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Charmonium radiative decays within the covariant confined quark model

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We have studied the dominant one-photon radiative transitions of the charmonium ground and orbitally excited states within an analytic confinement model. Along with two fixed basic model parameters (m_c and the cutoff value λ), we introduced only one adjustable parameter common to charmonium states: η_c, J/ψ, χ_{c0}, χ_{c1}, h_c and χ_{c2} to parameterize the quark distribution inside the hadron. Our estimates are in good agreement with the latest data.

Keywords: Charmed mesons; radiative transition; decay width; confinement; quark models.

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1. Introduction

The charmonium state, which represents a composite system of a charm quark and its counter-partner ($\bar{c}c$), has been the focus of intense research in various experiments [1]. A series of low-lying charmonium excited states have been observed, and they possess small binding energies [2]. These states can serve as a felicitous testing ground to check basic model assumptions, and a great number of theoretical approaches and frameworks have been devoted to the charmonium system.

Many phenomenological models and effective approaches vary in degrees of refinement with respect to the underlying QCD by describing hadronstructure within the Standard Model (SM). In particular, the considerable charm-quark mass exceeds many times the conventional confinement energy scale and the subtle binding energy allows for various perturbative calculations and other nonrelativistic approximations.

Nevertheless, many model predictions suffer from inaccuracies in the description of experimental data on the properties of charmonium. The latest average data on the fractional width of decay $J/\psi \to \gamma\eta_c$, in particular, is nearly twice as small [1] as the predictions of the Coulomb gauge approach [3] and the nonrelativistic potential model [4].

In a series of our studies, we have developed a relativistic quantum field model with specific forms of propagators of analytically confined quark and gluon and then applied it to various aspects of low-energy hadron physics [5–7].

Following the concept of analytical confinement, we present below our last theoretical estimates for radiative transitions in charmonium states performed within the *Covariant Confined Quark Model* (CCQM) [8, 9].

2. Model

An effective quark confinement mechanism is implemented in the CCQM by using convolutions of local quark propagators and hadron vertex functions. Hereby, a universal infrared cutoff of the scale integration ensures the complete removal of any singularities from the matrix elements (see, *e.g.* [9–11]). The CCQM is consistent with the Standard Model's SM's symmetries, clearly Lorentz invariant, and capable of describing various composite systems using the same theoretical principles. Our approach has been successfully applied to different modern problems in particle physics (see *e.g.*, [11, 13]). In particular, we have shown the equivalence of Fermi-type and Yukawa-type theories under constraint conditions that fix the coupling constants and particle mass. We obtained a new smooth behavior of the Fermi coupling in dependence on meson mass [12].

Our model Lagrangian describes the interaction of a hadron field $H(x)$ satisfying the corresponding equation of motion with an interpolating quark current $J_H(x)$ possessing corresponding quantum numbers for the hadron. It reads as follows in the case of the meson:

$$\begin{aligned}\mathcal{L}_{int} &= g_H \cdot H(x) \cdot J_H(x) + \text{H.c.}\,,\\ J_H(x) &= \int dx_1 \int dx_2 \delta^{(4)}(x - w_1 x_1 - w_2 x_2)\\ &\quad\times \Phi_H\left[(x_1 - x_2)^2\right] \bar{q}_2(x)\Gamma_H\, q_1(x)\,, \qquad (1)\end{aligned}$$

where the vertex function Φ_H effectively describes the quark distribution within the meson, Γ_H is the Dirac matrix ensuring the meson quantum numbers, and $w_i = m_i/(m_1 + m_2),\ i = 1, 2$ is the fractional quark mass.

The ultraviolet convergence of the loop integrals is ensured by the Fourier transform of Φ_H, which falls off in momentum space in the Euclidean region by the Gaussian law as follows:

$$\tilde{\Phi}_H\left(-p^2\right) = \exp\left(p^2/\Lambda_H^2\right)\,, \qquad (2)$$

where we introduce an adjustable size-related parameter Λ_H.

