

Roper-like singly heavy baryons in a chiral model

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We investigate the Roper-like singly heavy baryons such as $\Lambda_c(2765)$ and $\Xi_c(2967)$ in a chiral model. Based on chiral symmetry of diquarks inside the baryons, we propose that the Roper-like baryons are mostly pentaquark states ($Qqq\bar{q}q$) while the corresponding ground-state ones $\Lambda_c(2286)$ and $\Xi_c(2470)$ are three-quark states (Qqq). Besides, the mass spectrum of negative-parity Λ_c and Ξ_c which are heavy quark spin-singlet is predicted by the linear representation of chiral symmetry. We also derive a sum rule of the heavy baryon masses and an extended Goldberger-Treiman relation. In addition to them, we demonstrate the parity-partner structure of the Λ_c 's and Ξ_c 's by showing mass degeneracies of them at the chiral restoration point. We expect that the present investigation leads to a better understanding of the diquarks inside hadrons from the viewpoint of chiral symmetry.

Keywords: Singly heavy baryons; chiral symmetry; pentaquark picture.

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1. Introduction

Recent development of hadron experiments at, *e.g.*, KEK, LHC, and SLAC has been providing us with clues to understand properties of singly heavy baryons. Since those baryons include one heavy (c or b) quark acting as a spectator for the remaining light quarks, in particular, we can examine a role of diquarks inside the hadrons from them.

Among singly heavy baryons, the Roper-like baryons such as $\Lambda_c(2765)$ and $\Xi_c(2967)$ which are heavy quark spin-singlet and $J^P = (1/2^+)$ [1, 2] are attracting attention. Their masses are larger than the masses of the corresponding ground-state $\Lambda_c(2286)$ and $\Xi_c(2470)$ by about 500 MeV [3], which is similar to the Roper resonance $N(1440)$ for the nucleon sector [4]. Besides, the decay widths of those baryons are tens of MeV, and it is known that such large widths cannot be explained by the conventional nonrelativistic quark model [5].ⁱ This discrepancy is also similar to the nucleon sector [8].

Chiral models based on the chiral representation of the light quarks are useful in studying hadron properties. Thus far, investigation of hadrons by a chiral model has been broadly done not only for light hadrons but also for the heavy-light mesons [9, 10]. Moreover, recently the model is applied to singly heavy baryons by focusing on the chiral representation of the diquarks inside them [11–15]. In the present work, we extend the chiral model for singly heavy baryons to include the Roper-like baryons $\Lambda_c(2765)$ and $\Xi_c(2967)$ in addition to the ground-state ones $\Lambda_c(2286)$ and $\Xi_c(2470)$. In particular, in order to treat both the baryons, we introduce the *mirror diquark* which can be understood as a tetra-diquark state ($qq\bar{q}q$) as well as the *conventional diquark* (qq) [16].

This write-up is organized as follows. In Sec. 2, we introduce the mirror and conventional diquarks, and construct an effective Lagrangian for the singly heavy baryons based on their chiral representations. In this section we also explain input parameters. In Sec. 3 we present the resultant

mass spectrum of the heavy baryons. Our present analysis is based on the linear representation of chiral symmetry, so the parity-partner structure of the positive-parity and negative-parity heavy baryons is expected to be realized. Thus, in Sec. 4 we demonstrate mass degeneracies of the parity partners at the chiral restoration point. And in Sec. 5 we conclude the present study.

