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Charmonium spectrum and the color-octet heavy-quark potential from the instanton vacuum

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The properties of quarkonia are discussed in the framework of the nonrelativistic potential approach based on the instanton-induced interactions from the instanton vacuum. We have obtained the spectrum of charmonia and discuss the results. We also examine the instanton effects on the color-octet heavy-quark potential.

Keywords: Instanton-induced interactions; heavy-quark potential; quarkonia.

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1. Introduction

A conventional quarkonium consisting of one heavy quark and the corresponding heavy anti-quark is the simplest hadron that provides a great opportunity to scrutinize various theoretical methods. Since the constituents of the quarkonium are much heavier than those of light hadrons, potential models have been successfully applied for the description of the quarkonium structure [1, 2]. The heavy-quark potential contains mainly two different terms: The Coulomb-like one that originates from one-gluon exchange, and the confining one that simulates the quark confinement. However, the value of the strong coupling constant in the Coulomb-like term rather deviates from that in perturbative quantum chromodynamics (QCD), given the scale at the corresponding mass of the heavy quark. Moreover, certain effects from non-perturbative QCD are absent in this heavy-quark potential.

In the present talk, we introduce the non-perturbative contribution arising from the instanton vacuum [3–5] and examine its effects on the charmonium spectrum [6, 7], in addition to the existing two terms mentioned previously. Furthermore, we also include yet another non-perturbative instanton effect on one-gluon exchange. We will show that these non-perturbative instanton effects allow one to use more physical values of the parameters in the heavy-quark potential.

2. Heavy-quark potential from the instanton vacuum

The direct non-perturbative effects on the $Q\bar{Q}$ potential were discussed by Diakonov, Petrov and Pobylitsa [3]. Considering an instanton packing parameter λ to be very small ($\lambda = \rho^4/R^4 \sim 0.01$), which is defined as the fourth power of the ratio of the average instanton size $\bar{\rho}$ and the instanton inter-distance R, they derived the central part of the heavy-quark potential from the instanton vacuum systematically. Recently, Musakhanov *et al.* [6] also took into account the instanton effects on the Coulomb-like potential arising from one-gluon exchange (OGE). They evaluated a Yukawa-type heavy-quark potential, using the double expansion series in terms of the strong coupling constant α_s and the packing parameter λ. To keep the self-consistency, they found that α_s should be proportional to $\sqrt{\lambda}$.

The background field of the gluon is defined as a sum $A(\xi) = \sum_i A_i(\xi_i)$, where $\xi_i = (z_i, U_i, \rho_i)$ denotes a collective coordinate of the instantons describing positions z_i, U_i the color orientations, and ρ_i the sizes of the instantons. In the large N_c limit, where the width of the instanton distribution is of order $1/N_c$, ρ_i can be taken to be equal to the average size $\bar{\rho}$, $\rho_i = \bar{\rho}$. Having carried out lengthy manipulations, we obtain the quark and antiquark Lagrangians [6]:

$$L_Q = Q^\dagger(\theta^{-1} - ga - gA + \cdots)Q, \tag{1}$$

$$L_{\bar{Q}} = \bar{Q}^\dagger(\theta^{-1} - g\bar{a} - g\bar{A} + \cdots)\bar{Q}, \tag{2}$$

where a and A are perturbative gluon and instanton background fields, respectively. The dots are the next-to-leading order terms in the expansion of the inverse heavy-quark mass. $\langle t|\theta|t'\rangle = \theta(t-t')$ is given by the usual step function θ, so $\theta^{-1} = d/dt$. The fields a and A represent the fourth components of the fields, $a = ia_4$ and $A = iA_4$.

