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Visualization of internal forces inside the proton in a classical relativistic model

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A classical model of a stable particle of finite size is studied. The model parameters can be chosen such that the described particle has the mass and radius of a proton. Using the energy-momentum tensor (EMT), we show how the presence of long-range forces alters some notions taken for granted in short-range systems. We focus our attention on the D-term form factor. The important conclusion is that a more careful definition of the D-term may be required when long-range forces are present.

Keywords: Energy-momentum tensor; D-term; classical model.

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1. Introduction

In this proceeding, we review the results for EMT densities from Ref. [1] based on Białynicki-Birula's classical model of the proton (BB-model) [2]. The EMT can be studied through generalized parton distribution functions in hard exclusive reactions, and is of interest because it contains information about the basic properties of a particle: the mass, spin, and D-term [3–10]. Although less well-known, the D-term is of equal importance as the other basic properties [11]. From the D-term form factor $D(t)$ and other EMT form factors, one can learn about the 2D and 3D distributions of energy, angular momentum and internal forces [12–16]. Recently, first experimental insights on the D-term became available [17–20].

In Ref. [1] the BB-model [2] was used to understand the impact of long-range forces not considered in other theoretical studies in systems with exclusively short-range forces [21-43]. The BB-model [1] yields qualitatively similar results to experimental insights [18, 20]. Even though it is classical, the BB-model is well suited for our purpose since it lets us investigate the impact of long-range forces without worrying about technical difficulties that arise in quantum field theory. We will later show that our conclusions about the impact of the long-range forces are model independent.

2. EMT Tensor in the classical model

The BB-model [2] consists of "dust particles" in a spherically symmetric region of radius R bound by three fields: a massive scalar field, ϕ, a massive vector field, V^μ, and an electromagnetic field, A^μ. The particles couple to these fields via the coupling constants g_S, g_V, and the electric charge e. The classical field equations are relativistic and can be found in Refs. [1, 2]. The parameters g_S, m_S, g_V, m_V correspond, respectively, to the coupling constants and masses of sigma and omega mesons as used in nuclear models [2].

In this work, we will focus on 3D EMT densities which are well-defined concepts in the large-N_c limit, for nuclei [13, 32], and of course in classical models [1]. For discussions of 2D densities we refer to [14–16]. In Fig. 1a) we show the energy density which yields the mass of the system when integrated over the volume. $T_{00}(r)$ is always positive. The characteristic discontinuity at $r = R$ is due to dust particles which by the construction of the BB-model are confined within the radius $r \leq R$. The solutions to the field equations are static with $V^\mu = (V_0, 0, 0, 0)$ and analogous for the Coulomb field. At $r > R$, only the fields contribute to the energy density which decay exponentially like $\phi(r) \sim (1/r)\, e^{-m_S r}$ and $V_0(r) \sim (1/r)\, e^{-m_V r}$ for $r \gg R$, while the Coulomb potential is $A_0(r) \sim (1/r)$ for $r > R$. The exact expressions for the fields and dust particle distribution can be found in [1, 2].

The pressure $p(r)$ and shear force $s(r)$ are defined through the components of the stress tensor, i.e. the T_{ij} components of the EMT, as

$$T^{ij} = \left(e_r^i e_r^j - \frac{1}{3}\delta^{ij}\right) s(r) + p(r)\,\delta^{ij}\,, \tag{1}$$

where e_r^i is the unit vector in the radial direction. The total pressure, $p(r) = p_{\rm scal}(r) + p_{\rm vec}(r) + p_{\rm Coul}(r)$, receives contributions from fields which are given by

$$p_{\rm scal}(r) = -\frac{1}{6}\,\phi'(r)^2 - \frac{1}{2}\,m_S^2\,\phi(r)^2, \tag{2}$$

$$p_{\rm vec}(r) = \frac{1}{6}\,V_0'(r)^2 + \frac{1}{2}\,m_V^2 V_0(r)^2, \tag{3}$$

$$p_{\rm Coul}(r) = \frac{1}{6}\,A_0'(r)^2. \tag{4}$$

As can be seen in Eqs. (2–4), the scalar meson contribution is always negative, which corresponds to attractive forces directed towards the inside. On the other hand, the contributions of the vector mesons and the Coulomb field are always positive, which corresponds to repulsive forces directed towards the outside. When we integrate