According to the CCQM, the hadron renormalization coupling g_H in Eq. (1) is strictly fixed by the "compositeness condition" as follows:

$$Z_H = 1 - g_H^2 \tilde{\Pi}'_H\left(M_H^2\right) = 0\,,$$
$$\tilde{\Pi}'_H\left(p^2\right) = \frac{d}{dp^2}\tilde{\Pi}^{(1)}_H\left(p^2\right)\,, \tag{3}$$

and does not constitute further free parameters. Therefore, any bare states are removed totally from consideration, the mass and wave function of the hadron are renormalized, and the physical state is dressed. Note that $\tilde{\Pi}^{(1)}_H(p^2)$ is the diagonal part of hadron self-energy.

In particular, the meson self-energy function reads

$$\tilde{\Pi}_H(p) = N_c \int \frac{dk}{(2\pi)^4 i}\widetilde{\Phi}^2_H\left(-k^2\right) \times \text{tr}\left[\Gamma_H \widetilde{S}_1(\hat{k}+w_1\hat{p})\Gamma_H \widetilde{S}_2(\hat{k}-w_2\hat{p})\right]\,, \tag{4}$$

where m_q is the constituent quark mass and the quark propagator's Schwinger parametrization is introduced:

$$\widetilde{S}(\hat{k}) = \left(m_q + \hat{k}\right)\int\limits_0^\infty d\alpha\, \exp\left[-\alpha\left(m_q^2 - k^2\right)\right]. \tag{5}$$

Many matrix elements describing radiative transitions of hadrons, their mass operators and other decay processes by using Feynman quark-loop diagrams may be represented in terms of convolutions of vertex functions and quark propagators as follows:

$$\Pi^0 = N_c \int\limits_0^\infty dt t^{n-1}\int\limits_0^1 d^n\alpha\,\delta\Big(1-\sum_{i=1}^n \alpha_i\Big) f(t\alpha_1,\ldots,t\alpha_n). \tag{6}$$

However, at certain relation of kinematic variables, there may appear possible branch points connected with the creation of free quarks, and then, the integral in Eq. (6) diverges. These threshold singularities can be removed by introducing a universal infrared cutoff parameter, λ as follows:

$$\Pi^0 \to \Pi^\lambda = N_c \int\limits_0^{1/\lambda^2} dt t^{n-1}\ldots \tag{7}$$

The model free parameters λ, m_q, Λ_H are fixed by fitting the latest experimental data and, if necessary, some lattice results. In particular, the updated basic parameters are [12, 14]:

$$m_c = 1.67 \pm 0.17 \text{ GeV}, \qquad \lambda = 0.181 \text{ GeV}. \tag{8}$$

Our calculations below have a relative precision of about $\pm 10\%$.

2.1. Charmonium dominant radiative transitions

There are several low-lying excited $c\bar{c}$ states with relatively narrow widths that were observed first in different experiments [2]. These mesonic states decay under strong and weak interactions, but the typical dominant mode is a one-photon radiative transition into the ground-state charmonia.

Recent studies at hadron colliders have revealed that the branching fractions of the radiative transitions of the triplet $\chi_{\{c0,c1,c2\}} \to \gamma J/\psi$ are relatively large [15].

On the other hand, a theoretical investigation of the properties of the low-lying charmonium states is important to analyze the underlying physical processes of charmonium production in b-hadron decays observed recently by the LHCb Collaboration [17].

Different phenomenological models and effective theories have been developed to study the quasiparticle properties of charmonium states (see, *e.g.*, [3, 18–20]).

Despite the sophistication of theoretical frameworks, substantial discrepancies exist between the experimental measurements and various model predictions (see, *e.g.*, [3, 4, 21–24]).

The CCQM being a relativistic quantum field framework for hadron physics, can serve as an effective theoretical approach to analyze the recent measurements of the radiative transitions in charmonium states [17].

Below, we estimate the renormalized couplings, fractional decay widths of the dominant radiative transitions of $J/\psi, \eta_h$ mesons and triplet $\chi_{\{c0,c1,c2\}}$ in the framework of the CCQM.

