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Kaon and nucleon states with hidden charm

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In this talk we discuss the formation of exotic hadrons with hidden charm arising from three-body interactions. To be more specific, in the strangeness sector, we predict the existence of a mesonic state, $K^*(4307)$, which is dynamically generated from the three-body interactions of the $KD\bar{D}^*$ system, has a mass around 4307 MeV and quantum numbers $I(J^P) = 1/2\,(1^-)$. In the baryonic sector, we predict the existence of N^* states, which are generated from the three-body interactions of the $ND\bar{D}^*$ system, with masses around $4400 \sim 4600$ MeV, widths of $2 \sim 20$ MeV and positive parity.

Keywords: Exotic states; heavy-quark symmetry; hidden charm states.

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1. Introduction

In the latest years, due to the available access to higher energy regions, claims of observation of exotic states have increased, drawing a lot of attention to the subject. A classical example of these exotic states are the so called X, Y and Z families (see, *e.g.*, Refs. [1–5]).

Interestingly, all heavy exotic hadrons found experimentally in the recent years have two characteristics in common: $1)$ the meson states can be interpreted as tetraquarks or as states obtained from the dynamics involved in two-meson systems, while the baryon states can be understood as pentaquarks or as states originated from meson-baryon interactions; $2)$ most states have hidden or explicit charm content but somehow strangeness seems to miss the focus.

In this talk we present the results obtained for two different systems with hidden charm which we have studied: $KD\bar{D}^*$ and $ND\bar{D}^*$. In the former case, such study was motivated by the fact that the $D\bar{D}^*$, KD, KD^* subsystems have attractive interactions in s-wave, generating the states $X(3872)$ and $Z_c(3900)$ (in case of the $D\bar{D}^*$ interaction), $D_{s0}(2317)$ and $D_{s1}(2460)$ (from the KD and KD^* interactions). It is also interesting to note that considering all interactions in s-wave, the quantum numbers of the generated state would be compatible with a K^*, but with a mass being in the charmonium sector. Finding of such a state can stimulate experimental studies of kaons which seem to have stopped at 3100 MeV [6], even though data at higher energies are available. In case of the $ND\bar{D}^*$ system and the possible formation of N^* states with hidden charm, recently the LHCb collaboration announced the existence of possible hidden charm pentaquarks with non-zero strangeness and mass around 4459 MeV [7]. Such masses are close to the thresholds of the $ND\bar{D}^*/N\bar{D}D^*$ system. As mentioned above, the interaction in the $D\bar{D}^*$ subsystem is attractive, generating the states $X(3782)$ and $Z_c(3900)$. Interestingly, the interactions in the ND and ND^* subsystems are also attractive and form, for example, the state $\Lambda_c(2595)$. In this way, it is quite probable that N^* states with a three-body nature arise as a consequence of the dynamics involved in the $ND\bar{D}^*$ system.

2. Formalism

We are interested in two different systems constituted of three hadrons. In order to study the dynamics of these systems we can solve the Faddeev equation to obtain the T-matrix for the system.

For a three-body system, if the third particle P_3 is lighter than the cluster composed of the two other particles (P_1 and P_2), and we are looking for the possible formation of bound states, we can rely on the fixed center approximation (FCA) to solve the Faddeev equations. Considering N (K) as P_3 for the $ND\bar{D}^*$ ($KD\bar{D}^*$) system, with $D\bar{D}^*$ clustering as $X(3872)$ or $Z_c(3900)$, with isospin 0 or 1 respectively, both our systems, $ND\bar{D}^*$ and $KD\bar{D}^*$, satisfy the above mentioned criteria, that is, for each of the systems P_3 is lighter than the cluster, such that we can use the FCA to solve the Faddeev equations.

In the following, we are going to briefly present the formalism. More details can be found in Refs. [8, 9].

