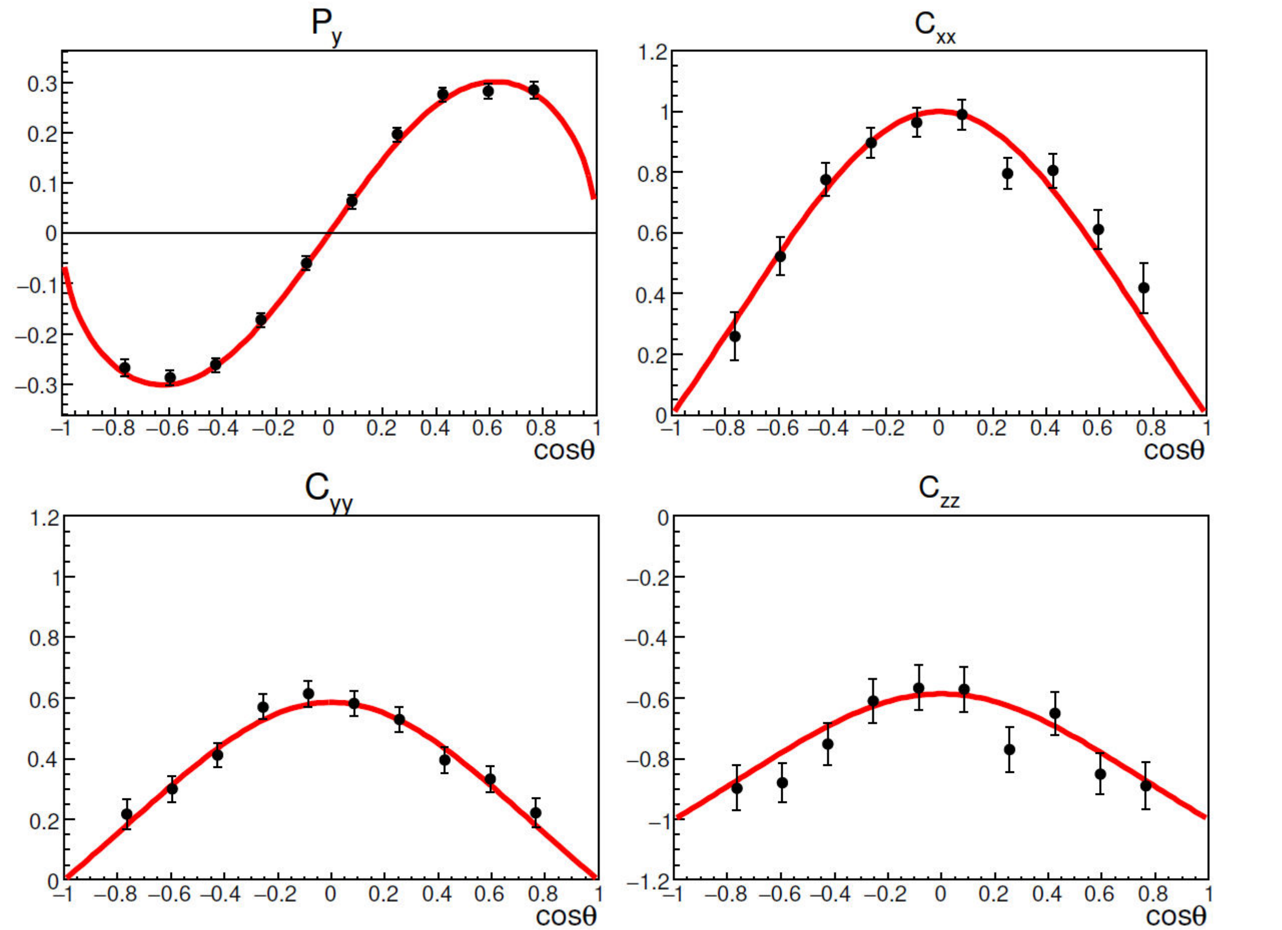


FIGURE 3. Polarisation and spin correlations and in the $e^+e^- \to \Xi^-\overline{\Xi}^+$ reaction. a) $P_y$, b) $C_{xx}$, c) $C_{yy}$ and d) $C_{zz}$ as functions of $\cos\theta$. The data points are determined independently in each bin. The red curves represent the expected angular dependence obtained with the values of $\alpha_\psi$ and $\Delta\Phi$ from the global fit. The errors bars indicate the statistical uncertainties.