2. Model

In this section, we construct an effective model describing the singly heavy baryons which are heavy quark spin-singlet and flavor-singlet, based on chiral symmetry of diquarks. For this purpose, we introduce the following two types of diquarks [16]:

$$\begin{aligned}
 (d_R)_i^a &\sim \epsilon_{ijk} \epsilon^{abc} (q_R^T)_j^b C (q_R)_k^c, \\
 (d_L)_i^a &\sim \epsilon_{ijk} \epsilon^{abc} (q_L^T)_j^b C (q_L)_k^c, \\
 (d'_R)_i^a &\sim \epsilon_{jkl} \epsilon^{abc} (q_R^T)_k^b C (q_R)_l^c [(\bar{q}_L)_i^d (q_R)_j^d], \\
 (d'_L)_i^a &\sim \epsilon_{jkl} \epsilon^{abc} (q_L^T)_k^b C (q_L)_l^c [(\bar{q}_R)_i^d (q_L)_j^d], \quad (1)
 \end{aligned}$$

where the superscripts (a, b, \dots) and subscripts (i, j, \dots) represent color and flavor indices, respectively, and $q_{R(L)} = (1 \pm \gamma_5/2)q$ is the left-handed (right-handed) quark. In Eq. (1), the former d_R and d_L are the conventional diquarks, while the latter d'_R and d'_L are the mirror diquarks which can be understood as tetra diquarks. It should be noted that the diquarks are Lorentz scalar since we focus on only heavy quark spin-singlet baryons. The diquarks (1) belong to the chiral representations as $d_R \sim (\mathbf{1}, \bar{\mathbf{3}})$, $d_L \sim (\bar{\mathbf{3}}, \mathbf{1})$, $d'_R \sim (\bar{\mathbf{3}}, \mathbf{1})$ and $d'_L \sim (\mathbf{1}, \bar{\mathbf{3}})$. Hence, when we define the heavy baryon fields by attaching a heavy quark Q to the diquarks (1) as $(B_{R(L)})_i \sim Q^a (d_{R(L)})_i^a$ and $(B'_{R(L)})_i \sim Q^a (d'_{R(L)})_i^a$, the chiral transformation laws of those baryons read

$$\begin{aligned}
B_R &\rightarrow B_R g_R^\dagger, & B_L &\rightarrow B_L g_L^\dagger, \\
B'_R &\rightarrow B'_R g_R^\dagger, & B'_L &\rightarrow B'_L g_R^\dagger.
\end{aligned} \quad (2)$$

It should be noted that $B_{R(L)}$ corresponds to the three-quark state while $B'_{R(L)}$ the pentaquark state.

From the chiral transformation laws in Eq. (2), a chiral Lagrangian which is invariant under $SU(3)_L \times SU(3)_R$ chiral and parity transformations is obtained as

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \sum_{\chi=L,R} \left(\bar{B}_\chi i v \cdot \partial B_\chi - \mu_1 \bar{B}_\chi B_\chi \right. \\
& + \bar{B}'_\chi i v \cdot \partial B'_\chi - \mu_2 \bar{B}'_\chi B'_\chi \left. \right) \\
& - \mu_3 (\bar{B}_R B'_L + \bar{B}'_L B_R + \bar{B}_L B'_R + \bar{B}'_R B_L) \\
& - g_1 (\bar{B}_L \Sigma^* B_R + \bar{B}_R \Sigma^T B_L) \\
& - g_2 (\bar{B}'_R \Sigma^* B'_L + \bar{B}'_L \Sigma^T B'_R) \\
& - g_3 (\bar{B}'_R \Sigma^* B_R + \bar{B}_L \Sigma^* B'_L + \text{h.c.}), \quad (3)
\end{aligned}$$

within the heavy baryon effective theory. In Eq. (3), v is a velocity of the heavy baryons and Σ is a light meson nonet. Besides, μ_1 , μ_2 and μ_3 are responsible for parts of the heavy baryon masses while g_1 , g_2 and g_3 are couplings between the baryons and light mesons. Under the spontaneous breakdown of chiral symmetry, the light meson nonet is replaced by its vacuum expectation value (VEV) $\langle \Sigma \rangle = \text{diag}(\sigma_q, \sigma_q, \sigma_s)$ with $\sigma_q = 93 \text{ MeV}$. We note that the value of σ_s is different from σ_q due to the small violation of $SU(3)_{L+R}$, and σ_s will be determined later.

