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Heavy quark hybrid decays

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In order to understand the nature of the XYZ particles, theoretical predictions of the various XYZ decay modes are essential. In this work, we focus on the semi-inclusive decay of heavy quarkonium hybrids into traditional quarkonium in the Born-Oppenheimer EFT (BOEFT) framework. We find that our numerical results of the decay rates are different from the previous studies. We also develop a systematic framework in which the theoretical uncertainty can be systematically improved.

Keywords: Quarkonium; heavy hybrids; exotic states; Born-Oppenheimer approximation; effective field theories.

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1. Introduction

In Standard Model (SM), the hadrons are described as bound states of quarks and gluons bounded by strong interactions. Traditionally in quark model, all known hadrons were classified as mesons (bound state of quark-antiquark pair) or baryons (bound state of 3-quarks). However, the theory of strong interactions (QCD) also allows for existence of more complex hadron structures known as *exotic hadrons* such as tetraquark (4-quark states), pentaquark (5-quark states), hybrids (hadrons with active gluonic degrees of freedom) and glueballs (bound state of gluons). In the heavy-quark sector, these exotic hadrons are known as **XYZ states**. These states do not fit the traditional charmonium $(c\bar{c})$ or bottomonium $(b\bar{b})$ spectrum and in some cases have exotic quantum numbers such as charged Z_c and Z_b states. In 2003, the first exotic state $X(3872)$ was discovered by the Belle experiment [1]. Since then, several of the new XYZ states have been observed by the different experimental collaborations (see Ref. [2] for recent review).

On the theoretical side, there has been several proposals to understand the nature of the XYZ states. One possible interpretation for at-least some of the XYZ states could be quarkonium hybrids. The other possible interpretations are hadroquarkonium, heavy meson molecule, tetraquark, and diquark-diquark model, etc. However, no single model can actually explain all the XYZ states. On the experimental side, several new exotic states have been observed for which the masses and the decay rates has been measured (see Ref. [3]). Several of these exotic states have been discovered from their decays to standard quarkonium. Therefore, a theoretical understanding of the decays of XYZ states might be an another avenue for understanding their structure. In this work, our objective is to study the inclusive decays of heavy quark hybrids to traditional quarkonium *i.e.* $H_m \to Q_n + X$, where H_m is a low-lying hybrid, Q_n is a low-lying quarkonium state and X denotes other final state particles.

Within QCD, one can use lattice studies and effective field theories (EFTs) to describe the traditional quarkonium and quarkonium hybrids and compute its spectra. Since the heavy quarks in such system are nonrelativistic, the appropriate framework to use is the NRQCD effective theory [4, 5]. More specifically, if one focuses only on the dynamics of two heavy quarks (as in quarkonium), then the appropriate framework is the pNRQCD effective theory [6, 7]. In the case of quarkonium hybrids, there are well-separated energy scales: m_Q (mass of heavy quark) $>>$ $m_Q v$ (relative momentum scale) $>>$ $\Lambda_{\rm QCD}$ (energy scale for gluonic excitations) $>>$ $m_Q v^2$ (dynamics of two heavy quark). The above momentum hierarchy suggests of an energy gap between the gluonic excitations and the excitations of the heavy quark-antiquark pair that has also been confirmed by the lattice data [8, 9]. This justifies the use of effective theory based on Born-Oppenheimer approximation (BOEFT) to describe the hybrids [10–14]. In this work, we will use the BOEFT for the hybrids and pNRQCD for the low-lying quarkonium states. In order to compute the decay rates, we perform a matching calculation between BOEFT and pNRQCD to obtain the imaginary terms in the BOEFT potential. In Sec. 2, we compute the quarkonium and the hybrid spectrum, in Sec. 3, we perform the matching calculation and compute the decay rates and we conclude in Sec. 4.