Our goal is to obtain the three-body T-matrices for the $KD\bar{D}^*$ and $ND\bar{D}^*$ systems. By using the FCA we are able to decompose the amplitude T as a sum of two partitions, T_{31} and T_{32}, which satisfy the following coupled equations

$$\begin{aligned} T &= T_{31} + T_{32}, \\ T_{31} &= t_{31} + t_{31} G_a T_{32}, \\ T_{32} &= t_{32} + t_{32} G_a T_{31}. \end{aligned} \tag{1}$$

In Eq. (1), t_{31} and t_{32} are two-body t-matrices describing the interactions in the $KD(ND)$ and $K\bar{D}^*(N\bar{D}^*)$ subsystems, respectively, while G_a is the propagator of the P_3 particle, that is, the propagator of K (N), in the cluster, and it is given by

$$\begin{aligned} G_K &= \frac{1}{2M_a}\int \frac{d^3q}{(2\pi)^3}\frac{F_a(\mathbf{q})}{q_0^2-\mathbf{q}^2-m_K^2+i\epsilon}, \\ G_N &= \frac{1}{2M_a}\int \frac{d^3q}{(2\pi)^3}\frac{m_N}{\omega_N(\mathbf{q})}\frac{F_a(\mathbf{q})}{q_0-\omega(\mathbf{q})+i\epsilon}. \end{aligned} \tag{2}$$

In Eq. (2), F_a is a form factor related to the molecular nature of the cluster and can be written as [10–12]

$$\begin{aligned} F_a(\mathbf{q}) &= \frac{1}{N}\int\limits_{|\mathbf{p}|,|\mathbf{p}-\mathbf{q}|<\Lambda} d^3\mathbf{p}\;\; f_a(\mathbf{p})f_a(\mathbf{p}-\mathbf{q}), \\ f_a(\mathbf{p}) &= \frac{1}{\omega_{a1}(\mathbf{p})\omega_{a2}(\mathbf{p})}\cdot\frac{1}{M_a-\omega_{a1}(\mathbf{p})-\omega_{a2}(\mathbf{p})}, \end{aligned} \tag{3}$$

with M_a being the mass of the cluster, $N = F_a(\mathbf{q}=0)$ is a normalization constant, Λ represents a cut-off $\sim$700 MeV and $\omega_{ai} = \sqrt{m_{ai}^2+\mathbf{p}^2}$.

Further, to solve Eq. (1), we need the t_{31} and t_{32} two-body t-matrices. To illustrate the method for calculating them, we consider, for example, the $KD\bar{D}^*$ system and first determine the expression for t_{31} in terms of different isospin contributions.

The $KD\bar{D}^*$ system has three possible $K-$cluster isospin configurations: $|KX, I=1/2, I_3=1/2\rangle$, $|KZ_c, I=1/2, I_3=1/2\rangle$ and $|KZ_c, I=3/2, I_3=3/2\rangle$. Considering for instance the state $|KX, I=1/2, I_3=1/2\rangle$, the amplitude t_{31} is given by

$$\langle KX, I=1/2, I_3=1/2|\, t_{31}\, |KX, I=1/2, I_3=1/2\rangle\,.$$

Remembering that t_{31} is related to the interaction between the K and the D, we need to express the KX state in terms of the isospin of the KD subsystem. This can be done by using Clebsch-Gordan coefficients to write

$$\begin{aligned} &|KX, I=1/2, I_3=1/2>=|KX,1/2,1/2\rangle \\ &= \frac{1}{2}\left[|KD,1,1\rangle\otimes\left|\bar{D}^*,\frac{1}{2},-\frac{1}{2}\right\rangle\right. \\ &\left.-\frac{1}{\sqrt{2}}\left(|KD,1,0\rangle+|KD,0,0\rangle\right)\otimes\left|\bar{D}^*,\frac{1}{2},\frac{1}{2}\right\rangle\right]. \end{aligned} \tag{4}$$

TABLE I. t_{31} amplitudes for the $KD\bar{D}^*$ system in terms of the two-body t-matrices for the KD subsystem.