In Eq. (3) the heavy baryon fields are expressed in terms of the chiral indices L and R . The parity eigenstates are given by $B_\pm = (1/\sqrt{2})(B_R \mp B_L)$ and $B'_\pm = (1/\sqrt{2})(B'_L \pm B'_R)$ [13], however, still these B_\pm and B'_\pm are not mass eigenstates due to mixings between them. In order to diagonalize the mixings, we define mass eigenstates $B_{\pm,i}^L$ and $B_{\pm,i}^H$ by introducing mixing angles $\theta_{B_{\pm,i}}$ as

$$\begin{pmatrix} B_{\pm,i}^L \\ B_{\pm,i}^H \end{pmatrix} = \begin{pmatrix} \cos \theta_{B_{\pm,i}} & \sin \theta_{B_{\pm,i}} \\ -\sin \theta_{B_{\pm,i}} & \cos \theta_{B_{\pm,i}} \end{pmatrix} \begin{pmatrix} B_{\pm,i} \\ B'_{\pm,i} \end{pmatrix}. \quad (4)$$

The corresponding mass eigenstates are given by

$$\begin{aligned}
M(B_{+,i}^{H/L}) &= m_B + \frac{1}{2} \left[m_{+,i} + m'_{+,i} \right. \\
&\quad \left. \pm \sqrt{(m_{+,i} - m'_{+,i})^2 + 4\tilde{m}_{+,i}^2} \right], \\
M(B_{-,i}^{H/L}) &= m_B + \frac{1}{2} \left[m_{-,i} + m'_{-,i} \right. \\
&\quad \left. \pm \sqrt{(m_{-,i} - m'_{-,i})^2 + 4\tilde{m}_{-,i}^2} \right], \quad (5)
\end{aligned}$$

with $m_{\pm,i} = \mu_1 \mp g_1 \sigma_i$, $m'_{\pm,i} = \mu_2 \mp g_2 \sigma_i$ and $\tilde{m}_{\pm,i} = \mu_3 \mp g_3 \sigma_i$, and m_B is an arbitrary mass parameter for defining the heavy baryon effective theory that we can fix by

hand. For later use we determine its value to be $m_B = 2868 \text{ MeV}$. For the VEV's we have defined $\sigma_1 = \sigma_2 \equiv \sigma_q$ and $\sigma_3 \equiv \sigma_s$. In Eq. (4) or in Eq. (5) the superscript H/L represents the higher/lower mass eigenvalue; H (L) corresponds to the $+$ ($-$) sign in the right hand side. Thus, $B_{+,i}^L$ refers to the ground-state $\Lambda_c(2286)$ (for $i = 3$) or $\Xi_c(2470)$ (for $i = 1, 2$), while $B_{+,i}^H$ the Roper-like state $\Lambda_c(2765)$ or $\Xi_c(2967)$. Besides, in Eq. (4) the mixing angles satisfy $\tan \theta_{B_{\pm,i}} = (2\tilde{m}_{\pm,i})/(m_{\pm,i} - m'_{\pm,i})$.

In our model (3), we have seven parameters: $\mu_1, \mu_2, \mu_3, g_1, g_2, g_3$, and σ_s . To fix them, we use masses of the observed baryons $\Lambda_c(2286)$, $\Xi_c(2470)$, $\Lambda_c(2765)$ and $\Xi_c(2967)$ as inputs [3]:

$$\begin{aligned}
M(B_{+,i=3}^L) &= 2286 \text{ MeV}, & M(B_{+,i=3}^H) &= 2765 \text{ MeV}, \\
M(B_{+,i=1,2}^L) &= 2470 \text{ MeV}, \\
M(B_{+,i=1,2}^H) &= 2967 \text{ MeV}. \quad (6)
\end{aligned}$$