2. Spectrum

2.1. Quarkonium

The conventional quarkonium states, $(Q\bar{Q})$ are color singlet bound states of a heavy quark and antiquark in the ground state static potential $V_{\Sigma_g^+}(r)$. The shape of the static potential $V_{\Sigma_g^+}(r)$ is well described by the Cornell potential. We use the form of the potential from Ref. [11]

$$V_{\Sigma_g^+}(r) = -\frac{\kappa_g}{r} + \sigma_g r + E_g^{Q\bar{Q}}, \tag{1}$$

where $\kappa_g = 0.489$, and the string tension parameter $\sigma_g = 0.187\,\mathrm{GeV}^2$ are determined from the fit to the lattice data.

The constant $E_g^{Q\bar{Q}}$ is different for both charmonium and bottomonium and is determined by comparison to the experimental spin-averaged mass from PDG 2020 [3]

$$E_g^{c\bar{c}} = -0.254\,\mathrm{GeV},\; E_g^{b\bar{b}} = -0.195\,\mathrm{GeV}, \tag{2}$$

where we have used the RS-scheme charm and bottom mass: $m_c = 1.477\,\mathrm{GeV}$ and $m_b = 4.863\,\mathrm{GeV}$ to compute the quarkonium spectrum.

2.2. Hybrids

Hybrid $(Q\bar{Q}g)$ are color singlet bound states of a color octet $Q\bar{Q}$ source coupled to gluonic excitations. In BOEFT description, the $Q\bar{Q}$ pair binds together in the background static potential generated by gluons. The hybrid quantum numbers are defined by the irreducible representations of the $D_{\infty h}$ symmetry group (like in diatomic molecules) in the static limit. In this limit, the spectrum is characterized by gluonic static energies that are nonperturbative quantities computed on lattice. In $r \to 0$ limit, where $\mathbf{r}$ is the relative coordinate of $Q\bar{Q}$, the gluon static energies are degenerate and quantum numbers are characterized by representations of symmetry group $O(3) \times C$ instead of $D_{\infty h}$ [6, 12, 13]. We focus here on the low-lying hybrids coming from Σ_u^- and Π_u static potentials and we closely follow the notations in Ref. [12].

In Ref. [12], the quarkonium hybrid spectrum was obtained by solving the coupled Schrödinger equations that are given by

$$\sum_{n=\Sigma,\,\Pi^\pm} \hat{\boldsymbol{n}}'^{\,*}(\theta,\varphi)\cdot\left(-\frac{\boldsymbol{\nabla}_r^2}{m_Q}+E_n^{(0)}(r)\right) \times \hat{\boldsymbol{n}}(\theta,\varphi)\,\Psi_n^{(m)}(\boldsymbol{r}) = \mathcal{E}_m\,\Psi_{n'}^{(m)}(\boldsymbol{r})\,, \tag{3}$$

where $\mathcal{E}_m$ is the hybrid energy, $E_n^{(0)}$ denotes the gluon static energies, $\Psi_n^{(m)}(\boldsymbol{r})$ denotes the wave-functions, the upper index (m) denotes the set of quantum numbers and the unit vectors $\hat{n}\,(\theta,\phi)$ are the projection operators that correctly reproduce the hybrid quantum numbers in $D_{\infty h}$ representation. We choose $\hat{n}\,(\theta,\phi) = \hat{r}$ for projecting onto the Σ_u^- state and the unit vectors $\hat{n}\,(\theta,\phi) = \hat{r}^{\pm} = \left(\hat{\theta} \pm i\hat{\phi}\right)/\sqrt{2}$ (where $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$ are the usual spherical unit vectors) for projecting onto the two components of the $\Pi_u^\pm$ state. Since, there are projection operators on both sides of $\boldsymbol{\nabla}_r^2$ in Eq. (3), the contributions from Σ_u^- and Π_u potentials mix together that results in pairs of solutions with same angular momentum quantum number but opposite parity [12]. The hybrid mass is given by $M_{Q\bar{Q}g} = 2m_Q + \mathcal{E}_m$ for $Q = (c\,,b)$.

