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Nonlinear Regge trajectories in the context of bottom-up holography

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Motivated by the non-holographic phenomenology, where the mesonic constituent mass breaks linearity in Regge trajectories, we discuss how to implement nonlinear Regge trajectories by deforming the static (monoparametric) quadratic dilaton into a non-quadratic one. This deformation adds an extra parameter into the dilaton, which measures the constituent mass effect, accounting for nonlinearity in the hadronic trajectory. We applied this model to the description of the isovector multiplet spectrum. The set of isovector parameters defines a set of hadronic calibration curves for the dilaton slope and linearity deviation parameter, allowing us to extrapolate the model to other vector states, such as heavy-light mesons or vector non-q$\bar{q}$ hadrons. In other words, this approach allows us to consider the hadronic inner structure by modifying the dilaton profile.

Keywords: AdS/QCD; hadron spectroscopy; Regge trajectories.

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1. Introduction

One of the most interesting problems in non-perturbative QCD is hadronic spectroscopy. The existence of hadronic bounded states is direct evidence of confinement. These emerging hadronic spectra are taxonomically organized in structures called *Regge Trajectories* (RT). Each known hadronic state belongs to a given trajectory, defined in its quantum numbers: radial or angular momentum [1, 2].

Confinement emerges at the holographic bottom-up level due to breaking the conformal symmetry slightly at the bulk [3]. In pure AdS, free normalizable modes form a continuous Eigenspectrum. By deforming the background AdS geometry [4, 5] or adding a dilaton field [3], we can obtain discrete eigenspectra, dual to hadronic Regge trajectories at the boundary. This manuscript will explore the latter: by using a dilaton field in the bulk action, we will generate a radial mass spectrum $M_n^2(n)$, where n is the radial quantum number, dual to hadronic RTs.

This work is organized as follows: first, in Sec. 2, we will give a brief discussion about confinement emergence and AdS geometry to motivate the bottom-up AdS/QCD idea. Then, we move to Sec. 3, where we will introduce the holographic non-linear RT from a static quadratic dilaton deformation, following Ref. [6]. We will test our approach with the isovector meson family. Then, in Sec. 4, we will apply this idea to other mesonic species as the vector kaons and vector heavy-light mesons. We will also use this non-quadratic dilaton idea to describe holographically the vector tetraquark candidate [7] $Z_c(3900)$ by analyzing its inner structure. Finally, in Sec. 5, we conclude our work.

2. AdS space, Confinement and Hadrons

In its original form, AdS/CFT does not hold with the idea of confinement. Normalizable free bulk fields acquire a continuous spectrum since the associated Schrodinger-like potential does not produce bounded states, a clear signal of confinement emergence. We have to break the bulk conformal invariance softly to circumvent this issue. We can deform the bulk geometry or introduce extra bulk fields to do so. In this manuscript, we will follow the latter.

As it was proved in Ref. [3], by including a static dilaton $\Phi(z)$ in the AdS bulk action

$$I_{\text{Hadron}} = \int d^5x\sqrt{-g}\; e^{-\Phi(z)}\,\mathcal{L}_{\text{Hadron}}, \tag{1}$$

we can obtain a discrete spectrum, which is dual to the hadrons living at the boundary. The Lagrangian density $\mathcal{L}_{\text{Hadron}}$ carries all of the holographic information concerning hadrons at the conformal boundary. Thus, equations of motion coming from $\mathcal{L}_{\text{Hadron}}$ will allow us to construct Regge Trajectories (RT) as their mass eigenvalues.

Following this idea, we will start from the Poincare patch metric, defined as

$$dS^2 = \frac{R^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\,dx^\mu\,dx^\nu\right], \tag{2}$$

where R is the AdS curvature radius, $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric and z is the holographic coordinate. In this geometry, the conformal boundary lies at $z \to 0$ [8].

3. Non-linear Regge Trajectories

Regge trajectories (RT) are defined as a taxonomic form to systematically organize hadronic masses in their quantum numbers, such as radial number, angular momentum, or spin. In general, Regge trajectories can be written as

$$M_n^2 = A\,(n+B)^{\nu}\,, \tag{3}$$

where A is the Regge slope, B is the intercept, and ν is the linearity of the trajectory. The Bethe-Salpeter analysis makes it possible to connect the linearity with the constituent mass, implying that in the massless limit, $\nu \rightarrow 1$.