In addition to them, we also employ masses of the conventional diquarks measured by the lattice simulation [17] together with the chiral model [13] as additional inputs:

$$\begin{aligned}
M(d_{+,i=1,2}) &= 906 \text{ MeV}, & M(d_{-,i=1,2}) &= 1142 \text{ MeV}, \\
M(d_{+,i=3}) &= 725 \text{ MeV}, \\
M(d_{-,i=3}) &= 1265 \text{ MeV}, \quad (7)
\end{aligned}$$

where the subscripts \pm and i refer to the parity eigenvalue and flavor index of the diquarks, respectively. Those diquark masses were measured without corrections from the mirror diquarks, hence, the masses are related to those of the heavy baryons of three-quark states B_\pm with no corrections from the pentaquark ones B'_\pm . In our model, the masses of such uncorrelated three-quark states are read by switching off the pentaquark states in Eq. (3) as $B_{\pm,i} = 0$, yielding $M(B_{\pm,i}) = m_B + m_{\pm,i}$. When we regard the heavy quark inside the baryons as a spectator with respect to the remaining diquark, the mass difference between $M(B_{-,i})$ and $M(B_{+,i})$ is expected to coincide with that of $M(d_{-,i})$ and $M(d_{+,i})$. That is, we have

$$M(B_{-,i}) - M(B_{+,i}) = 2g_1 \sigma_i = M(d_{-,i}) - M(d_{+,i}), \quad (8)$$

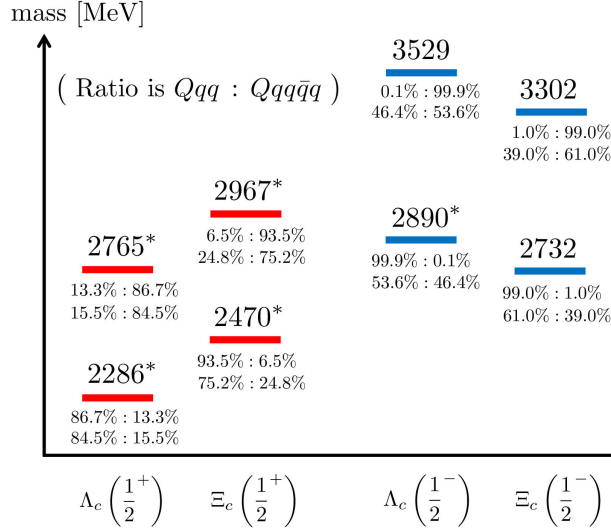
which fixes σ_s and g_1 . The above procedure enables us to fix six parameters, however, one parameter remains undetermined. In order to fix it, we further employ a mass of the negative-parity and heavy quark spin-singlet Λ_c baryon estimated by a quark model as an input: $M(B_{-,3}^L) = 2890 \text{ MeV}$ [18].

3. Mass spectrum

In Sec. 2 we have constructed our chiral model. In this section, based on the model we show the resultant mass spectrum of positive-parity and negative-parity Λ_c 's and Ξ_c 's which are heavy quark spin-singlet.

TABLE I. Two parameter sets (I) and (II), where the asterisk (*) stands for the inputs. The arbitrary mass parameter m_B is fixed to be $m_B = 2868$ MeV.

	μ_1 [MeV]	μ_2 [MeV]	μ_3 [MeV]	g_1	g_2	g_3	σ_q [MeV]	σ_s [MeV]
Set (I)	-247	247	∓ 91.0	1.27	1.94	± 0.34	93*	212
Set (II)	94.1	-94.1	± 246	1.27	1.94	± 0.34	93*	212


 FIGURE 1. Mass spectrum of Λ_c 's and Ξ_c 's for $J^P = 1/2^+$ (red) and $J^P = 1/2^-$ (blue). The asterisk (*) represents the inputs. The ratio stands for $Qqq : Qqqqq$ of each heavy baryon, where the upper and lower ones are corresponding to the parameter set (I) and set (II), respectively.