The static potential that we use for computing the hybrid spectrum is split into a short-distance part and long-distance part [12]:

$$E_n^{(0)}\,(r) = \begin{cases} V_o^{\mathrm{RS}}(\nu_f)+\Lambda_H^{RS}(\nu_f)+b_n r^2, & r < 0.25\,\mathrm{fm} \\ \mathcal{V}(r), & r > 0.25\,\mathrm{fm} \end{cases}, \tag{4}$$

where for the short-distance part $(r < 0.25\,\mathrm{fm})$ we have used the RS-scheme octet potential $V_o^{\mathrm{RS}}(r)$ up to order α_s^3 in perturbation theory and the RS-scheme gluelump mass $\Lambda_H^{RS} = 0.87(15)$ GeV at the subtraction scale $\nu_f = 1$ GeV [9, 15]. The form of $V_o^{RS}(r)$ is given in Ref. [9]. The long-distance $(r > 0.25\,\mathrm{fm})$ part of the potential $\mathcal{V}(r)$ is given by

$$\mathcal{V}(r) = \frac{a_1}{r} + \sqrt{a_2\,r^2 + a_3} + a_4. \tag{5}$$

We choose this particular form so as to reproduce the short and long distance behavior of the Cornell potential. The parameters b_n in Eq. (4) and a_1, a_2, a_3 and a_4 in Eq. (5) are different for both Σ_u^- and Π_u potentials. They are determined by performing a fit to the lattice data in Refs. [8, 9] and demanding that the short-range and the long-range pieces in Eq. (4) are continuous upto first derivatives (see Ref. [12] for details). The result for the spectrum is given in Table III of Ref. [12].

We denote the hybrid wave-function by

$$\Psi_\lambda^{(m)}(\boldsymbol{r}) \equiv \Psi_\lambda^{(mjls)}(\boldsymbol{r}) = \psi_\lambda^{m}(r)\Phi_\lambda^{(jls)}\,(\theta,\phi)\,, \tag{6}$$

where λ labels the projection vectors: $\lambda = 0$ for $\hat{r}$ and $\lambda = \pm 1$ for $\hat{r}^{\pm}$ and m is the principle quantum number. The quantum number j is the eigenvalue of the total angular momentum operator: $\mathbf{J} = \mathbf{L} + \mathbf{S}$, where $\mathbf{S}$ is the $Q\bar{Q}$ spin (denoted by quantum number s) and $\mathbf{L} = \mathbf{L}_{Q\bar{Q}} + \mathbf{K}$ (denoted by quantum number L), where $\mathbf{K}$ is the gluon angular momentum, and $\mathbf{L}_{Q\bar{Q}}$ is the heavy quark angular momentum. We denote the hybrid state by the spectroscopic notation $m\,^{2s+1}L_j$, where $L = 0, 1$, etc is represented by $\mathcal{S}, \mathcal{P}$, etc., Hybrid states with same L and opposite parity (see Ref. [12]) will be denoted by a prime superscript such as $\mathcal{S}'$.

3. Inclusive decay rate

We want to study the inclusive decays of low-lying quarkonium hybrids to traditional quarkonium, *i.e.* $H_m \to Q_n + X$, where X denotes other final state particles. Let $\Delta E = \mathcal{E}_m - E_n^Q$ denote the energy (mass) difference, where E_n^Q is the quarkonium energy. For low-lying hybrid and quarkonium states, the following hierarchy of energy scale emerges: $m_Q v >> \Delta E >> \Lambda_{\mathrm{QCD}} >> m_Q v^2$. The hierarchy suggests that the theory at the scale ΔE is the weakly coupled pNRQCD (which is obtained from NRQCD by integrating out gluons with momentum and energy of order $\sim m_Q v$ and quarks with energy of order $\sim m_Q v$). Hence, starting with pNRQCD effective theory, we can integrate out gluons with $4-$momnetum of order $\sim \Delta E$ and $\sim \Lambda_{\mathrm{QCD}}$ in loops and match it to Born-Oppenherimer theory (BOEFT) that describes system at energy scale $m_Q v^2$. This matching will give rise to an imaginary potential in BOEFT that is related to the inclusive decay rate of hybrids using optical theorem.

