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The anomalous magnetic moment of the muon: short overview

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We critically review the current status of the muon $g-2$, where the Standard Model prediction deviates at 4.2σ from the experimental average, which is tantalizing for new physics interpretations.

Keywords: Standard model; muon $g-2$.

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1. Introduction

The anomalous magnetic moment of charged leptons ℓ, $a_\ell = (g_\ell - 2)/2$, being g_ℓ the ℓ gyromagnetic factor, was for $\ell = e$ one of the most important observables in the validation and consolidation of quantum field theory at the level of quantum (loop) corrections. For $\ell = \mu$ is nowadays among the most stringent new physics tests. The short τ lifetime has prevented significant measurements of a_τ so far.

Foley and Kusch [1, 2] first measured a non-vanishing a_e, consistent with a permille effect, that was explained by Schwinger [3] as a result of the dominant one-loop photon exchange correction to the QED interaction vertex, $a_e = \alpha/(2\pi) + \mathcal{O}(\alpha^2)$. To this day, a_e is the most precisely measured and predicted [4–9] observable in Nature, with an amazing precision at the level of 3×10^{-13}.[i]

The current world average for a_μ is obtained combining the final BNL result [11]

$$a_\mu^{\rm BNL} \times 10^{11} = 116592089(63), \qquad (1)$$

with the recent FNAL measurement [12]

$$a_\mu^{\rm FNAL} \times 10^{11} = 116592040(54)\,, \qquad (2)$$

to get

$$a_\mu^{\rm Exp} \times 10^{11} = 116592061(41)\,. \qquad (3)$$

This experimental average is 4.2σ away from the Standard Model (SM) prediction of a_μ, as published in the White Paper (WP) [13][ii]

$$a_\mu^{\rm SM} \times 10^{11} = 116591810(43)\,. \qquad (4)$$

Very importantly, however, the lattice QCD evaluation of the dominant hadronic contribution to a_μ by the BMW Coll. [67] reduces the previous discrepancy to 1.6σ. Thus, the difference between $a_\mu^{\rm SM}$ and $a_\mu^{\rm BMW}$ needs to be understood, in anticipation of future measurements that could increase the $a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ discrepancy at the level of an indirect new physics discovery.

In the remainder of this contribution we will first review the basic aspects behind $a_\mu^{\rm SM}$ according to the WP, highlighting then the Mexican predictions for its different contributions and recent related developments, ending with our conclusions and outlook.

2. The SM prediction of a_μ

At the current level of experimental and theoretical precision, all SM interactions: QED, electroweak and QCD are relevant to understand a_μ^{SM}. QED has been computed up to five-loop order $a_\mu^{QED} = 116584718.9(1) \times 10^{-11}$ [8, 13, 44], with the error budget saturated by the effect of uncomputed six-loop contributions. These are however 400 times smaller than experimental or hadronic uncertainties on a_μ. The electroweak (understood as being non-QED and non-QCD) contributions have been calculated up to two loops [45, 46], including an error estimate for the next order, whose leading effect is accounted for, and are $a_\mu^{EW} = 153.6(1.0) \times 10^{-11}$ [46]. The latter uncertainty is 40 times smaller than current errors of $a_\mu^{\rm EXP,SM}$.

Clearly, QCD (in fact hadronic) contributions to $a_\mu^{\rm SM}$ saturate its error. These are of two types: the so-called hadronic vacuum polarization (HVP) and the hadronic light-by-light ($HLbL$) contributions. HVP, at leading order (LO), amounts to include one hadronic blob in the middle of the photon propagator in the Schwinger diagram. In the $HLbL$ case, three photons are radiated from the muon line and connected to the hadronic blob, from which the additional (external) photon is radiated.

The known running of $\alpha_S(Q^2)$ immediately tells that the low-energy regions of the hadronic loops will dominate completely $a_\mu^{HVP,HLbL}$, which complicates greatly their accurate computation. Lattice QCD continuous and fast improvements are not covered here, the interested reader is addressed to the dedicated sections in Ref. [13] for their detailed account.