	KX	KZ_c
KX	$\frac{1}{4}\left(3t_{KD}^{I=1}+t_{KD}^{I=0}\right)$	$\frac{\sqrt{3}}{4}\left(t_{KD}^{I=1}-t_{KD}^{I=0}\right)$
KZ_c	$\frac{\sqrt{3}}{4}\left(t_{KD}^{I=1}-t_{KD}^{I=0}\right)$	$\frac{1}{4}\left(t_{KD}^{I=1}+3t_{KD}^{I=0}\right)$

Using the preceding equation, the t_{31} amplitude for the $KX\to KX$ transition in isospin $1/2$ can be written as follows

$$\langle KX|\, t_{31}\, |KX\rangle \equiv t_{1(11)} = \frac{1}{4}(3t_{KD}^{I=1}+t_{KD}^{I=0}), \tag{5}$$

where, to simplify the notation, we use the subscript 11 to denote $KX\to KX$. In Eq. (5) $t_{KD}^{I=a}$ is the two-body t-matrix for the KD subsystem with isospin $I=a$, where $a=0,1$. The system KZ_c can also have total isospin $1/2$, thus, it can couple to KX. This means that we also need to consider the transitions $KX\to KZ_c$ and $KZ_c\to KZ_c$ in order to obtain t_{31}. Repeating the process previously explained to get Eq. (5), we can summarize the different contributions to t_{31} as presented in Table I.

Analogously, in case of t_{32} we obtain the same results as in Table I but changing $D\to\bar{D}^*$ and adding a global minus sign to the non-diagonal terms.

Similarly, the results for the $ND\bar{D}^*$ system are completely analogous to those obtained for the $KD\bar{D}^*$ system since the same clusters are formed and both K and N have isospin $1/2$.

As we can see in Table I, to determine t_{31} and t_{32}, and thus solve Eq. (1), we need the two-body t-matrices for the ND/KD, $N\bar{D}/K\bar{D}$, ND^*/KD^*, $N\bar{D}^*/K\bar{D}^*$ subsystems. These amplitudes can be obtained by solving the Bethe-Salpeter equation

$$t_{AB} = V_{AB} + V_{AB}G_{AB}t_{AB}, \tag{6}$$

where G_{AB} is the two body loop function for a channel made of hadrons A and B, and V_{AB} is the corresponding kernel, which is obtained from an effective Lagrangian based on appropriate symmetries. The loop function G_{AB} needs to be regularized either with cut-off or with dimensional regularization.

To determine the KD, $K\bar{D}^*$ two-body t-matrices, we have considered the following effective Lagrangian based on heavy-quark spin symmetry [16–18]

$$\mathcal{L} = \frac{1}{4f^2}\left\{\partial^\mu P\left[\phi,\partial_\mu P\right]P^\dagger - P\left[\phi,\partial_\mu\right]\partial^\mu P^\dagger\right\}, \tag{7}$$

where P and ϕ are given by

$$P = \left(D^0\; D^+\; D_s^+\right), \tag{8}$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (9)$$

In the case of the $ND/N\bar{D}^*/ND^*/N\bar{D}$ two-body t-matrices, we consider two models: the first one is based on the $SU(4)$ and heavy-quark spin symmetries [13, 14], while the second one is based on the $SU(8)$ spin flavor symmetry [15].

It is interesting to note that different to the KD and KD^* systems, the ND and ND^* systems can be coupled in s-wave: the state ND (ND^*) in s-wave, *i.e.*, orbital angular momentum $L = 0$, has spin-parity $J^P = 1/2^-$ ($J^P = 1/2^-, 3/2^-$). To consider this fact, we follow Ref. [14] and obtain the transition amplitude $DN \to D^*N$ through box diagrams describing the processes $ND \to ND$ and $ND^* \to ND^*$ (for more details we refer the reader to Ref. [14]).

In case of the model based on the $SU(8)$ spin flavor symmetry, the $DN \to D^*N$ transition is already included in the $SU(8)$ effective Lagrangian used in Ref. [15].

3. Results

In Figs. 1 and 2 we show the results obtained for $|T|^2$ as a function of $\sqrt{s}$ for the $KD\bar{D}^*$ system and the configurations KX and KZ_c. As we can see, in both cases, a peak around 4300 MeV shows up in the processes $KX \to KX$ and $KZ_c \to KZ_c$, considering KX and KZ_c as coupled channels when solving Eq. (1). In the KZ_c case, we have included the width of the Z_c state by making the following transformation $M \to M - i\Gamma/2$, with $\Gamma \sim 28$ MeV, in the corresponding form factor of Eq. (3). We have also varied the cut-off present in Eq. (3) from 700 to 750 MeV, but as can be seen in the Figs. 1 and 2 the results do not depend strongly on the cut-off.