The pNRQCD Lagrangian upto NLO in multipole expansion or in $1/m_Q$ is given by

$$L_{\rm pNRQCD} = \int\limits_{\boldsymbol{R}}\int\limits_{\boldsymbol{r}} \Big({\rm Tr}\left[S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right] + g{\rm Tr}\left[S^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,O + O^\dagger\boldsymbol{r}\cdot\boldsymbol{E}\,S\right] + \frac{gc_F}{m_Q}{\rm Tr}\left[S^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,O + O^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,S\right] + \cdots\Big), \quad (7)$$

where $\int_{\boldsymbol{R}} \equiv \int d^3\boldsymbol{R}$, S and O denotes the singlet and the octet fields and ellipses represents higher order terms as well as terms including light quarks and gluons. The singlet and octet Hamiltonian densities are given by

$$h_s = -\boldsymbol{\nabla}_r^2/m_Q + V_s(r),\; h_o = -\boldsymbol{\nabla}_r^2/m_Q + V_o(r), \quad (8)$$

where V_s and V_o are the perturbative singlet and the octet potentials. The $\mathbf{r}\cdot\mathbf{E}$ vertex in Eq. (7) is responsible for spin-conserving decays of hybrid whereas the $\mathbf{S}\cdot\mathbf{B}$ vertex is responsible for spin-flipping decays. The BOEFT Hamiltonian $(H_{\rm BO})$ that describes the hybrid states is given by

$$H_{\rm BO} = \int\limits_{\boldsymbol{R}}\int\limits_{\boldsymbol{r}} {\rm Tr}\left[H^{i\dagger}\left(h_o\delta^{ij} + V^{ij}_{soft} + \Delta V\delta^{ij}\right)H^j\right], \quad (9)$$

where $V^{ij}_{soft} = \Lambda_{\rm glue} + b^{ij}r^2 + \cdots$ (ellipses represent higher order terms in multipole expansion), H^i denotes the hybrid fields (index i is the vector index), and ΔV is the effective potential that is determined by matching condition. Using the optical theorem, the decay rate is given by $\Gamma_{H_m\to Q_n} = -2\langle H_m|{\rm Im}\,\Delta V|H_m\rangle$.

Starting with pNRQCD, we want to integrate out gluons with $4-$momentum $\sim\Delta E$ and $\sim\Lambda_{\rm QCD}$ in two steps, and obtain the BOEFT theory at $m_Q v^2$. This is implemented by the matching condition and we do that by computing the two-point function in both the theories and equating them. For spin-conserving decays of hybrid to quarkonium, the two-point function in pNRQCD is expanded up to $\mathcal{O}\left(r^2\right)$ in the multipole expansion using the NLO pNRQCD Lagrangian in Eq. (7) which is equated to the corresponding two-point function in BOEFT computed using Eq. (9). For the spin-flipping decay of hybrids, the two-point function is expanded up to $\mathcal{O}\left(1/m_Q^2\right)$ using the pNRQCD Lagrangian in Eq. (7). For the decay of hybrid H_m to a specific quarkonium state $\Phi_n^Q(\boldsymbol{r})$, the spin-conserving decay rate is thus given by (see details of the calculation in Ref. [16])

$$\Gamma(m\to n) = \sum_{n'}\left|\int d^3r\,\Phi^{s\dagger}_{n'}(\boldsymbol{r})\Phi^Q_n(\boldsymbol{r})\right|^2\Gamma_{mn'}\,, \quad (10)$$

where in the above expression we have included the overlap between the quarkonium $\left(\Phi^Q_n\right)$ and the Coulomb singlet $\left(\Phi^s_{n'}\right)$ wave-functions and $\Gamma_{mn'}$ is given by

$$\Gamma_{mn'} \equiv \frac{4\alpha_s T_F}{3N_c}\int\frac{d^3\boldsymbol{l}}{(2\pi)^3}\int\frac{d^3\boldsymbol{l'}}{(2\pi)^3}\int d^3\boldsymbol{r}\int d^3\boldsymbol{r'}\int d^3\boldsymbol{r''}\int d^3\boldsymbol{r'''}\left[\Psi^{i\dagger}_m(\boldsymbol{r})\Phi^o_{\boldsymbol{l}}(\boldsymbol{r})\right]\left[\Phi^{o\dagger}_{\boldsymbol{l}}(\boldsymbol{r'})r'^i\Phi^s_{n'}(\boldsymbol{r'})\right] \times\left[\Phi^{s\dagger}_{n'}(\boldsymbol{r''})r''^i\Phi^o_{\boldsymbol{l'}}(\boldsymbol{r''})\right]\left[\Phi^{o\dagger}_{\boldsymbol{l'}}(\boldsymbol{r'''})\Psi^i_m(\boldsymbol{r'''})\right]\left(\Lambda_{\rm glue} + E^o_{\boldsymbol{l}}/2 + E^o_{\boldsymbol{l'}}/2 - E^s_n\right)^3, \quad (11)$$

