Suplemento de la Revista Mexicana de Física **3** 0308092 (2022) 1–6

Dressed quark-gluon vertex form factors from gauge symmetry

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Received 11 January 2022; accepted 14 February 2022

We present preliminary results on the longitudinal *and* transverse form factors of the quark-gluon vertex as functions of the incoming and outgoing quark momenta and an angle $\theta = 2\pi/3$ between them. The expressions for these form factors were previously derived from Slavnov-Taylor identities, gauge covariance and multiplicative renormalizability that firmly constrain the fermion-boson vertex.

Keywords: Quantum chromodynamics; strong interactions; nonperturbative techniques; gap equation; quark-gluon vertex; dynamical chiral symmetry breaking.

DOI: https://doi.org/10.31349/SuplRevMexFis.3.0308092

The origin of *dynamical chiral symmetry breaking* (DCSB), the mass-generating mechanism responsible for the overwhelming contribution to the nuclei's masses, lies in the non-Abelian nature of the theory of strong interactions known as Quantum Chromodynamics (QCD). Starting with the seminal work by Nambu and Jona-Lasinio [1], this mechanism has been gradually elucidated in QCD over the past decades. While its role in generating hadron masses two orders of magnitude larger than those of the light current quarks is nowadays widely recognized, its likely deeper connection to *confinement* still remains speculative.

A common approach to investigate DCSB is to study the gap equation of the quark, i.e. its two-point Green function, and its nonperturbative formulation in terms of the Dyson-Schwinger equation (DSE) [2]. The latter is a Euclidean-space description of the quark's equation of motion in relativistic quantum field theory and can be derived from the generating functional in QCD [3]. The self-energy term in this integral equation involves other Green functions, namely the gluon propagator and the quark-gluon vertex, which are irreducible two- and three-point functions, respectively. Both contribute, along with the strong coupling α_s, to the integral kernel's "strength" in the DSE.

Indeed, it is this strength that controls the emergence of a mass gap. While the strong coupling and the form factor associated with the gluon dressing form an overall strength factor, the twelve tensor structures of the quark-gluon vertex reveal a more intricate story. Their contributions are codified in so-called longitudinal and transverse form factors and their convolution with the strong coupling and the gluon dressing function gives rise to a constituent quark mass scale.

Since the full structure of the dressed quark-gluon vertex, and in particular of the associated form factors, still poses a serious computational challenge in functional as well as in lattice-QCD approaches, a common expedient in applications to hadron physics is to retain merely its perturbative γ_μ term. Folding its form factor with that of the gluon propagator and the strong coupling, one arrives at the nowadays well known *rainbow-ladder* truncation of the gap equation, in which a single analytic function mimics the infrared and ultraviolet behavior and the strength of the strong interaction in an effective manner [4, 5].

While this approach has certainly proven to be successful in the computation of the light meson and baryon spectrum and their electromagnetic properties [6–15], it fails to correctly describe the scalar and axialvector meson masses and does not produce a satisfying mass ordering of higher radially excited mesons. It also gives rise to a spurious spectrum of unobserved light mesons [6] with "exotic" quantum numbers, and admits $\bar{3}_c$ colored diquark bound-states. Still, the latter feature can be favorably used to derive approximate Faddeev wave functions of baryons. The shortcoming of this leading approximation is also observed in solving the Bethe-Salpeter equation for pseudoscalar and vector D and B mesons [6, 9], but can be overcome introducing a flavor dependence in the quark-gluon interaction [16–18]. However, this comes at the cost of additional parameters for the charm and bottom mesons.

Important improvements, based on the three-particle irreducible (3PI) effective QCD action [19] or on a model ansatz for the quark-gluon vertex [20,21] amongst others, have been obtained over the past decade, and one may assert that functional QCD approaches to light and flavored mesons, heavy quarkonia and baryons based on the DSE in conjunction with either the Bethe-Salpeter equation (BSE) or Faddeev equation are overall very successful. This includes the mass spectrum of light and heavy mesons, the nucleon and Δ baryons, their parity partners and radial excitations, as well as Compton scattering, elastic and transition form factors. Extensions to tetraquark states have also been studied within this approach [22].