Holographically, RTs emerge from the eigenvalue problem $-\psi'' + V(z)\,\psi = M_n^2\,\psi$, calculated with the Schrödinger-like potential defined as

$$\begin{aligned} V(z) &= \frac{1}{4}\left(\frac{\beta}{z}+\Phi'(z)\right)^2 \\ &\quad + \frac{1}{2}\left(\frac{\beta}{z^2}-\Phi''(z)\right) + \frac{M_5^2\,R^2}{z^2}, \end{aligned} \tag{4}$$

where $\beta = -3 + 2\,S$ defines the hadronic spin S, R is the AdS curvature radius, and the bulk field mass M_5^2 defines the hadronic identity through the dimension Δ of the operator creating hadrons. In general we have for the bulk mass

$$M_5^2\,R^2 = (\Delta - S)(\Delta + S - 4). \tag{5}$$

Notice that $\psi(z)$ is a normalizable field defined from the bulk fields via the so-called *Bogoliubov transformation* [3,6]. This normalizable $\psi(z)$ mode is dual to hadrons of spin S at the boundary. In our particular case, we will focus on the radial isovector family (ω, ϕ, J/ψ and Υ mesons), characterized by the quantum numbers $I^G\,J^{PC} = 0^-(1^{--})$. Holographically, the isovector family is defined by the following parameter choice: $M_5 = 0$ and $\beta = -1$.

As it was proved in [3], when the dilaton is chosen to be quadratic, *i.e.*, $\Phi(z) = \kappa^2\,z^2$, the holographic trajectory is linear, implying $\nu = 1$. In this formalism, κ defines the Regge slope and carries energy (GeV) units.

Following the Bethe-Salpeter frame [1,9], non-vanishing constituent quark masses m_q cause the trajectory linearity to deviate from one. Furthermore, when $m_q \rightarrow \infty$, we get $\nu \rightarrow 2/3$.

Holographically, this phenomenology is reflected in the dilaton profile deformation $\Phi(z) = (\kappa\,z)^{2-\alpha}$, where linearity in RTs is recovered when $\alpha = 0$, implying massless constituent quarks. When the constituent mass is increased, α increases also. Table I summarizes our holographic results compared with experimental data (PDG) [10].

TABLE I. Summary of results for different families of isovector radial mesonic states considered in this work. All of the mass spectra displayed in this table are calculated with the parameters mentioned on each sub-table header, using (4) with the deformed dilaton $\Phi(z) = (\kappa\,z)^{2-\alpha}$. The Regge trajectories are also presented in units of GeV^2. The last column on each set of data is the relative error per state. Experimental results are read from PDG [10].

ω **with** $\alpha = 0.04$ **and** $\kappa = 498$ MeV				ϕ **with** $\alpha = 0.07$ **and** $\kappa = 585$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**	**n**	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	782.65 ± 0.12	981.43	25.4	1	1019.461 ± 0.016	1139.43	11.8
2	$1400 - 1450$	1374	3.6	2	1698 ± 20	1583	5.8
3	1670 ± 30	1674	0.25	3	$2135 \pm 8 \pm 9$	1921	10
4	1960 ± 25	1967	1.7	4	Not Seen	–	–
5	2290 ± 20	2149	6.2	5	Not Seen	–	–
$M^2 = 0.9514(0.012+n)^{0.9798}$ with $R^2 = 0.999$				$M^2 = 1.268(0.0244+n)^{0.9650}$ with $R^2 = 0.999$			
ψ **with** $\alpha = 0.54$ **and** $\kappa = 2150$ MeV				Υ **with** $\alpha = 0.863$ **and** $\kappa = 11209$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**	**n**	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	3096.916 ± 0.011	3077.09	0.61	1	9460.3 ± 0.26	9438.5	0.23
2	3686.109 ± 0.012	3689.62	0.1	2	10023.26 ± 0.32	9923.32	0.78
3	4039 ± 1	4137.5	2.44	3	10355 ± 0.5	10277.2	0.75
4	4421 ± 4	4499.4	1.77	4	10579.4 ± 1.2	10558.6	0.19
5	Not Seen	–	–	5	$10889.9^{+3.2}_{-2.6}$	10793.5	0.88
6	Not Seen	–	–	6	$10992.9^{+10.0}_{-3.1}$	10995.7	0.03
$M^2 = 8.07(0.287+n)^{0.6315}$ with $R^2 = 0.999$				$M^2 = 76.511(0.901+n)^{0.2369}$ with $R^2 = 0.999$			

TABLE II. Summary of results for the vector kaon K^* radial mesonic states, with $I(J^P) = 1/2(1^-)$. The last column is the relative error per state. Notice that we also show the linear and non-linear radial experimental RTs, altogether with the theoretical fit for the sake of clarity. The values of κ and α are extrapolated from Eqs. (6) and (7). See [6] for further details. Experimental results are read from PDG [10].