For the one-photon radiative transition $X_1 \to \gamma X_2$ we write the invariant matrix element as follows:

$$\mathcal{M}^\sigma_{X_1\to\gamma X_2} = i\,(2\pi)^4 \varepsilon_{X_1}\,\varepsilon_{X_2}\,\varepsilon_{\gamma\sigma}\,T^\sigma_{X_1\to\gamma X_2}(q_1,q_2)\,, \tag{9}$$

where p, q_1, q_2 are the momenta, and $\{\varepsilon_{X_1}, \varepsilon_{X_2}, \varepsilon_\gamma\}$ are the polarization vectors of the charmonium states X_1, X_2, and the photon (γ).

The gauge-invariant part of the amplitude fulfills the following requirement:

$$q_{2\sigma}\cdot T^{\text{inv};\sigma}_{X_1\to\gamma X_2}(q_1,q_2) = 0\,. \tag{10}$$

The LO contribution to the transition amplitude $T^\sigma_{X_1\to\gamma X_2}(q_1,q_2)$ provided by the "triangle" Feynman diagram within the CCQM is as follows [9]:

$$T^{\Gamma_1,\Gamma_2,\sigma}_{X_1\to\gamma X_2} = g_{X_1} g_{X_2} e_c e N_c \int \frac{d^4k}{(2\pi)^4 i}\widetilde{\Phi}_{X_1}\Big(-k^2\Big) \times \widetilde{\Phi}_{X_2}\Big(-(k+\tfrac{1}{2}q_2)^2\Big)\text{tr}\Big[\Gamma_2 S(\hat{k}+\tfrac{1}{2}\hat{p}) \times \Gamma_1 \widetilde{S}(\hat{k}-\tfrac{1}{2}\hat{p})\gamma^\sigma_\perp \widetilde{S}(\hat{k}-\tfrac{1}{2}\hat{p}+\hat{q}_2)\Big], \tag{11}$$

where $\Gamma_1 = \{\gamma^\mu, I, \gamma^\mu\gamma_5, \overleftrightarrow{\partial}_\nu\ \gamma^5, i(\gamma^\mu\ \overleftrightarrow{\partial}_\nu\ +\gamma^\nu\ \overleftrightarrow{\partial}_\mu)/2\}$ are the Dirac matrices for charmonium ingoing states $\{J/\psi, \chi_{c0}, \chi_{c1}, h_c, \chi_{c2}\}$ and $\Gamma_2 = \{i\gamma^5, \gamma^\mu\}$ for outgoing $\{\eta_c, J/\psi\}$, respectively, and $e_c = 2/3$, e is the electric charge of an electron ($\alpha = e^2/4\pi = 1/137.036$) and $N_c = 3$.

The renormalized couplings g_{X_1} and g_{X_2} of the participating charmonium states in Eq. (11) are strictly determined in accordance with Eq. (1).

Transition $J/\psi(^3S_1) \to \gamma\eta_c(^1S_0)$

The dominant one-photon decay of vector charmonium J/ψ ($\Gamma_1 = \gamma^\rho$) to pseudoscalar η_c ($\Gamma_2 = i\gamma^5$) is a typical electromagnetic M1 transition, and the corresponding gauge invariant transition amplitude takes the form:

$$T^{\mathrm{inv};\rho\sigma}_{J/\psi\to\gamma\eta_c} = g_{J/\psi}\, g_{\eta_c}\, \epsilon^{\mu\nu\rho\sigma} q_1^\mu q_2^\nu\, C(p^2, q_1^2, q_2^2)\,, \qquad (12)$$

where the form factor $C(p^2, q_1^2, q_2^2)$ is defined in Eq. (11).