Nonetheless, a less model dependent interaction kernel, based on *calculated* QCD Green functions, of the quark DSE and related bound-state equations is desirable. Only a detailed construction of the interaction kernel, which involves the fully dressed quark-gluon vertex, will allow to verify whether the known hadron spectrum can be completely described with functional methods in QCD. In addition, there are not merely phenomenological but also formal, field-theoretical motivations to study the analytic behavior of the fermion-boson vertex. After all, this vertex plays a pivotal role for DCSB in QED and in QCD. Its contribution to the infrared behavior of the quark propagator and to fragmentation functions, and therefore to the elucidation of the confinement mechanism, cannot be appreciated enough.

In this contribution we extend recent studies on the transverse quark-gluon vertex, which we derived from transverse Slavnov-Taylor identities and multiplicative renormalizability in Refs. [23,24]. In those studies we did not present figures of the different vertex form factors, so we here take the opportunity to fill this gap.

The dressed quark-gluon vertex is the essential three-point function which describes the nonperturbative coupling of a dressed quark to a dressed gluon. As such, it is the source of nonperturbative radiative gluon corrections to the current quark's relativistic motion. As in other relativistic quantum field theories, the related equation of motion can be expressed by a DSE conveniently derived within a functional approach to QCD [3]:

$$S^{-1}(p) = Z_2\, i\gamma\cdot p + Z_4\, m + Z_1 g^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\, \Delta^{ab}_{\mu\nu}(q)\, \gamma_\mu t^a S(k)\, \Gamma^b_\nu(k,p)\ . \quad (1)$$

In this integral equation, m is the renormalized current-quark mass and $Z_4(\mu,\Lambda)$ is its renormalization constant in the QCD Lagrangian, while $Z_1(\mu,\Lambda)$ and $Z_2(\mu,\Lambda)$ are vertex and wave-function renormalization constants, respectively. The integral in Eq. (1) expresses the quark's self-energy $\Sigma(p^2)$, where Λ is an ultraviolet Poincaré invariant cut-off and μ is the renormalization scale imposed, so that $S^{-1}(p)|_{p^2=\mu^2} = \gamma\cdot p + m(\mu)$, with the common choice $\Lambda \gg \mu$. Furthermore, $\Delta_{\mu\nu}(q)$ is the dressed gluon propagator in Landau gauge and $\Gamma^a_\mu(k,p) = \Gamma_\mu(k,p)\, t^a$ is the dressed quark-gluon vertex, where $t^a = \lambda^a/2$ are the SU(3)$_c$ group generators and a,b represent color indices.

The solutions of the DSE (1) can be most generally decomposed into scalar and vector pieces,

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}\ , \quad (2)$$

where $Z(p^2,\Lambda^2,\mu^2) = 1/A(p^2,\Lambda^2,\mu^2)$ and $M(p^2) = B(p^2,\mu^2,\Lambda^2)/A(p^2,\mu^2,\Lambda^2)$ are the flavor dependent, running wave renormalization and mass functions, respectively.

The complete vertex, $\Gamma_\mu(k,p)$, can be expanded in terms of four non-transverse and eight transverse covariant vector structures [25],

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k,p) L^i_\mu(k,p) + \sum_{i=1}^{8} \tau_i(k,p) T^i_\mu(k,p)\ , \quad (3)$$

in which p is the incoming and k the outgoing quark momentum and the gluon momentum, $q = k - p$, flows into the vertex. The transverse vertex is naturally defined by $q\cdot\Gamma^T(k,p) = 0$. We work with the vector base for $L^i_\mu(k,p)$ and $T^i_\mu(k,p)$ defined in Ref. [26].