where α_s is evaluated at the scale $\Delta E = \mathcal{E}_m - E^Q_n$, Ψ^i_m is the hybrid wave-function given in Eq. (6) (i is the vector index), $\Phi^o_{\boldsymbol{l}}$ is the octet wave-function, $E^o_{\boldsymbol{l}}$ is the octet energy, E^s_n is the singlet energy, and $\Lambda_{\rm glue} = 0.87(15)$ GeV in RS-scheme. For octet wave-function, we use the RS-scheme potential $V_o^{\rm RS}(r)$, while for Coulomb singlet wave-function, we use Eq. (1) without the linear piece. For Coulomb singlet, the constant $E_g^{Q\bar{Q}}$ in Eq. (1) is chosen such that we reproduce spin-averaged $1s$ charmonium and bottomonium mass.

Suppose we assume that the overlap between hybrid and octet wave-functions $\int d^3r\,\Psi^{i\dagger}_m(\boldsymbol{r})\Phi^o_{\boldsymbol{l}}(\boldsymbol{r})$ is nonzero only for hybrid energy $\mathcal{E}_m$: $\mathcal{E}_m \approx E^o_{\boldsymbol{l}} + \Lambda_{\rm glue}$ (which holds for short-distance approximation where contribution from $b^{ij}r^2$ piece in Eq. (9) is ignored), the overlap function of quarkonium and Coulomb singlet wave-function satisfy $\int d^3r\,\Phi^{s\dagger}_n(\boldsymbol{r})\Phi^Q_n(\boldsymbol{r})\approx 1$ and the singlet and quarkonium energy satisfy $E^Q_n \approx E^s_n$, then Eq. (10) is simplified to

$$\Gamma^{\rm sim}(m\to n) \approx \frac{4\alpha_s T_F}{3N_c}T^{ij}(T^{ij})^*\left(\mathcal{E}_m - E^Q_n\right)^3, \quad (12)$$

$$T^{ij} \equiv \int d^3r\,\Psi^{i\dagger}_m(\boldsymbol{r})r^j\Phi^Q_n(\boldsymbol{r}). \quad (13)$$

The simplified decay rate in Eq. (12) is identical to Eq. (17) in Ref. [11] and Eq. (62) in Ref. [17]. However, in Ref. [11], the authors only consider the diagonal elements T^{ii} of the matrix element instead of the full tensor structure T^{ij} in Eq. (13). This led to a selection rule that hybrid states with $L = L_{Q\bar{Q}}$ does not decay. This is incorrect as such decays are allowed by considering the tensor structure T^{ij} of the matrix element in Eqs. (10) and (12). The results are shown in table below.

In Table I, we see that for most of the cases, the values of the decay rate obtained using the simplified expression in

TABLE I. Inclusive decay rate (spin-conserving) for hybrid decays to traditional quarkonium states: $H_m \to Q_n + X$. The hybrid states are denoted by $m\left(L_{Q\bar{Q}}\right)_L$ whereas the quarkonium states are denoted by $nL^{'}$. The decay rate in third column is computed using Eq. (12) and in last column using Eq. (10). The upper error bar is from changing the scale to $\Delta E/2$ in α_s while the lower error bar is from changing the scale to $2\Delta E$ in α_s.