The corresponding one-photon radiative-decay width within the CCQM reads:

$$\Gamma(J/\psi \to \gamma\eta_c) = \frac{\alpha\, g^2_{J/\psi}\, g^2_{\eta_c}}{24}\, M^3_{J/\psi}\left(1 - \frac{M^2_{\eta_c}}{M^2_{J/\psi}}\right)^3 \times \left[C(M^2_{J/\psi}, M^2_{\eta_c}, 0)\right]^2 . \qquad (13)$$

Transition $\chi_{c0}(^3P_0) \to \gamma J/\psi(^3S_1)$

The orbitally excited scalar charmonium $\chi_{c0}(^3P_0)$ decays into the vector ground-state by radiating a photon and the corresponding gauge-invariant transition amplitude takes the form:

$$T^{\mathrm{inv};\rho\sigma}_{\chi_{c0}\to\gamma J/\psi} = g_{\chi_{c0}} g_{J/\psi}\left(q_1^\sigma q_2^\rho - g_{\rho\sigma}(q_1\, q_2)\right) d(p^2, q_1^2, q_2^2)\,. \qquad (14)$$

Then, the fractional decay ($\chi_{c0} \to \gamma J/\psi$) can be calculated by using the following expression:

$$\Gamma(\chi_{c0} \to \gamma J/\psi) = \frac{\alpha\, g^2_{\chi_{c0}}\, g^2_{J/\psi}}{24}\, M^3_{\chi_{c0}}\left(1 - \frac{M^2_{J/\psi}}{M^2_{\chi_{c0}}}\right)^3 \times \left[d(M^2_{\chi_{c0}}, M^2_{J/\psi}, 0)\right]^2 . \qquad (15)$$

Note that the form factor $d(p^2, q_1^2, q_2^2)$ is determined by Eq. (11).

Transition $\chi_{c1}(^3P_1) \to \gamma J/\psi(^3S_1)$

We can parameterize the gauge-invariant amplitude of the transition $\chi_{c1} \to \gamma J/\psi$ by using four seemingly independent Lorentz form factors as follows:

$$\begin{aligned} T^{\mathrm{inv};\mu\rho\sigma}_{\chi_{c1}\to\gamma J/\psi} &= g_{\chi_{c1}} g_{J/\psi}[\epsilon^{q_2\mu\sigma\rho}(q_1\, q_2) W_1 \\ &+ \epsilon^{q_1 q_2\sigma\rho} q_1^\mu W_2 + \epsilon^{q_1 q_2 \mu\rho} q_2^\sigma W_3 \\ &+ (\epsilon^{q_1 q_2\mu\sigma} q_1^\rho - \epsilon^{q_1\mu\sigma\rho}(q_1\, q_2)) W_4]. \end{aligned} \qquad (16)$$

The form factors W_1, W_2, W_3 and W_4 are determined according to Eq. (11) by taking into account the on-mass-shell conditions $p^2 = M^2_{\chi_{c1}}$, $q_1^2 = M^2_{J/\psi}$, $q_2^2 = 0$.

The fractional decay width of the axial-vector charmonium χ_{c1} is calculated by using the formula:

$$\Gamma(\chi_{c1} \to \gamma J/\Psi) = \frac{\alpha\, g^2_{\chi_{c1}} g^2_{J/\psi}}{12\pi}\, \frac{|\vec{q}_2|}{M^2_{\chi_{c1}}}\left(|H_L|^2 + |H_T|^2\right), \qquad (17)$$

where we have introduced two independent helicity amplitudes as follows:

$$\begin{aligned} H_L &= i\frac{M^2_{\chi_{c1}}}{M_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{M^2_{J/\psi}}{M_{\chi_{c1}}|\vec{q}_2|}W_4\right], \\ H_T &= -i\, M_{\chi_{c1}}|\vec{q}_2|^2\left(W_1 + W_2 - \left[1 + \frac{M^2_{J/\psi}}{M_{\chi_{c1}}|\vec{q}_2|}\right] W_4\right), \\ |\vec{q}_2| &= \left(M^2_{\chi_{c1}} - M^2_{J/\psi}\right)/\, 2M_{\chi_{c1}}\,. \end{aligned} \qquad (18)$$

Transition $h_c(^1P_1) \to \gamma\eta_c(^1S_0)$

The latest experimental data on the full decay width of the orbitally excited charmonium state $h_c(3525)$ suffers a large uncertainty: $\Gamma_{tot}(h_c) = 0.7 \pm 0.4$ MeV [1]. Nevertheless, we also consider the dominant one-photon decay of $h_c(3525)$ into the ground state. We have $\Gamma_1 = k^\mu\gamma^5$ for h_c and $\Gamma_2 = \gamma^\rho$ for η_c.