The form factors of the quark-gluon vertex, in particular of the longitudinal components, have been explored in pQCD and in nonperturbative approaches, see for instance Refs. [21, 27–37] or Ref. [24] for a more detailed bibliography on the fermion-boson vertex. Recently, we employed two transverse Slavnov-Taylor identities [38], which express color gauge invariance and Lorentz covariance and constrain the transverse quark-gluon vertex, to derive the eight $\tau_i(k,p)$ form factors in QCD [24]. Along with the known expressions for $\lambda_i(k,p)$ [31,32] we found that $\Gamma_\mu(k,p)$ in Eq. (3) can be described by the following set of form factors:

$$\lambda_1(k,p) = \frac{1}{2}\, G(q^2) X_0(q^2) \left[A(k^2) + A(p^2)\right]\ , \quad (4)$$

$$\lambda_2(k,p) = G(q^2) X_0(q^2)\, \frac{A(k^2) - A(p^2)}{k^2 - p^2}\ , \quad (5)$$

$$\lambda_3(k,p) = G(q^2) X_0(q^2)\, \frac{B(k^2) - B(p^2)}{k^2 - p^2}\ , \quad (6)$$

$$\lambda_4(k,p) = 0 \quad (7)$$

$$\tau_1(k,p) = -\frac{Y_1}{2(k^2-p^2)\nabla(k,p)}\ , \quad (8)$$

$$\tau_2(k,p) = -\frac{Y_5 - 3Y_3}{4(k^2-p^2)\nabla(k,p)}\ , \quad (9)$$

$$\tau_3(k,p) = \frac{1}{2}\, G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2}\right] + \frac{Y_2}{4\nabla(k,p)} - \frac{(k+p)^2 (Y_3 - Y_5)}{8(k^2-p^2)\nabla(k,p)}\ , \quad (10)$$

$$\tau_4(k,p) = -\frac{6Y_4 + Y_6^A}{8\nabla(k,p)} - \frac{(k+p)^2 Y_7^S}{8(k^2-p^2)\nabla(k,p)}\ , \quad (11)$$

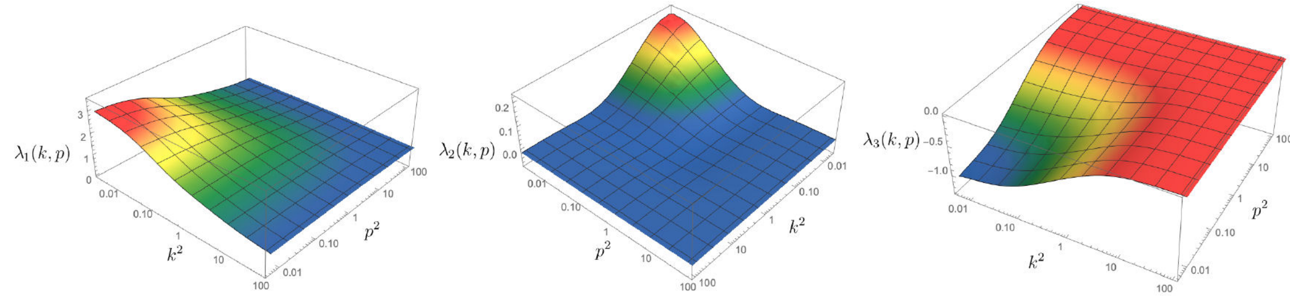

FIGURE 1. Form factors of the longitudinal vertex $\Gamma^L_\mu(k,p)$ (3) as functions of the quark momenta, k^2 and p^2 [in GeV^2], and for the angle $\theta = 2\pi/3$.