$m\left(L_{Q\bar{Q}}\right)_L \to nL^{'}$	ΔE (GeV)	$\Gamma^{\rm sim}$ (MeV)	Γ (MeV)
	charmonium hybrid decay		
$1p_0 \to 1s$	1.522	$327\,^{+137}_{-71}$	$117\,^{+49}_{-25}$
$1p_0 \to 2s$	0.912	$194\,^{+118}_{-53}$	$71\,^{+43}_{-19}$
$2p_0 \to 1s$	1.986	$45\,^{+16}_{-9}$	$15\,^{+5}_{-3}$
$1p_1 \to 1s$	1.218	$156\,^{+76}_{-37}$	$146\,^{+71}_{-35}$
$2p_1 \to 1s$	1.599	$65\,^{+27}_{-14}$	$9\,^{+4}_{-2}$
$2(s/d)_1 \to 1p$	1.013	$113\,^{+63}_{-29}$	$7\,^{+4}_{-2}$
$4(s/d)_1 \to 1p$	1.381	$99\,^{+44}_{-22}$	$8\,^{+4}_{-2}$
	bottomonium hybrid decay		
$1p_0 \to 1s$	1.622	$69\,^{+28}_{-14}$	$102\,^{+41}_{-22}$
$1p_0 \to 2s$	1.055	$159\,^{+86}_{-40}$	$20\,^{+11}_{-5}$
$2p_0 \to 1s$	1.909	$34\,^{+12}_{-7}$	$15\,^{+5}_{-3}$
$2p_0 \to 2s$	1.342	$42\,^{+19}_{-10}$	$63\,^{+29}_{-14}$
$3p_0 \to 1s$	2.174	$19\,^{+6}_{-4}$	$12\,^{+4}_{-2}$
$3p_0 \to 2s$	1.607	$20\,^{+4}_{-8}$	$25\,^{+10}_{-5}$
$4p_0 \to 1s$	2.421	$12\,^{+4}_{-2}$	$7\,^{+2}_{-1}$
$4p_0 \to 2s$	1.854	$11\,^{+4}_{-2}$	$30\,^{+11}_{-6}$
$1p_1 \to 1s$	1.404	$29\,^{+13}_{-7}$	$80\,^{+35}_{-18}$
$2p_1 \to 1s$	1.617	$28\,^{+11}_{-6}$	$26\,^{+11}_{-6}$
$3p_1 \to 1s$	1.828	$22\,^{+8}_{-4}$	$16\,^{+6}_{-3}$
$2(s/d)_1 \to 1p$	1.068	$15\,^{+8}_{-4}$	$163\,^{+87}_{-41}$
$3(s/d)_1 \to 1p$	1.264	$73\,^{+35}_{-17}$	$90\,^{+43}_{-21}$
$3(s/d)_1 \to 2p$	0.907	$22\,^{+14}_{-6}$	$83\,^{+51}_{-23}$
$4(s/d)_1 \to 1p$	1.300	$155\,^{+72}_{-36}$	$103\,^{+48}_{-24}$

Eq. (12) and the general expression (involving several overlap expressions) in Eq. (10) differs with each other even considering the error bars. This raises the questions on the validity of the approximations that were used to obtain the simplified expression in Eq. (12). We find that the overlap between hybrid and octet wave-function $\int d^3r\, \Psi_m^{i\dagger}(\boldsymbol{r})\Phi_{\boldsymbol{l}}^o(\boldsymbol{r})$ is nonzero over wide range of octet energies, if we don't assume $\mathcal{E}_m \approx E_{\boldsymbol{l}}^o + \Lambda_{\rm glue}$ (which holds for short-distance approximation. See Ref. [16]). Also, we find that the approximation about the singlet and quarkonium energy $E_n^Q \approx E_n^s$ is only valid for 1s charmonium and bottomonium.

4. Conclusions

In this work, we use the framework of nonrelativistic effective theory to study the inclusive decays of heavy quark hybrids to traditional quarkonium, We obtain an expression of the decay rate (Eq. (10)) by doing one-loop matching between pNRQCD and BOEFT. The decay rate depends on the overlap function of hybrid, octet, Coulomb singlet and quarkonium wave-functions. We find that using certain assumptions, the decay rate in Eq. (10) reduces to a simplified expression in Eq. (12) that was earlier derived in Refs. [11, 17]. However, the results in Table I raises questions on the validity of those approximations.

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