K^* **with** $\bar{m} = 413$ **MeV,** $\alpha = 0.055$**, and** $\kappa = 531.24$ MeV				
n	**State**	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	$K^*(892)$	895.55 ± 0.8	1038.4	16.2
2	$K^*(1410)$	1414 ± 15	1451.0	2.6
3	$K^*(1680)$	1718 ± 18	1754.5	2.1
Experimental Linear R. T.:		$M^2 = 1.075(-0.2157 + n)$ with $R^2 = 0.9992$.		
Experimental Non-Linear R. T.:		$M^2 = 1.157(-0.6102 + n)^{0.718}$ with $R^2 = 1$.		
Theoretical Non-Linear R. T.:		$M^2 = 1.175(-0.0911 + n)^{0.902}$ with $R^2 = 1$.		

TABLE III. Summary of results for vector heavy-light mesonic states contrasting our theoretical results with the available experimental data. The last column is the relative error per state. Experimental results are read from PDG [10]. The values of κ and α are extrapolated from Eqs. (6) and (7). See [6] for further details.

State	$I(J^P)$	$q_1\, q_2$	$\bar{m}$ **(MeV)**	κ **(MeV)**	α	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
$K^*(782)$	$1/2(1^-)$	$d\,\bar{s}$	413	531.24	0.055	895.55 ± 0.8	1038.4	16.2
$D^{*0}(2007)$	$1/2(1^-)$	$c\,\bar{u}$	943	1070.8	0.261	2006.85 ± 0.05	1902.5	5.20
$D^{+0}(2010)$	$1/2(1^-)$	$c\,\bar{d}$	945	1073.6	0.262	2010.26 ± 0.05	1906.4	5.16
D_s^{*+}	$0(?^?)$	$c\,\bar{s}$	1018	1179.1	0.296	2112.2 ± 0.4	2051.7	2.86
B^{*+}	$1/2(1^-)$	$u\,\bar{b}$	2533	4681.2	0.800	5324.70 ± 0.22	4561.2	14.3
B^{*0}	$1/2(1^-)$	$d\,\bar{b}$	2535	4687.3	0.801	5324.70 ± 0.22	4564.4	14.27
B_s^{*0}	$0(1^-)$	$s\,\bar{b}$	2608	4901.2	0.809	$5415^{+1.8}_{-1.5}$	4683.0	13.52

4. Other hadronic families

The parameters used to describe the isovector family, *i.e.*, κ and α, define a *running behavior* with the constituent mass

$$\alpha(\bar{m}) = 0.8454 - 0.8485\, e^{-0.4233\,\bar{m}^2}, \qquad (6)$$

$$\kappa(\bar{m}) = 15.21 - 14.81\, e^{-0.0524\bar{m}^2}. \qquad (7)$$

Thus, if we properly determine a consistent parametrization for the constituent mass, we can extrapolate the RTs for other hadronic species.

In this spirit, we will define the following parametrization for constituent masses, regarding the constituent type, as follows

$$m_{\text{cons}} = \sum_i^{\text{quarks}} P_i^q\, m_i + \sum_i^{\text{gluons}} P_i^g\, m_g + \sum_i^{\text{meson cores}} P_i^M\, m_M, \qquad (8)$$

where we are considering hadronic structures made of quarks, gluons and meson cores, as in the non-$q\bar{q}$ hadrons. Notice that $P_i^{\text{constituent}}$ defines the probability to have a particular constituent inside the hadron, thus $\sum_{i,\text{constituent}} P_i^{\text{constituent}} = 1$, and $m_{\text{constituent}}$ is the constituent mass.

We will apply these conditions to the heavy-light meson sector and to possible tetraquark structures.

4.1. Vector Kaons

Vector kaons, identified as $I(J^P) = 1/2(1^-)$, with $S = \pm 1$ and $C = B = 0$, are constructed by considering $P_i^g = P_i^M = 0$. Thus, the constituent mass $\bar{m}$ is given by

$$\bar{m}_{K^*} = \frac{m_s + m_d}{2}. \qquad (9)$$

Using this constituent mass trigger in Eqs. (6) and (7) we calculate the parameters α and κ for the vector kaons. As in the unflavored case, the bulk mass is fixed to $M_5^2\, R^2 = 0$. Results in this case are summarized in Table II.