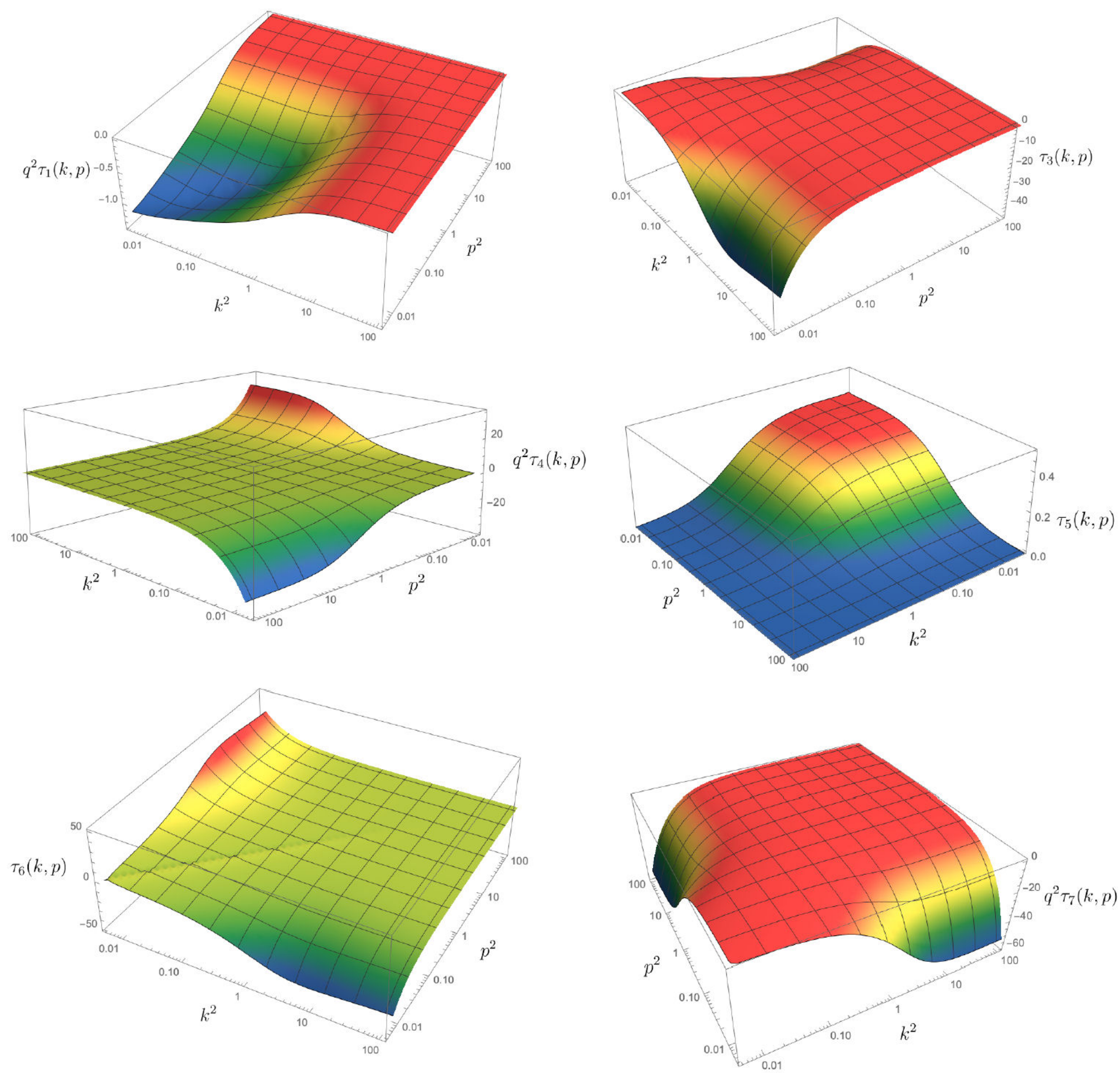

FIGURE 2. Form factors of the transverse vertex $\Gamma^T_\mu(k,p)$ (3) as functions of the quark momenta, k^2 and p^2 [in GeV^2], and for the angle $\theta = 2\pi/3$.