Following Ref. [6], we can define the heavy quark-antiquark correlator in the instanton background as

$$W = \int D\xi \exp\left[\frac{1}{2}\sum_{i\neq j=1}^{2}\left(\frac{\delta}{\delta a_\alpha^{(i)}} S_{\alpha\beta}^{(ij)} \frac{\delta}{\delta a_\beta^{(j)}}\right)\right] \times \frac{1}{D^{(1)} - ga^{(1)}} \frac{1}{D^{(2)} - g\bar{a}^{(2)}}\Bigg|_{a=0}, \tag{3}$$

which is identified as a Wilson loop. Here, $S_{\alpha\beta}^{(ij)}$ is a gluon propagator and $D\xi = \prod_{i=1} d\xi_i = V^{-1}\prod_{i=1} dz_i dU_i$. In the heavy-(anti)quark propagator, $D^{(i)} = \theta^{-1} - gA^{(i)}(\xi)$. The superscripts (1) and (2) in the propagators in Eq. (3) represent the lines along the time direction of the Wilson loop as shown in Fig. 1.

To obtain the heavy-quark potential, one can use the Pobylitsa equation [8] and the Fourier transformation [3]:

$$\langle t_1^{(1)}|W|t_1^{(2)}\rangle = \int \frac{d\omega}{2\pi} e^{i\omega(t_1^{(1)} - t_1^{(2)})} \frac{1}{W^{-1}(\omega)}, \tag{4}$$

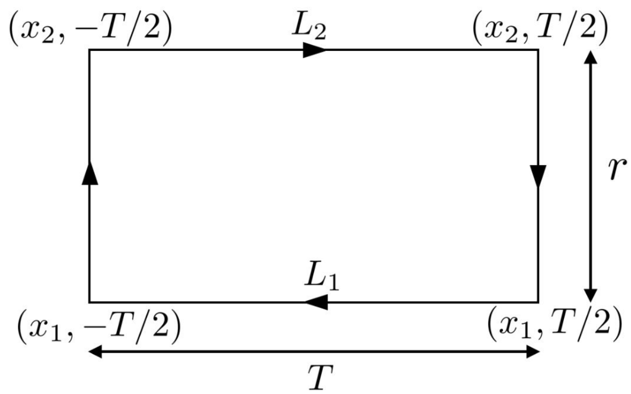

FIGURE 1. The rectangular Wilson loop. T is a long time interval and r is a distance between a quark and an antiquark.

where $t_1^{(1)}$ and $t_1^{(2)}$ are $-T/2$ in L_1 and $-T/2$ in L_2, respectively. Considering the infinite time limit $T \to \infty$ in Eq. (4), we get

$$\langle t_1^{(1)} | W | t_1^{(2)} \rangle \approx \exp\left[-(V_I^{(dir)} + V_I^{(per)}) T \right], \qquad (5)$$

where $V_I^{(\mathrm{dir})}$ and $V_I^{(\mathrm{per})}$ are the instanton-induced direct and OGE potentials, respectively. They have forms

$$V_I^{(dir)} = \frac{N}{2VN_c} \sum_{\pm} \int_d^3 z_{\pm} \times \mathrm{Tr}_c \left(1 - P \exp\left[i \int_{-\infty}^{\infty} dx_4 A_{\pm,4} \right] \times P \exp\left[i \int_{-\infty}^{\infty} dy_4 A_{\pm,4} \right] \right), \qquad (6)$$

$$V_I^{(\mathrm{per})} = g^2 \frac{\lambda_a}{2} \frac{\bar{\lambda}_a}{2} \left(1 - \frac{2r}{\pi} \int_0^{\infty} dq j_0(qr) \times \frac{3\pi^2 \lambda K_1^2(q\bar{\rho})}{1 + 3\pi^2 \lambda K_1^2(q\bar{\rho})} \right). \qquad (7)$$

Here $M_g(q)$ denotes the momentum dependent gluon mass

$$M_g(q) = \frac{2\pi}{\bar{\rho}} \left(\frac{6\lambda}{N_c^2 - 1} \right)^{1/2} q\bar{\rho} K_1(q\bar{\rho}), \qquad (8)$$

where K_1 is the modified Bessel function of the second kind. For more details, see Refs. [3, 6]. In Eq. (6), the singular gauge instanton field $A_{\pm,\mu}$ is defined as

$$A_{\pm,\mu}(x, z_{\pm}) = \frac{\eta^{\mp}_{\mu\nu}(x - z_{\pm})_{\mu} \lambda^a \bar{\rho}^2}{(x - z_{\pm})^2((x - z_{\pm})^2 + \bar{\rho}^2)}, \qquad (9)$$

where $\eta^{\mp}_{\mu\nu}$ are the 'tHooft symbols.