A fundamental advance towards precision computations of a_μ came through the model-independent relation of $a_\mu^{HVP,LO}$ to $e^+e^- \to \rm hadrons$ data [68, 69], which relies on unitarity and analyticity. In this way, the very accurate measurement of the bare[iii] hadronic cross-section (more than

35 exclusive channels are added below 2 GeV and inclusive measurements are used onwards), normalized to the corresponding point cross-section, is integrated from the $\pi^0\gamma$ threshold to infinite, in the variable $s = (p_{e^-} + p_{e^+})^2$, with a kernel that enhances the low-energy region as $1/s$. The profile of the narrow quarkonium resonances is taken into account minutely and at some high-enough energy (11 GeV for sure, although a lower value is sometimes taken) the four-loop QCD prediction [70] is used for $\sigma(e^+e^- \to \mathrm{hadrons})$.

For $\sqrt{s} \leq 1$ GeV, agreement between the evaluations [16, 18, 19] (in turn, below) at the quoted uncertainties is remarkable:

$$a_\mu^{HVP,LO}|_{\sqrt{s}\leq 1\mathrm{GeV}} \times 10^{10} = 495.0(2.6), \quad 497.4(3.6),\ 493.8(1.9)\,. \tag{5}$$

Detailed channel-by-channel comparison including uncertainties on uncertainties and their correlations, yields [13]

$$a_\mu^{HVP,LO} \times 10^{10} = 693.1(4.0)\,. \tag{6}$$

The previous uncertainty receives two equal contributions. The first one is experimental and is dominated by the 2π (1.9×10^{-10}) and 3π channels (1.5×10^{-10}). The second one, of systematic nature, arises from the incompatibility between the 2π BABAR [71] and KLOE [72] data sets. Other sources of error are negligible at the quoted precision. Citing the WP [13], Eq. (6) is a conservative but realistic assessment of the current situation' (it was published in December 2020). We must recall, however, the BMW result [67], $a_\mu^{HVP,LO,BMW} \times 10^{10} = 707.5 \pm 5.5$, 2.1σ away from the WP value.

Higher-order iterations of HVP have also been evaluated, yielding [13] ([19] at NLO and [20] at NNLO)

$$a_\mu^{HVP,NLO} \times 10^{10} = -9.83(7), \quad a_\mu^{HVP,NNLO} \times 10^{10} = 1.24(1)\,, \tag{7}$$

with neglibible errors compared to the one in Eq. (6). Previous results add up to give

$$a_\mu^{HVP} \times 10^{10} = 684.5(4.0)\,. \tag{8}$$

As opposed to HVP at LO, there is not a closed formula relating the LO HLbL to measurements. HLbL turns out to be much more difficult than HVP because in the latter the only varying scale is the virtuality of the photon, while in HLbL there are now three photons in the loop with two additional changing scales, these being a couple of Madelstam variables related to the light-by-light scattering (for one on-shell external photon). Despite of that, impressive progress has been achieved defining consistently contributions to HLbL in terms of cuts with increasing number of on-shell particles, which are now related to observables via separate dispersion relations for each possible intermediate state [73].

In the WP, the contribution from the lightest pseudoscalar poles is obtained combining the dispersive evaluation of the π^0 contribution [33] with the Canterbury Approximants computation of the η and η' poles [31], yielding

$$a_\mu^{\pi^0+\eta+\eta'} \times 10^{11} = 93.8^{+4.0}_{-3.6}\,. \tag{9}$$

Other HLbL contributions cancel to a large extent, as the result quoted in the WP for the whole LO contribution is

$$a_\mu^{HLbL,LO} \times 10^{11} = 92 \pm 18\,, \tag{10}$$

where the uncertainty reduced from 19 to 18 when lattice evaluations were also accounted for [13].