$$\int_0^\infty dr\, r^2 p_i(r),$$

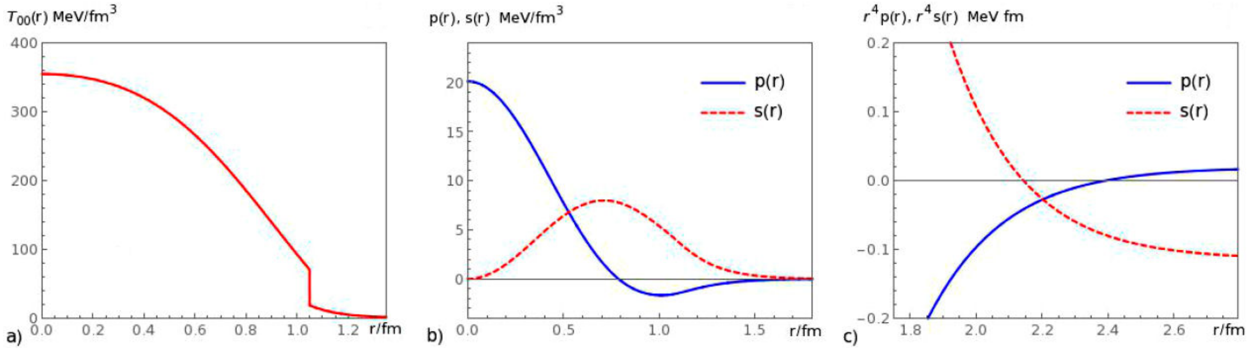

FIGURE 1. EMT densities in the BB-model [1]. a) $T_{00}(r)$ (total) vs. r. b) $p(r)$ and $s(r)$ (total) vs. r in the region of smaller r ($r \lesssim 2\,\mathrm{fm}$). c) $r^4 p(r)$ and $r^4 s(r)$ at very large r ($r \gtrsim 2\,\mathrm{fm}$), where we see the new features (the power r^4 is included to enhance the features).

we get $-10.916\,\mathrm{MeV}$ from the scalar fields, $10.891\,\mathrm{MeV}$ from the vector field, and a miniscule $0.025\,\mathrm{MeV}$ from the Coulomb field. This reflects that the proton is a bound state of strong forces and the electromagnetic contribution plays a minor role. But no contribution, no matter how small, can be neglected as these numbers must add up exactly to zero and fulfill von Laue condition,

$$\int_0^\infty dr\, r^2 p(r) = 0, \tag{5}$$

which shows that the internal forces balance each other and is a necessary condition for mechanical stability [21]. The von Laue condition is exactly satisfied in the BB-model [1, 2].

The shear force is $s(r) = \phi'(r)^2 - V_0'(r)^2 - A_0'(r)^2$. Notice that the dust particles do not contribute to $s(r)$ and $p(r)$. The pressure and shear force are not independent but connected by $(2/3)\,s'(r) + (2/r)\,s(r) + p(r) = 0$ due to EMT conservation. The model results are shown in Fig. 1b). The pressure inside the proton obtained from this model is an order of magnitude smaller than in the chiral quark soliton model [21] or that inferred from experiment [18]. This is because the BB-model is based on “residual nuclear forces” which are about an order of magnitude weaker than the strong forces among quarks inside the proton.

The results for $s(r)$ and $p(r)$ in Fig. 1b) are qualitatively very similar to what was found in other theoretical studies [21-42]. In order to see the impact of long-range forces, it is necessary to look more closely at the region of large r which we shall do in the next section.

3. Effects of long-range forces on the EMT

In previous studies of strongly interacting systems governed by short-range forces, three common features were observed. The first feature is that the shear force is always positive. The second feature is that the pressure has one node at some point r_0 with $p(r) > 0$ when $r < r_0$ and the pressure is less than zero for $r > r_0$. This property arises from the fact that the pressure must have at least one node to satisfy the von Laue condition, and the ground state exhibits a single node. Finally, the combination of $(2/3)\,s(r) + p(r)$, which is normal force per unit area, is always positive.

The BB-model is different from other studies, as it includes long-range Coulomb forces. From the model expressions for $T_{00}(r)$, $s(r)$ and $p(r)$, we obtain the long-distance behavior which holds numerically for $r \gtrsim 2\,\mathrm{fm}$,

$$T_{00}(r) = \frac{1}{2}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots\,, \tag{6}$$

$$s(r) = -\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots\,, \tag{7}$$

$$p(r) = \frac{1}{6}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots\,, \tag{8}$$

where the dots indicate contributions from the strong fields which are exponentially suppressed, and α is the fine-structure constant. We observe that $T_{00}(r)$ is always greater than zero which is in agreement with all prior studies. Because of the $1/r^4$ decay of $T_{00}(r)$, the total energy converges but the mean square radius of the energy density diverges.

In Fig. 1b) we saw that $s(r)$ is positive, which agrees with prior studies. But this is true only up to about $2.1\,\mathrm{fm}$ at which point $s(r)$ changes sign as shown in Fig. 1c). Similarly, the picture of the pressure in the BB-model in Fig. 1b) agrees with observations in other studies with $p(r)$ turning from positive to negative around $0.8\,\mathrm{fm}$. However, looking more closely in the region of larger r we see that $p(r)$ exhibits a second node around $2.4\,\mathrm{fm}$, and then remains positive. For completeness, we remark that the normal force, $(2/3)\,s(r) + p(r)$, exhibits an unusual feature and turns negative in the large r region [1].