One can see that the direct potential does not have any color operators. We will show details in the next section. On the other hand, the perturbative part has a prefactor defined by a color state, *e.g.* in the singlet state, the factor is

$$\left(\frac{\lambda_a}{2} \frac{\bar{\lambda}_a}{2} \right)_S = -\frac{N_c^2 - 1}{2N_c},$$

while the factor becomes

$$\left(\frac{\lambda_a}{2} \frac{\bar{\lambda}_a}{2} \right)_O = \frac{1}{2N_c}$$

in the octet state.

3. The direct color-octet potential from the instanton vacuum

We use the insertion of the color exchange point to analyze the direct octet potential from the instantons [9] (see Fig. 2).

Using the Wilson loop in Fig. 2, we can rewrite the correlation function in Eq. (3)

$$W = \int D\xi \exp\left[\frac{1}{2} \sum_{i \neq j = 1}^{2} \left(\frac{\delta}{\delta a_\alpha^{(i)}} S_{\alpha\beta}^{(ij)} \frac{\delta}{\delta a_\beta^{(j)}} \right) \right] \times \frac{1}{D^{(1)} - g a^{(1)}} T_c \frac{1}{D^{(2)} - g\bar{a}^{(2)}} T_c \Bigg|_{a=0}, \qquad (10)$$

where the repeated color index c is not summed over. From the average over the color orientations, one can rewrite Eq. (10):

$$W = \Big\langle \int dU U_{i_1 i_2} W^{(L_2)}_{i_2 j_1}(\mathbf{x}_2, \Delta t) U^{\dagger}_{j_1 j_2} (T_c)_{j_2 k_1} \times U_{k_1 k_2} W^{(L_1)\dagger}_{k_2 l_1}(\mathbf{x}_1, \Delta t) U^{\dagger}_{l_1 l_2} (T_c)_{l_2 i_1} \Big\rangle, \qquad (11)$$

where U_{ij} and $W^{(L_i)}$ denote a color-orientation operator and Wilson line of L_i, respectively. The integration over the color-orientation gives [10]

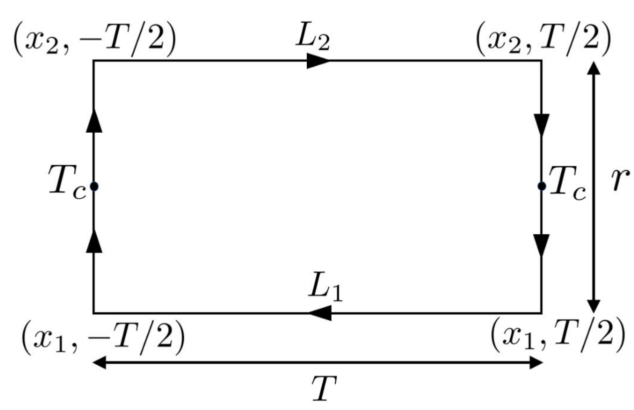

FIGURE 2. Inserted color operator T_a is in the spatial direction of the modified Wilson loop.

$$\int dU U_{i_1 i_2} U^{\dagger}_{j_1 j_2} U_{k_1 k_2} U^{\dagger}_{l_1 l_2}$$
$$= \frac{1}{N_c^2 - 1}\left\{\delta_{i_1 j_2}\delta_{k_1 l_2}\left(\delta_{j_1 i_2}\delta_{l_1 k_2} - \frac{1}{N_c}\delta_{j_1 k_2}\delta_{l_1 i_2}\right)\right.$$
$$\left. +\delta_{i_1 l_2}\delta_{k_1 j_2}\left(\delta_{j_1 k_2}\delta_{l_1 i_2} - \frac{1}{N_c}\delta_{j_1 i_2}\delta_{l_1 k_2}\right)\right\}. \tag{12}$$