We find the gauge-invariant transition amplitude in the following form:

$$T^{\mathrm{inv};\rho\sigma}_{h_c\to\gamma\eta_c} = g_{h_c} g_{\eta_c}(q_2^\rho q_1^\sigma - g^{\rho\sigma}(q_1\, q_2)) h(p^2, q_1^2, q_2^2), \qquad (19)$$

with a form factor $h(p^2, q_1^2, q_2^2)$ determined by Eq. (11).

With spin $S = 1$, the one-photon radiative decay width of h_c is calculated as follows:

$$\Gamma(h_c \to \gamma\eta_c) = \frac{\alpha\, g^2_{h_c}\, g^2_{\eta_c}}{24\,(1+2S)}\, M^3_{h_c}\left(1 - \frac{M^2_{\eta_c}}{M^2_{h_c}}\right)^3 \times |h(M^2_{h_c}, M^2_{\eta_c}, 0)|^2. \qquad (20)$$

Transition $\chi_{c2}(^3P_2) \to \gamma J/\psi(^3S_1)$

For the radiative transition of the orbital excitation χ_{c2} we have $\Gamma_1 = \gamma^\mu k^\nu + \gamma^\nu k^\mu$ and $\Gamma_2 = \gamma^\rho$.

The gauge-invariant transition amplitude expressed by using two independent form factors reads:

$$\begin{aligned} T^{\mathrm{inv};\mu\nu\rho\sigma}_{\chi_{c2}\to\gamma J/\psi} &= g_{\chi_{c2}} g_{J/\psi}\Big(A\Big[g^{\mu\rho}\Big\{g^{\sigma\nu}(q_1\, q_2) - q_1^\sigma\, q_2^\nu\Big\} \\ &+ g^{\nu\rho}\Big\{g^{\sigma\mu}(q_1\, q_2) - q_1^\sigma\, q_2^\mu\Big\}\Big] \\ &+ B\Big[g^{\sigma\rho}\Big\{q_1^\mu\, q_2^\nu + q_1^\nu\, q_2^\mu\Big\} \\ &- g^{\mu\sigma} q_1^\nu\, q_2^\rho - g^{\nu\sigma} q_1^\mu\, q_2^\rho\Big]\Big)\,, \end{aligned} \qquad (21)$$

where $A(p^2, q_1^2, q_2^2)$ and $B(p^2, q_1^2, q_2^2)$ are defined by Eq. (11).

We obtain the fractional decay width of χ_{c2} as follows:

$$\Gamma(\chi_{c2} \to \gamma J/\psi) = \frac{\alpha\, g^2_{\chi_{c2}}\, g^2_{J/\psi}}{4(1+2S)}\, M^3_{\chi_{c2}}\left(1 - \frac{M^2_{J/\psi}}{M^2_{\chi_{c2}}}\right) \times \left(C_A A^2 + C_{AB}\, A\, B + C_B\, B^2\right), \qquad (22)$$

where $S = 2$ is the spin value and the numerical constants $C_A = 0.7584$, $C_{AB} = -0.02576$ and $C_B = 0.00226$ are polynomials of the mass relation $M^2_{J/\psi}/M^2_{\chi_{c2}}$.

2.2. Numerical results and discussion

For the charmonium states under consideration, we modify the CCQM by replacing a set of six free "size" parameters (Λ_{η_c}, $\Lambda_{J/\psi}$, $\Lambda_{\chi_{c0}}$, $\Lambda_{\chi_{c1}}$, Λ_{h_c}, $\Lambda_{\chi_{c2}}$) with a single common adjustable parameter, $\varrho > 0$, as shown below [9]:

$$\widetilde{\Phi}_X\left(-p^2\right)=\exp\left(\frac{1}{\varrho^2}\frac{p^2}{M_X^2}\right), \quad \varrho \equiv \frac{\Lambda_X}{M_X}=\text{const.} \qquad (23)$$

Note, in our calculation we keep the basic model parameters represented in Eq. (8).