These other contributions which basically compensate among them are the $\pi\pi$ [32] and KK [40] box contributions, the $\pi\pi$ contributions in S [32, 37, 39] or D [37, 38] waves, axial-resonance contributions (which correspond to mostly three but also four meson cuts) [10, 37, 41] and purely short-distance contributions [30, 35, 36]. The effect of higher-order HLbL corrections is small, *e.g.* $a_\mu^{HLbL,NLO} \times 10^{-11} = 2 \pm 1$ [13, 42].

Adding up all previous pieces renders a_μ^{SM}, Eq. (4).

2.1. Mexican contributions

These will be reviewed chronologically next.

2.1.1. *An upper bound on the HLbL contribution*

A parton-level estimation of a_μ^{HLbL} was given in Ref. [74]. This approach is exact in the limit of infinitely heavy quarks and overestimates the real contribution for light quarks, where non-perturbative effects dominate. As result, it gives a model-independent upper bound, which is

$$a_\mu^{HLbL} \times 10^{9} = 1.59\,, \tag{11}$$

at 95% C. L., in perfect agreement with Eq. (10). Approaches with a similar philosophy are *e.g.* Refs. [75, 76].

2.1.2. *The discrepancy between τ and e^+e^- spectral functions revisited and the consequences for the HVP contribution*

Hadronic tau decays ALEPH data [77] enabled to improve the precision in the $a_\mu^{HVP,LO}$ evaluation [78]. At such level of accuracy, isospin-breaking corrections in the conserved vector current relation between the hadronic e^+e^- cross-section and the spectral function in hadronic tau decays became important and were studied in Refs. [79, 80], using Resonance Chiral Theory [81, 82] for the structure-dependent contributions. The analysis in Ref. [83] was based upon the computation of these effects using vector meson dominance [84, 85]. The long-distance electromagnetic correction factor (G_{EM}) was larger in Refs. [84, 85] than in Refs. [79, 80]. It was already clear at that time that the ρ-ω-π effective interactions (present in Refs. [84, 85] but absent in

Refs. [79, 80]) were responsible for this discrepancy[iv]. Conservatively, Ref. [83] decided to enlarge the corresponding error so that both groups of results were covered by the uncertainty band. Ref. [83] result was 1.9σ lower than the BNL measurement, Eq. (1) (tau based determinations are always closer to $a_\mu^{\rm EXP}$ than the e^+e^- ones). Ref. [83] was subsequently updated in Refs. [86, 87]. In the White Paper [13], τ-based evaluations of $a_\mu^{HVP,LO}$ were not used because at the level of precision of e^+e^- data, the uncertainty on isospin-breaking corrections was not small enough.

2.1.3. Light pseudoscalar meson contributions to HLbL

Within Resonance Chiral Theory, Ref. [88] computed the π^0 exchange contribution to a_μ^{HLbL} and we [89] refined this evaluation, taking into account a consistent set of short-distance constraints [90] and extending it to the η, η' mesons. These results were updated, for the corresponding pole contributions and refined (including $U(3)$ flavor symmetry breaking effects) in Ref. [91], where we found

$$a_\mu^{\pi^0+\eta+\eta',\,HLbL} \times 10^{10} = 8.47 \pm 0.16_{\rm sta} \pm 0.09_{1/N_C}{}^{+0.50}_{-0}{}_{\rm asym}\,, \qquad (12)$$

in which the first error comes from the fit, the second one from subleading corrections in the large-N_C limit [92] and the last one is an asymmetric error coming from the fact that -with only one multiplet of vector resonances- it is not possible to comply with the limit of QCD in which both photon virtualities are asymptotically large. Accounting for this latter uncertainty, our result is in agreement with Eq. (9). In the chiral and $N_C \to \infty$ limits we get [89, 91]

$$a_\mu^{\pi^0+\eta+\eta',\,HLbL} \times 10^{10} = 8.27 \pm 0.16_{sta}{}^{+0.29}_{-0.09}{}_{1/N_C+\rm chiral}{}^{+0.50}_{-0}{}_{\rm asym}\,, \qquad (13)$$

in fine agreement with our Eq. (12).