## 5. $\psi(3686) \to \Omega^-\bar{\Omega}^+$

To spin of $\Omega^-$ is predicted to be 3/2 according to quark model, but it is not unambiguous determined in experiments. Based on $448.1 \times 10^6$ $\psi(3686)$ events collected by BESIII, the process of $\psi(3686) \to \Omega^-\bar{\Omega}^+$, with the sequential decays of $\Omega^- \to K^-\Lambda, \Lambda \to p\pi^-$ and $\bar{\Omega}^+ \to K^+\bar{\Lambda}, \bar{\Lambda} \to \bar{p}\pi^+$ has been studied [22]. To increase the statistics, a single-tag method is implemented in which only the $\Omega^-$ or the $\bar{\Omega}^+$ is reconstructed via $\Omega^- \to K^-\Lambda \to K^-p\pi^-$ or $\bar{\Omega}^+ \to K^+\bar{\Lambda} \to K^+\bar{p}\pi^+$, and the $\bar{\Omega}^+$ or $\Omega^-$ on the recoil side is inferred from the missing mass of the center mass system. With different spin hypothesis (3/2 or 1/2), an unbinned maximum likelihood fit is performed to measure the free parameters with multi-dimensional angular distributions. By comparing the likelihood values with two assumptions to the around 4000 selected signal events, we determine the significance of the $J = 3/2$ hypothesis over the $J = 1/2$ to be larger than $14\sigma$ as Fig. 4 shown. With the helicity amplitudes measured, we calculated the $\cos\theta_{\Omega^-}$ dependence of the multipolar polarization operators as shown in Fig. 5. For the process of $\psi(3868) \to \Omega^-\bar{\Omega}^+$, we not only observe vector polarization

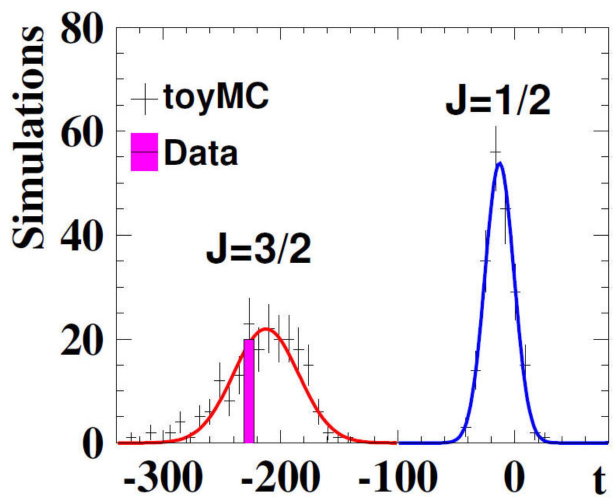


FIGURE 4. Distribution of the test statistic $t = S^{J=1/2} = S^{J=3/2}$ for a series of MC simulations performed under the $J = 1/2$ (right peak) and J = 3/2 (left peak) hypotheses. The lines represent Gaussian fits to the simulated data points. The $t$ value obtained from experimental data is indicated by the vertical bar.

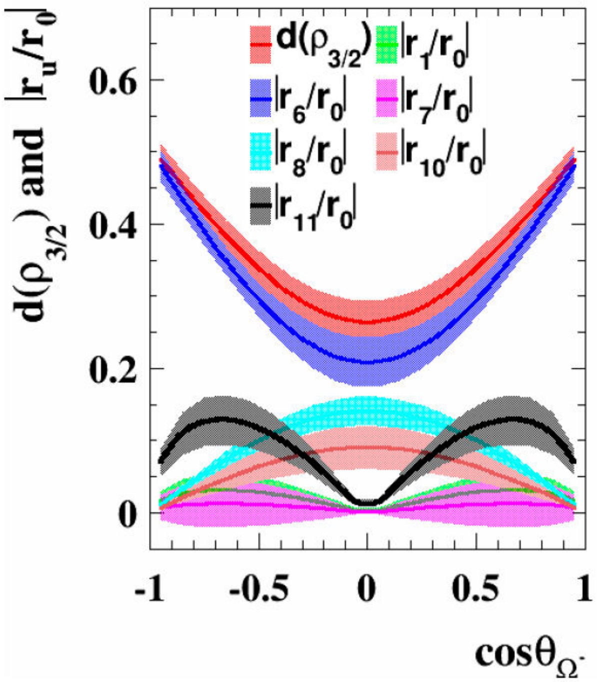


FIGURE 5. The $\cos\theta_{\Omega^-}$ dependence of the multipolar polarization operators. The solid lines represent the central values, and the shade areas represent $\pm$ one standard deviations.

($r_1$), but also quadrupole ($r_6, r_7, r_8$) and octupole ($r_{10}, r_{11}$) polarization contributions as the predictions [23, 24].

## 6. Summary

With the world's largest $J/\psi$ and $\psi(3686)$ data samples produced at that time, $J/\psi$ and $\psi(3686)$ decays into light hyperon pairs have been studied, including $J/\psi \to \Lambda\bar{\Lambda}$, $J/\psi(\psi(3686)) \to \Sigma^+\bar{\Sigma}^-$, $J/\psi \to \Xi^-\bar{\Xi}^+$, and $\psi(3686) \to \Omega^-\bar{\Omega}^+$. The hyperon pair produced in $e^+e^-$ collision is in a quantum entanglement system, and the transverse polarization of the hyperon is non-zero. This make the precise measurement of the decay parameters possible. They are measured by exploiting the full decay chain of the process by performing multi-dimensional fit to the angular observables. With the decay parameters, the CP conservation in the hyperon sector are tested. Now, BESIII experiment has collected ten times larger $J/\psi$ sample and seven times larger $\psi(3686)$ sample, measurements with higher precision and in more decay modes are expected in the near future.


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