Then Eq. (11) becomes

$$W = \frac{1}{2(N_c^2-1)}\langle \mathrm{Tr}_c W^{(L_2)}(\mathbf{x}_2,\Delta t)\mathrm{Tr}_c W^{(L_1)\dagger}(\mathbf{x}_1,\Delta t)\rangle$$
$$- \frac{1}{2N_c(N_c^2-1)}\langle \mathrm{Tr}_c W^{(L_2)}(\mathbf{x}_2,\Delta t) W^{(L_1)\dagger}(\mathbf{x}_1,\Delta t)\rangle$$
$$\equiv \frac{N_c^2}{2(N_c^2-1)} w^{(L_2)} w^{(L_1)} - \frac{1}{2(N_c^2-1)} W_s, \tag{13}$$

where W_s corresponds to the averaged Wilson loop in the color-singlet state and $w^{(L_i)}$ is the averaged Wilson line of L_i. In the definition of Eq. (13), we assumed that $\langle\langle W^{(L_2)}\rangle\langle W^{(L_1)\dagger}\rangle\rangle = \langle\langle W^{(L_2)}\rangle\rangle\langle\langle W^{(L_1)\dagger}\rangle\rangle$ since each Wilson line is isolated from the other line. From the definition of the heavy-quark potential, we can get the color-octet potential

$$V_{O,I}^{(\mathrm{dir})} = -\lim_{T\to\infty}\frac{1}{T}\ln\langle T|W|T\rangle$$
$$= -\lim_{T\to\infty}\frac{\frac{d}{dT}\langle T|W|T\rangle}{\langle T|W|T\rangle}. \tag{14}$$

The correlation function can be written as

$$\langle T|W|T\rangle = \frac{N_c^2}{2(N_c^2-1)}\langle T|w^{(L_2)}|-T\rangle\langle -T|w^{(L_1)}|T\rangle$$
$$- \frac{1}{2(N_c^2-1)}\langle T|W|T\rangle = \frac{N_c^2}{2(N_c^2-1)} e^{-2\Delta M T}$$
$$- \frac{1}{2(N_c^2-1)} e^{-V_{S,I}^{(\mathrm{NP})}T}. \tag{15}$$

From Eq. (13), one can get the result that is identical to the free energy of a single-quark system from lattice calculation [11, 12]. Here the free energy $2\Delta M$ is known as an asymptotic value of heavy-quark and antiquark static potential from the instanton vacuum. From Eq. (14), we have

$$V_{O,I}^{(\mathrm{dir})} = V_{S,I}^{(\mathrm{dir})} \equiv V_I^{(\mathrm{dir})}. \tag{16}$$

It means that the direct instanton effects are not affected by the color-states.

4. Charmonium spectrum

In this section, we will show the charmonium spectrum following the definitions in Ref. [6]. Consequently, the direct instanton induced Eq. (6) and instanton affected OGE Eq. (7) potentials can be parametrized as

$$V_I^{(\mathrm{dir})}(r) = \frac{4\pi\lambda}{N_c\rho}\mathcal{I}_{\mathrm{dir}}\left(\frac{r}{\bar\rho}\right), \tag{17}$$

$$V_{S,I}^{(\mathrm{per})}(r) = -\frac{4\alpha_s}{3r}\left(1 - \frac{2r}{\pi\bar\rho}\mathcal{I}_{\mathrm{scr}}\left[\frac{r}{\bar\rho}\right]\right)$$
$$\equiv V_C(r) + V_{S,I}^{(\mathrm{scr})}(r), \tag{18}$$

where the interpolation functions $\mathcal{I}_{\mathrm{dir}}$ and $\mathcal{I}_{\mathrm{scr}}$ are given as

$$\mathcal{I}_{\mathrm{dir}} = I_0^d\left\{1 + \sum_{i=1}^{2}\left[a_i^d x^{2(i-1)} + a_3^d(-b_3^d x)^i\right]e^{-b_i^d x^2}\right.$$
$$\left. + \frac{a_3^d}{x}\left(1 - e^{-b_3^d x^2}\right)\right\}, \tag{19}$$