Within a Schwinger-Dyson eqs. approach to QCD, in Ref. [93], using the Qin-Chang interaction [94] and improving rainbow-ladder truncation to comply with the (non-Abelian) axial anomaly, we could fulfil all known long- and short-distance constraints, describing the whole data accurately, achieving

$$a_\mu^{\pi^0+\eta+\eta',\,HLbL} \times 10^{10} = 8.97 \pm 0.48\,, \qquad (14)$$

in nice accord with Eq. (9). The meson transition form factors employed in this work were obtained in Refs. [95, 96]. In our work, we also first predicted the η_c and η_b poles contribution using the results in Refs. [97, 98]. The latter will remain to be negligible for the precision of the present generation of experiments, but the former, $(0.09 \pm 0.01) \times 10^{-10}$, will need to be considered by the time of the final FNAL result. Our Eq. (14) agrees nicely with a previous Schwinger-Dyson evaluation [99].

2.1.4. Axial-vector meson contributions to HLbL and short-distance constraints

In Ref. [41] we first showed (and solved) an issue with the existing bases employed to compute the contribution of axial-vector mesons to the a_μ^{HLbL} and provided a dictionary between them. The problematic shows whenever axial-vector mesons are offshell, which is the case in the HLbL. We definitely settled these issues in our later work, [49], in connection with the short distance behavior. In particular, we advocated a preferred choice of basis, which avoids the need for including nonvanishing offshell unphysical form factors to fulfil the axial anomaly [100, 101][v]. Our results for the axial-vector meson contributions to a_μ^{HLbL} [49]

$$a_\mu^{\rm Axials,HLbL} \times 10^{11} = 16.0^{+5.1}_{-4.5}\,, \qquad (15)$$

favor those obtained in holographic QCD [63, 103–105]

$$a_\mu^{\rm Axials,HLbL} \times 10^{11} = 17.1 \pm 1.3\,, \qquad (16)$$

for the transverse contribution coming from axial-vector mesons. We stress Eqs. (15) and (16) are substantially larger than the value published in the WP[vi]: the quoted $6(6)\times 10^{-11}$ should be $(17.1\pm 1.3)\times 10^{-11}$ and this needs to be taken into account for the WP update.

2.1.5. New τ-based evaluation of $a_\mu^{HVP,LO}$

The ρ-ω-π interactions were absent in Refs. [79, 80], as these included up to next-to-leading order effects in the chiral expansion [106–108] and such effective vertices arise at the next order. These interactions (subleading at low energies) were included in Ref. [51], adding the extended resonance Lagrangian in Refs. [88, 109] to the original one [81, 82]. We [51] confirmed the importance of such ρ-ω-π effective vertices, in agreement with Refs. [84, 85]. Our tau-based evaluation yields [51] $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm th} = (12.5 \pm 6.0) \times 10^{-10}$, (including up to next-to-leading order terms in the chiral expansion), and $\Delta a_\mu = \left(17.5^{+6.8}_{-7.5}\right) \times 10^{-10}$ at one order further, corresponding to 2.1 and 2.3σ deviations, respectively. Another recent determination of $a_\mu^{HVP,LO}$ using tau data can be found in Ref. [110].

2.1.6. Pion and Kaon box contributions from Schwinger-Dyson eqs

The fundamental non-perturbative ingredient to compute these contributions is the pion (Kaon) electromagnetic form factor, which we take from the Schwinger-Dyson eqs. evaluation in Refs. [111-114]. References [111, 112] truncate the infinite series of coupled integral equations according to the rainbow ladder prescription and Refs. [113, 114] goes beyond that by including meson cloud effects. Within rainbow ladder truncation, in addition to the direct evaluation, integration was also made using perturbation theory integral representations [111, 112], with fully compatible results.