$$\tau_5(k,p) = -G(q^2)X_0(q^2)\left[\frac{B(k^2)-B(p^2)}{k^2-p^2}\right] - \frac{2Y_4+Y_6^A}{2(k^2-p^2)}\,, \tag{12}$$

$$\tau_6(k,p) = \frac{(k-p)^2Y_2}{4(k^2-p^2)\nabla(k,p)} - \frac{Y_3-Y_5}{8\nabla(k,p)}\,, \tag{13}$$

$$\tau_7(k,p) = \frac{q^2(6Y_4+Y_6^A)}{4(k^2-p^2)\nabla(k,p)} + \frac{Y_7^S}{4\nabla(k,p)}\,, \tag{14}$$

$$\tau_8(k,p) = -G(q^2)X_0(q^2)\left[\frac{A(k^2)-A(p^2)}{k^2-p^2}\right] - \frac{2Y_8^A}{k^2-p^2}\,. \tag{15}$$

In Eqs. (8) to (15) the Gram determinant is defined by $\nabla(k,p) = k^2p^2 - (k\cdot p)^2$. The form factors $\lambda_i(k,p)$, $i = 1,2,3$, and $\tau_i(k,p)$, $i = 3,5,8$, are proportional to the ghost-dressing function $G(q^2)$ which is renormalized as $G(\mu^2) = 1$. Moreover, $X_0(q^2)$ is the leading form factor of the quark-ghost scattering amplitude, $H^a(k,p) = H(k,p)t^a$; see, *e.g.*, Refs. [32, 33] for details. The a priori unknown scalar functions, $Y_i^{A,S}$, are form factors we introduce to decompose a four-point function that appears in the transverse Slavnov-Taylor identities and which involves a non-local vector vertex and a Wilson line to preserve gauge invariance. We refer to the discussion in Ref. [38] and merely stress that the $Y_i^{A,S}$ functions have been constrained by us in Ref. [39] with the vertex ansatz of Ref. [29] and insisting on multiplicative renormalizability.

We solve the DSE (1) with numerical input from lattice QCD [40, 41] for the gluon and ghost dressing functions, $\Delta(q^2)$ and $G(q^2)$, and with the vertex defined by Eqs. (3) to (15). The solution for the vector and scalar components of the quark propagator, $A(p^2)$ and $B(p^2)$ respectively, are then used to compute the form factors $\lambda_i(k,p)$ and $\tau_i(k,p)$. We present them in Figs. 1 and 2 as functions of the momenta squared k^2 and p^2 and for the kinematic configuration: $\cos\theta = k\cdot p/|k||p| = -1/2$, $\theta = 2\pi/3$.

The functional form of the $\lambda_i(k,p)$ form factors is similar to that found in Refs. [32, 33] though with differences in magnitude, as those studies exclusively concentrated on $\Gamma^L_\mu(k,p)$. Therefore, some strength is shifted from the transverse to the longitudinal vertex and a direct comparison is difficult. As observed in Ref. [24], the dominating contribution of the transverse vertex in the gap Eq. (1), and therefore to DCSB, is due to the form factors $\tau_4(k,p)$ and $\tau_7(k,p)$. We multiply them and also $\tau_1(k,p)$ by q^2 in Fig. 2 in order to regularize a singular behavior at the origin. Note that this poses no problem in the numerical treatment of the quark DSE since kinematic factors in the integral kernel have an analogous regularizing effect.

A more comprehensive treatment of the these form factors taking into account other angles θ, which include the soft-gluon and symmetric quark limit, is underway. This also requires the contributions of the sub-leading form factors $X_1(k,p)$, $X_2(k,p)$ and $X_3(k,p)$ that parametrize the quark-ghost scattering amplitude.

Acknowledgments

B.E.'s research activity is funded by FAPESP grant No. 2018/20218-4 and CNPq grant No. 428003/2018-4. F.E.S. is a CAPES-PNPD postdoctoral fellow, grant no. 88882.314890/2013-01, L.A. was a FAPESP postdoctoral fellow supported by grant No. 2018/17643-5, and R.C.S. is a CAPES PhD fellow. A.B. acknowledges funding from the Coordinación de la Investigación Científica (CIC) of the University of Michoacán and CONACyT, Mexico, through grant Nos. 4.10 and CB2014-22117, respectively.

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