In view of what has been learned from other studies based on short-range forces, these three features are counter-intuitive. It is an important observation that the presence of long-range interactions introduces new features which have not been observed in prior studies of EMT densities. One important practical implication is the divergence of D-term which we shall review in the next section.

4. Divergence of the D-term

The D-term, "the least known global property [13]", is given in terms of two equivalent definitions (arising from EMT conservation) in terms of shear force and pressure,

$$D = -\frac{4}{15}\,M \int d^3r\; r^2 s(r) = M \int d^3r\; r^2 p(r)\,. \qquad (9)$$

The Coulomb contributions to $s(r)$ and $p(r)$ are minuscule in the region $r < 2\,\mathrm{fm}$, see Fig. 1b), giving the impression that the electromagnetic interaction plays a very small role for the description of the structure of a charged hadron. However small, the Coulomb contribution cannot be ignored, as it tells is that there is an electric charge. Especially at large r, the long-range $1/r$ behavior of the Coulomb contribution takes over which has an important impact on the D-term. Because of the asymptotic behavior of $s(r)$ and $p(r)$ at large r in Eqs. (7, 8), both expressions for the D-term in (9) diverge. The fact that the D-term diverges due to long-range forces is a new result, which has not been seen in prior studies.

In order to obtain a finite ("regularized") value for the D-term, one can introduce a regularization prescription. A unique regularization method can be derived by observing that, if the integrals were finite, then any linear combination of the two equivalent expressions in Eq. (9) would give the same expression for D. However, the divergence can be removed by considering one and only one linear combination which leads to finite regularized result for D, namely

$$D_{\rm reg} = M \int d^3r\; r^2 \left[\frac{4}{9}\,s(r) + \frac{8}{3}\,p(r)\right]. \qquad (10)$$

Numerically, we find $D_{\rm reg} = -0.317$, *i.e.* this regularization method preserves the negative sign of the D-term that has been observed in all prior studies. The numerical value is about an order of magnitude smaller than *e.g.* in the quark soliton model [21], which is expected as the BB-model is based on "residual nuclear forces" that are weaker than the strong interactions among quarks. It would be interesting to see if other methods exist to regularize these divergences.

The form factor $D(t)$ in the BB-model is negative in a wide range of t. Only when $(-t) \lesssim 2.8 \times 10^{-4}\,\mathrm{GeV}^2$ does it become positive, and diverges like $D(t) \sim 1/\sqrt{-t}$ for still smaller t [1]. Such small momentum transfers are currently beyond experimental reach. Noteworthy, the regularized value $D_{\rm reg}$ together with a quadrupole fit, provide a very good approximation to the exact numerical model results for $D(t)$ which confirms the practical usefulness of the regularization method [1].

5. Model independent conclusions

Our results for the EMT densities are model dependent in the region $r\ < 2\text{-}3\,\mathrm{fm}$, where the strong forces dominate. However, at $r \gg 3$ fm, exact QED calculations yield the same EMT density results as us, since QED has to reproduce Maxwell's classical theory at long distances. In particular, the results in Eqs. (6, 7, 8) are model independent and were obtained in QED calculations [44, 45]. The divergence of $D(t)$ at small t due to QED effects was also found in chiral perturbation theory calculations for charged pions [46]. When comparing our results for $D(t)$ to those found using effective field theory techniques, we find that in the region $(-t) < 10^{-6}$ GeV^2, the model exactly reproduces QED [44, 46].

6. Conclusion

In Ref. [1], we used a classical model [2] which includes long-range forces through the Coulomb contribution to calculate the D-term. The classical character of the model was not an impediment. It allowed us to investigate properties affected by the presence of long-range forces without worrying about the technical difficulties which arise when studying more complicated quantum systems. We found that the D-term of the proton diverges, in direct contrast to the convergent results of previous studies. This feature is due to the infinite range of the electromagnetic interaction and model independent. In fact, the model gives $T_{00}(r)$, $s(r)$, $p(r) \sim (\alpha/r^4)$ at large r [1] which agrees with QED calculations [44, 45]. In the model, we were able to derive a unique regularization prescription to obtain a meaningful, finite, negative value for the D-term in agreement with other studies. Without such a regularization, the form factor $D(t)$ changes sign and diverges at very small momentum transfers below $-t < 10^{-4}$ GeV^2. While this t-region is currently out of reach experimentally, it indicates that it may be necessary to refine the definitions of the EMT properties in the presence of long-range forces. It is currently an open question how to do this in a model-independent way, or whether the divergence of $D(t)$ may be remedied by considering QED radiative corrections.

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