4.2. Heavy-Light vector mesons

In the case of the heavy-light sector, we consider $P_i^g = P_i^M = 0$. Thus, the constituent mass $\bar{m}$ contribution is fixed as the average constituent quark mass. Therefore, we have

$$\bar{m}_{HL} = \frac{m_H + m_L}{2}. \qquad (10)$$

TABLE IV. This table summarizes the holographic results for three different structures for the $Z_c(3900)$ candidate to tetraquark. As a holographic prediction, the preferred structure is the hadronic molecule, with the smallest relative error. Experimental results are taken from PDG [10].

Holographic test for $Z_c(3900)$ **with** $I^G(J^{CP}) = 1^+(1^{+-})$	
Experimental mass: 3887.2 ± 2.3 MeV.	
$\alpha = 0.539$ **and** $\kappa = 2151$ **MeV**	
$\Delta = 6$ **and** $\bar{m}_{\textbf{diquark-antidiquark}}$	
Theoretical mass: 4004.8 MeV	Relative error: 3.0%
$\Delta = 6$ **and** $\bar{m}_{\textbf{hadronic molecule}}$	
Theoretical mass: 3816.3 MeV	Relative Error: 1.82%
$\Delta = 5$ **and** $\bar{m}_{\textbf{Hybrid meson}}$	
Theoretical mass: 3721.9 MeV	Relative error: 4.24%.

With this constituent mass trigger, we can use Eqs. (6) and (7) to calculate the non-quadratic dilaton parameters α and κ. As in the case of isovector mesons, the bulk mass $M_5^2\, R^2 = 0$. Numerical results are exposed in Table III.

4.3. Tetraquark candidates

Another interesting possibility we can explore using the (α, κ)-running Eqs. (6) and (7) is the holographic modeling of non-$q\,\bar{q}$ hadronic states.

At the holographic level, the only hadronic fingerprint we have at hand is the bulk mass, which exclusively depends on the constituent number. However, the bulk mass is not sensitive to inner configuration. Thus, holographic tests of possible multiquark states are difficult. It is necessary to introduce another observable that breaks this bulk mass *degeneracy.* In our non-quartic dilaton proposal we can use the average constituent mass (8) to explore these multiquark structures with fixed number of constituent quarks.

Let us consider the $Z_c(3900)$ meson with $I^G(J^{CP}) = 1^+(1^{+-})$, which is a vector tetraquark candidate (See [7] for further details). This particular state can be modeled as a diquark-antidiquark pair, hadrocharmonium, hadronic molecule or hybrid mesons.

These structures can be parametrized in terms of the constituent mass equation as

$$\bar{m}_{\text{diquark-Antidiquark}} = \bar{m}_c, \tag{11}$$

$$\bar{m}_{\text{hadronic molecule}} = 0.283\, m_{J/\psi} + 0.717\, m_\rho, \tag{12}$$

$$\bar{m}_{\text{hybrid meson}} = 0.49\, m_q\, 0.49\, m_{\bar{q}} + 0.02\, m_G, \tag{13}$$

where we have used $\bar{m}_u(d) = 336$ MeV, $\bar{m}_c = 1550$ MeV, $\bar{m}_G = 700$ MeV, $m_{J/\Psi} = 3077.9$ MeV and $m_\rho = 770$ MeV as constituent quark and core meson masses (see PDG [10]). Numerical results for each structure are summarized in Table IV. Further analysis concerning other non-$q\bar{q}$ hadrons can be found in [6].

In Table IV we have considered multiquark structures, which have associated the $\Delta = 6$ conformal dimension, and gluonic excitations with $\Delta = 5$. With these values of Δ we can compute the corresponding bulk mass using Eq. (5). Our holographic analysis for the $Z_c(3900)$ meson suggests the hadronic molecule structure is preferred since its hadronic mass has the smallest relative error.

5. Conclusions

In this work we have introduce a different dilaton profile to address isovector meson masses, *i.e.*, $\Phi(z) = (\kappa\, z)^{2-\alpha}$. This non-quadratic profile induces *non-linear Regge Trajectories* at the conformal boundary. This non-linearity, measured with the parameter ν, defines $(\alpha, \kappa)-$parameter running with the hadronic constituent mass. This feature allows us to explore vector kaons and heavy light systems with a single input parameter, the constituent mass $\bar{m}$, given in Eq. (8). The RMS error obtained by fitting 27 vector meson states with 15 parameters is near to 13%.

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