Since the mass formulae in Eq. (5) include square roots, we can get several parameter sets. In particular, we have obtained two physically distinct parameter sets, as summarized in Table I. We note that the arbitrary mass parameter m_B is fixed to be $m_B = 2868$ MeV as mentioned before.

The resultant mass spectrum of Λ_c 's and Ξ_c 's for $J^P = 1/2^+$ (red) and $J^P = 1/2^-$ (blue) are shown in Fig. 1. In this figure the ratio indicated below the bars stands for $Qqq : Qqqqq$ of each heavy baryon, where the upper and lower ones are corresponding to the parameter set (I) and set (II) in Table I, respectively. Besides, the asterisk (*) means the input mass. The figure shows that the Roper-like $\Lambda_c(2765)$ and $\Xi_c(2967)$ are mostly dominated by the pentaquark state composed of the mirror diquark ($qq\bar{q}q$), while the ground-state $\Lambda_c(2286)$ and $\Xi_c(2470)$ by the three-quark state composed of the conventional diquark (qq) for both the parameter sets. On the other hand, the ratio of negative-parity baryons are largely dependent on the parameter set.

Since our analysis is based on chiral symmetry of the diquarks, we can obtain an extended Goldberger-Treiman relation that the couplings among the heavy baryons and light pseudo-scalar mesons satisfy:

$$\sum_{n=H,L} M(B_{-,i}^n) - \sum_{n=H,L} M(B_{+,i}^n) = 2(g_1 + g_2)\sigma_i. \quad (9)$$

In addition to this relation, field-theoretical treatment of the mass spectrum in the present study yields a sum rule of the baryon masses

$$\sum_{p=\pm, n=H,L} M(B_{p,i=1,2}^n) = \sum_{p=\pm, n=H,L} M(B_{p,i=3}^n), \quad (10)$$

which cannot be obtained by quark models. Furthermore, our analysis naturally explains the suppression of the direct decay process in $\Lambda_c(2765) \rightarrow \Lambda_c(2286)\pi\pi$ [1], because a coupling of $\Lambda_c(2765)$ - $\Lambda_c(2286)$ - σ (this σ is the lightest scalar meson) vanishes due to the diagonalization to get the mass formulae in Eq. (5).

4. Mass degeneracy of parity partners

One of the peculiarities of chiral models is a mass degeneracy of the parity partners at the chiral restoration point. The mirror diquarks proposed in the present study can be regarded as an analogue of the mirror nucleons for negative-parity nucleon $N^*(1535)$ in the parity doublet model [19,20]. Within this model, it has been suggested that the ground-state $N(940)$ and the excited negative-parity $N^*(1535)$ tend to degenerate as chiral symmetry restores, showing the parity-partner structure [21–25]. Thus, in the present model for the singly heavy baryons as well, we can expect that mass degeneracies of the parity partners occur in a similar way. In this section, we show such mass degeneracies by simply changing the VEV's σ_q and σ_s in Eq. (5) to demonstrate the parity-partner structure of the heavy baryons.

The resultant mass changes of Λ_c 's (left) and Ξ_c 's (right) with respect to σ_q and σ_s are shown in Fig 2. It should be noted that $\sigma_s = 212$ MeV and $\sigma_q = 93$ MeV correspond to the vacuum while $\sigma_s = \sigma_q = 0$ the chiral restoration point. The figure clearly shows that

$$\begin{aligned} M(B_{+,i}^L) &= M(B_{-,i}^L), \\ M(B_{+,i}^H) &= M(B_{-,i}^H), \end{aligned} \quad (11)$$

holds at the chiral restoration point. That is, the parity-partner structure of $\{B_{+,i}^L, B_{-,i}^L\}$ and $\{B_{+,i}^H, B_{-,i}^H\}$ is indeed demonstrated.