$$\mathcal{I}_{\mathrm{scr}} = I_0^s\left\{\sum_{i=1}^{2}\left[a_i^s x^{2(i-1)} + a_3^s(-b_3^s x)^i\right]e^{-b_i^s x^2}\right.$$
$$\left. + \frac{a_3^s}{x}\left(1 - e^{-b_3^s x^2}\right)\right\}, \tag{20}$$

with the parameters $\mathcal{I}_0^{d,s}$, $a_i^{s,d}$ and $b_i^{s,d}$ [6]

$$\mathcal{I}_0^d = 4.41625\,, \qquad \mathcal{I}_0^s = 0.578695,$$

$$a^d = \begin{pmatrix} -1 \\ 0.128702 \\ -1.1047 \end{pmatrix}, \qquad a^s = \begin{pmatrix} 1 \\ 0.121348 \\ 2.71619 \end{pmatrix}, \tag{21}$$

$$b^d = \begin{pmatrix} 0.404875 \\ 0.453923 \\ 0.420733 \end{pmatrix}, \qquad b^s = \begin{pmatrix} 0.144123 \\ 0.189758 \\ 0.144123 \end{pmatrix}. \tag{22}$$

In Eq. (18), $V_{S,I}^{(\mathrm{scr})}$ has the meaning of the screening potential, which screens the coulomb-like potential $V_C(r)$ at large distances.

The spin-dependent interactions have forms [13, 14]

$$V_{SD}(r) = V_{SS}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\bar Q} + V_{LS}(r)\mathbf{L}\cdot\mathbf{S}$$
$$+ V_T(r)[3(\mathbf{S}_Q\cdot\hat n)(\mathbf{S}_{\bar Q}\cdot\hat n) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar Q}], \tag{23}$$

where the radial parts are given by

$$V_{SS}(r) = \frac{2}{3m_Q^2}\nabla^2 V_V, \tag{24}$$

$$V_{LS}(r) = \frac{1}{2m_Q^2 r}\left(3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right), \tag{25}$$

$$V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}\right). \tag{26}$$

Here $V_V = V_{S,I}^{(\mathrm{per})}(r)$ and $V_S = kr + V_0$ are the vector and the phenomenological scalar potential. The point-like spin-spin interaction term can be represented by the Gaussian approximation:

$$V_{SS}(r) = \frac{32\pi\alpha_s}{9m_Q^2}\delta(r) \approx \frac{32\alpha_s\sigma^3}{9m_Q^2\sqrt{\pi}}e^{-\sigma^2 r^2}, \tag{27}$$

where σ is a fitting parameter. The spin-dependent parts of the direct instanton potential is given in Ref. [13]

$$V_{SD}^{I}(r) = V_{SS}^{I}(r)\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} + V_{LS}^{I}(r)\mathbf{L}\cdot\mathbf{S}, \\ + V_T^I(r)[3(\mathbf{S}_Q\cdot\hat{n})(\mathbf{S}_{\bar{Q}}\cdot\hat{n}) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}], \quad (28)$$

where the radial parts are defined as

$$V_{SS}^{I}(r) = \frac{1}{3m_Q^2}\nabla^2 V_I(r), \quad (29)$$

$$V_{LS}^{I}(r) = \frac{1}{2m_Q^2}\frac{1}{r}\frac{dV_I(r)}{dr}, \quad (30)$$

$$V_T^I(r) = \frac{1}{3m_Q^2}\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)V_I(r). \quad (31)$$

Here $V_I(r) = V_I^{(\mathrm{dir})}(r)$. Then whole potential becomes

$$V_{Q\bar{Q}} = V_{S,I}^{(\mathrm{per})} + V_I^{(\mathrm{dir})} + V_{SD} + V_{SD}^I + V_S. \quad (32)$$

Next, by solving the Schrödinger equation with the sets in Table I, we obtain the charmonium spectrum in Table II.

To fit the potential parameters in Table I, we use the 5 S-wave states in Table II: J/ψ, $\eta_c(1^1S_0)$, $\psi(2^3S_1)$, $\eta_c(2^1S_0)$ and $\psi(4^3S_1)$. A similar study was conducted in Ref. [15] by considering only 4 input states in the case of without instantons. One can see that the spectrum can be obtained reasonably well with the small number of input data.