First, we used Eq. (3) to calculate the renormalization couplings g_X. For all charmonium members, we reveal that $g_X(\varrho)$ decreases monotonically as ϱ increases.

Second, we have fitted the latest data on the dominant radiative decays of the triplet states χ_{c0}, χ_{c1} and χ_{c2} and have fixed the model parameters as follows [9]:

$$m_c = 1.80\,\text{GeV}, \qquad \varrho = 0.485. \qquad (24)$$

With the fixed model parameters, we have estimated the fractional decay widths of the one-photon radiative transitions of the charmonium states under consideration within the CCQM (see Table I) and then, compared our results with the recent experimental data in [1] and some theoretical predictions reported in Refs. [18, 20, 26].

We have the following remarks:

1) Recent lattice QCD simulations by using the twisted mass action with light dynamical quarks [18] considerably overestimated the fractional decay width $\Gamma(J/\psi \to \gamma\eta_c)$ while a constituent quark model [26] underestimated significantly the latest data [1]. The transition rate obtained within the CCQM is in agreement with the recent data.

2) The lattice QCD studies [23, 24] on the radiative transition properties of charmonium states χ_{c0} and χ_{c1} are still lack good descriptions due to some technical problems, while the Cornell potential model in the long-wavelength approximation [20] underestimates the latest data. Our calculations for these partial decay widths (in Table I) are in agreement with the recent LHCb data [1].

3) As mentioned above, the latest experimental data on the full decay width of the orbitally excited charmonium state $h_c(3525)$ suffers from a large uncertainty. A light front quark model [19] and a lattice QCD calculation [18] yielded similar predictions for the decay rate $\Gamma[h_c(1P) \to \gamma\eta_c]$, which was obviously much larger than the data cited in [1]. In contrast, a constituent quark model result [26] is more or less consistent with the latest data, taking into account its large uncertainties.

Our calculation within the CCQM (see in Table I) does not contradict the data.

By combining the latest fractional ratio from [1] with our estimate in Table 1 we have calculated the "theoretical expected full decay width" as $\Gamma_{h_c}^{\text{theor}} \simeq (0.57 \pm 0.12)$ MeV. We see that, compared with data $\Gamma_{h_c}^{\text{exp}} \simeq (0.7 \pm 0.4)$ MeV [1], our prediction is located in a more narrow interval.

To conclude, we have studied radiative transitions of the charmonium ground and orbitally excited states in the framework of modified CCQM. In doing so, we have introduced one common adjustable parameter $\varrho > 0$ instead of the six independent parameters used before. We have calculated the renormalized couplings and fractional decay widths for the charmonium states $J/\psi(^3S_1)$, $\chi_{c0}(^3P_0)$, $\chi_{c1}(^3P_1)$, $h_c(^1P_1)$, and $\chi_{c2}(^3P_2)$. Our results are in good agreement with the experimental data. We have also calculated the theoretical "full decay width" $\Gamma_{h_c}^{\text{theor}} \simeq (0.57 \pm 0.12)$ MeV that is located in a narrower interval than the experimental value [1].

TABLE I. The fractional decay widths (in units of keV) of the charmonium one-photon radiative transitions calculated within the CCQM compared with recent data and some theoretical predictions.

Decay	CCQM	Exper. [1]	[20]	[18]	[26]
$\Gamma(J/\psi \to \gamma\eta_c)$	1.771	1.58 ± 0.43	-	2.64(11)	1.25
$\Gamma(\chi_{c0} \to \gamma J/\psi)$	142.0	151 ± 14	128	-	128
$\Gamma(\chi_{c1} \to \gamma J/\psi)$	296.7	288 ± 22	266	-	275
$\Gamma(h_c \to \gamma\eta_c)$	290.8	357 ± 270	-	720(70)	587
$\Gamma(\chi_{c2} \to \gamma J/\psi)$	358.1	374 ± 27	353	-	467

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