The resulting values [115],

$$a_\mu^{\pi^\pm-\text{box}} \times 10^{11} = -15.56 \pm 0.47 \,, \quad (17)$$

$$a_\mu^{K^\pm-\text{box}} \times 10^{11} = -0.48 \pm 0.04 \,, \quad (18)$$

are in nice agreement with the WP results [13], with competitive uncertainties.

3. Conclusions and outlook

The uncertainty or the FNAL measurement, Eq. (2), will have a fourfold improvement (at least) by the time of their final results, in a few years from now. This clearly demands an equivalent reduction of the theory error, Eq. (4), which should reach $(10, 15) \times 10^{-11}$. This is basically one tenth of the difference between the WP [13] (data-driven) evaluation of $a_\mu^{HVP,LO}$ and the BMW [67] (lattice QCD) result. Obviously, understanding this discrepancy is the most urgent task of the Muon $g-2$ Theory Initiative (WP authors). This need is further reinforced by the future MUonE [116] and J-PARC [117] experiments. The main advantage of the J-PARC a_μ setup is that the experimental design is completely different (being a tabletop experiment with ultra-cold muons), with orthogonal systematics to the last a_μ measurements, that will provide an absolutely independent check on the FNAL and earlier BNL results, which share storage ring. MUonE will be extremely valuable as well, by accessing $a_\mu^{HVP,LO}$ from a measurement of VP effects at low Q^2 spacelike values, which shall complement nicely lattice simulations and perturbation theory at larger virtualities.

Temptingly, whether the WP result or the BMW value is confirmed, there needs to be new physics *somewhere*, according to electroweak precision and $\sigma(e^+e^- \to \text{hadrons})$ data, taking into account the influence of the latter on the hadronically-induced running of the fine-structure constant, one of the key inputs in the precision electroweak tests [118–122] (see also Ref. [67]). As the WP [13] merged the different $a_\mu^{HVP,LO}$ determinations to reach Eq. (6), its future update shall include a data-based conclusion on possible new physics according to HVP and electroweak precision observables. Incidentally, our one-sigma result [51] at next-to-next to leading order in the chiral expansion, is compatible at $1.2(0.8)\sigma$ with the WP (BMW) ranges, while WP and BMW differ at 2.1σ. Larger compatibility of τ datasets with a_μ^{SM} than of e^+e^- measurements with the SM prediction has long been established and seems robust. It will be crucially important for pinpointing the size and nature of beyond the SM contributions that new lattice evaluations can reach the remarkable precision of the BMW evaluation to verify their result.

Once this puzzle settles, the a_μ^{SM} uncertainty will still be four times larger than that foreseen for the final FNAL publication. Then, improved data on hadronic cross-sections, $\mu e \to \mu e$ scattering and semileptonic tau decays, as well as lattice QCD advances, will hopefully reduce the error of a_μ^{HVP}. Soon after, a_μ^{HLbL} errors will become relevant for the SM uncertainty. From this point on, the improvements that we have discussed on axial-vector contributions and short-distance constraints (as well as other contributions to HLbL) will be important in this continuous feedbacked marathon of improving the experimental and theoretical knowledge of a_μ, aiming to finally uncover the eagerly sought new physics beyond the SM.

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i. At this impressive accuracy, the hadronic contribution to a_e, $a_e^{had} = 1.693(12) \times 10^{-12}$ needs to be included, as opposed to the electroweak one, $a_e^{\text{EW}} \sim 3 \times 10^{-14}$ [10].

ii. This result is based on Refs. [8,10,14-46]. Later developments to Ref. [13] include Refs. [47-66].

iii. Vacuum polarization effects are already included in the running of α.

iv. This issue is further discussed in Sec. 2.1.5.

v. This optimal basis has recently been employed for predicting the axial-vector contribution to the hyperfine splitting of muonic atoms [102], a relevant result for the proton radius precision program.

vi. WP short-distance and longitudinal contributions are basically confirmed. See also Refs. [50, 61].

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