Although our analysis shows the partner structure in the simplest way, in more realistic situation the chiral restoration occurs at finite temperature, density, and volume where other corrections contribute. Hence, investigation including such effects together with the changes of σ_q and σ_s on the masses

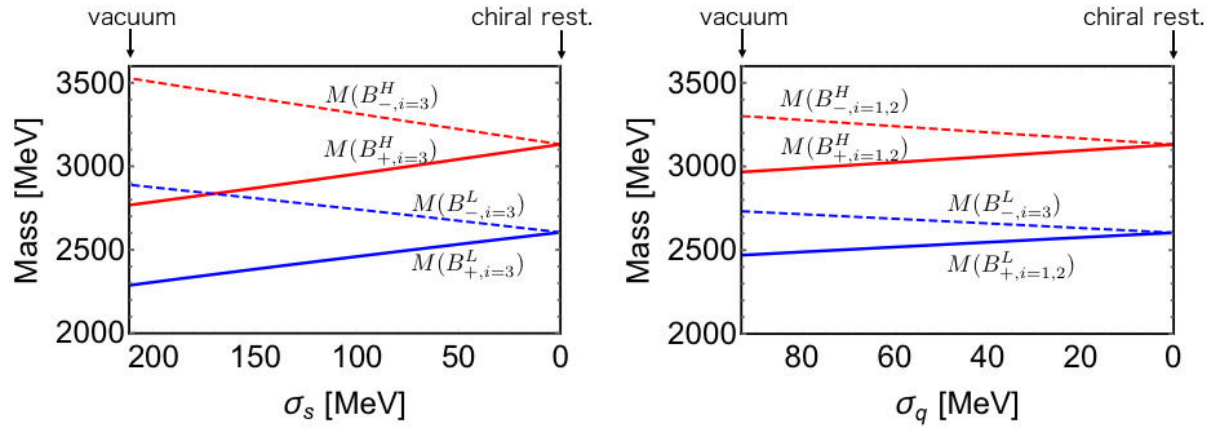


FIGURE 2. Mass changes of Λ_c 's (left) and Ξ_c 's (right) with respect to σ_s and σ_q . $\sigma_s = 212$ MeV and $\sigma_q = 93$ MeV correspond to the vacuum while $\sigma_s = \sigma_q = 0$ the chiral restoration point.

of Λ_c 's and Ξ_c 's are inevitable, and we leave such a study for future work. From such an investigation, a better understanding of the relation between the mirror diquark and the conventional diquark from the viewpoint of chiral symmetry is expected.

5. Conclusions

In the present work we have studied the Roper-like $\Lambda_c(2765)$ and $\Xi_c(2967)$ together with the ground-state $\Lambda_c(2286)$ and $\Xi_c(2470)$ in a chiral model, by introducing the mirror diquark ($qq\bar{q}q$) in addition to the conventional diquark (qq). As a result, we have found that the Roper-like $\Lambda_c(2765)$ and $\Xi_c(2967)$ are mostly pentaquark state ($Qqq\bar{q}q$) while the ground-state $\Lambda_c(2286)$ and $\Xi_c(2470)$ are three-quark state

(Qqq). Besides, chiral representation of the diquarks has yielded the mass spectrum of negative-parity Λ_c 's and Ξ_c 's in addition to the positive-parity ones. Furthermore, a sum rule of the heavy baryon masses and an extended Goldberger-Treiman relation have been derived, as a consequence of field-theoretical treatment based on chiral symmetry. In addition to those findings, we have demonstrated the parity-partner structure of the heavy baryons by showing mass degeneracies at the chiral restoration point.

We expect that our present investigation leads to a better understanding of the diquarks inside hadrons from the viewpoint of chiral symmetry. We also expect that our findings and predictions provide future experiments with useful information on unobserved negative-parity Λ_c and Ξ_c .

- i.* The relativistic corrections in the quark model was found to improve the discrepancy [6]. Also, the 3P_0 model can reproduce the decay width reasonably [7].
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