Let us analyze the value of the running coupling constant α_s. At the one-loop level in pQCD, it is given by the expression

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0}\frac{1}{\ln(\mu^2/\Lambda_{QCD}^2)}. \quad (33)$$

TABLE I. Set of potential parameters. The results denoted by sets "a" and "b" are obtained by using $V_0 = 0$ and $V_0 \neq 0$, respectively. Sets I and II are evaluated by using the instanton parameters "$\bar{\rho} = 1/3\,\mathrm{fm}$ and $R = 1\,\mathrm{fm}$" and "$\bar{\rho} = 0.36\,\mathrm{fm}$ and $R = 0.89\,\mathrm{fm}$", respectively.

Set	α_s(-)	k(GeV^2)	σ(GeV)	m_Q(GeV)	V_0(GeV)
SetIa	0.5141	0.1432	1.136	1.3634	0
SetIIa	0.4783	0.1375	1.174	1.3251	0
SetIb	0.5098	0.1444	1.166	1.3932	-0.0563
SetIIb	0.4773	0.1375	1.174	1.3211	0.0062

TABLE II. Charmonium spectrum.

State	Exp	SetIa	SetIIa	SetIb	SetIIb
$J/\psi(1^3S_1)$	3096.900 ± 0.006	3098	3097	3098	3097
$\eta_c\ (1^1S_0)$	2983.9 ± 0.5	2985	2988	2985	2988
$\psi\ (2^3S_1)$	3686.097 ± 0.025	3684	3685	3684	3685
$\eta_c\ (2^1S_0)$	3637.6 ± 1.2	3638	3639	3638	3639
$\psi\ (3^3S_1)$	4039 ± 1	4085	4086	4085	4086
$\eta_c\ (3^1S_0)$		4053	4054	4053	4053
$\psi\ (4^3S_1)$	4421 ± 4	4422	4420	4421	4421
$\eta_c\ (4^1S_0)$		4396	4394	4396	4395
$\chi_{c2}\ (1^3P_2)$	3556.17 ± 0.07	3550	3544	3550	3543
$\chi_{c1}\ (1^3P_1)$	3510.67 ± 0.05	3511	3510	3512	3510
$\chi_{c0}\ (1^3P_0)$	3414.71 ± 0.30	3414	3414	3415	3414
$h_c\ (1^1P_1)$	3525.38 ± 0.17	3519	3515	3520	3514
$\chi_{c2}\ (2^3P_2)$	3927.2 ± 2.6	3970	3967	3970	3966
$\chi_{c1}\ (2^3P_1)$		3936	3938	3937	3938
$\chi_{c0}\ (2^3P_0)$	3862^{+26+40}_{-32-13}	3873	3874	3873	3873
$h_c\ (2^1P_1)$		3941	3940	3942	3939
$\chi_{c2}\ (3^3P_2)$		4318	4313	4318	4313
$\chi_{c1}\ (3^3P_1)$		4286	4287	4287	4287
$\chi_{c0}\ (3^3P_0)$		4236	4235	4235	4235
$h_c\ (3^1P_1)$		4290	4287	4290	4287

Here $\beta_0 = (11N_c - 2N_f)/3$ and $\Lambda_{\mathrm{QCD}} = 0.217\,\mathrm{GeV}$ [16]. If the scale μ is very close to the charm-quark mass $m_c = 1.275$, then we get $\alpha_s(\mu) = 0.4258$. One can see that SetII instanton model gives quite reasonable values of α_s.

5. Summary and Outlook

In this work, we studied the charmonium spectrum, which was derived by including the heavy-quark potential from the instanton vacuum. The results showed that the instanton effects partially explain the origin of the strong coupling constant and allow us to use the value closer to that from QCD, compared with other phenomenological approaches.

We also discussed the color-octet potential, which was derived from the instanton vacuum. It was shown that the direct instanton potential was not affected by color states. The octet potential is important in studies of the chromoelectric polarizability.

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