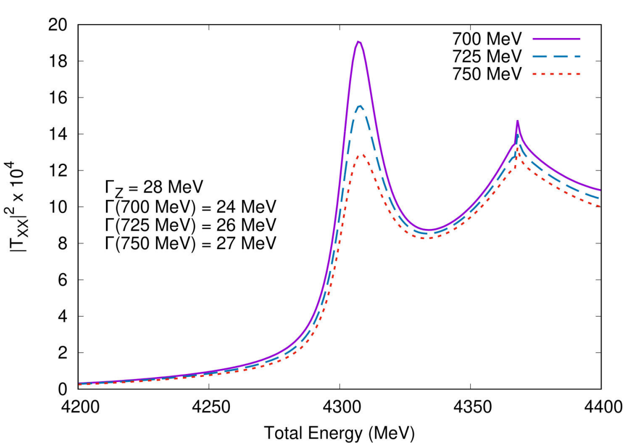

FIGURE 1. Modulus squared of the three-body T-matrix for the transition $KX \to KX$ considering the coupling between the KX and KZ_c channels.

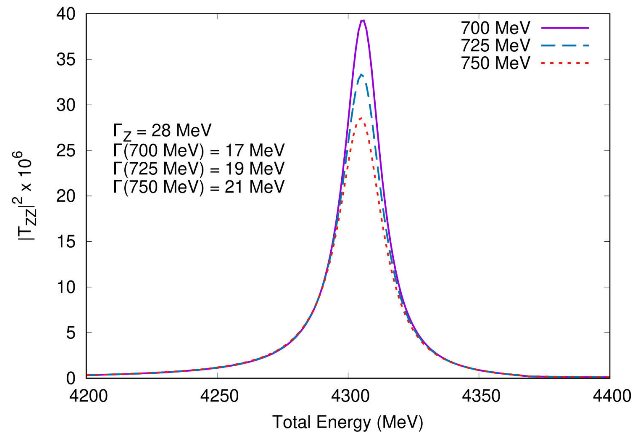

FIGURE 2. Modulus squared of the three-body T-matrix for the transition $KZ_c \to KZ_c$ considering the coupling between the KX and KZ_c channels.

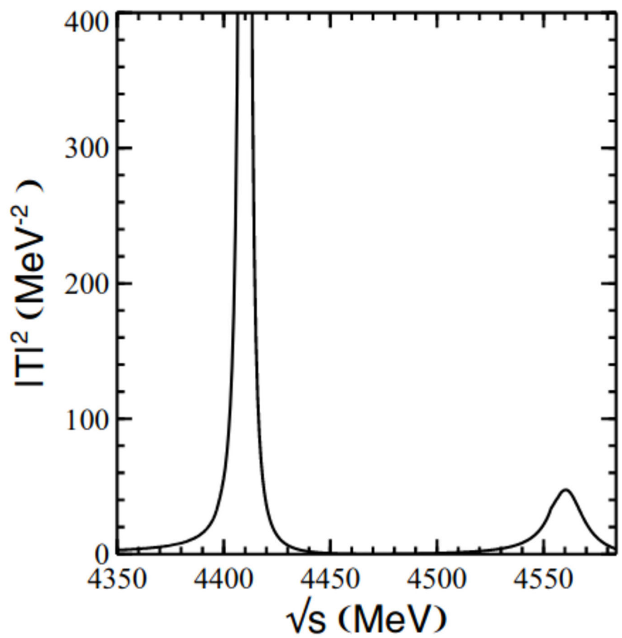

FIGURE 3. $|T|^2$ for the transition $NZ_c \to NZ_c$ with $I(J^P) = 1/2\,(1/2^+)$ as a functions of $\sqrt{s}$.

In $|T|^2$ of the $KX \to KX$ transition shown in Fig. 1 we can also see a peak around 4375 MeV, which is related to the threshold of the three-body system.

We have also computed $|T|^2$ for the transition $KZ_c \to KZ_c$ with isospin $3/2$ but no signal of formation of a bound state is found.

In Figs. 3-6, we show the results for $|T|^2$ versus $\sqrt{s}$ for the $ND\bar{D}^*$ system as obtained with the inputs from the model of Refs. [13, 14]. In case of Figs. 3 and 6, the results shown correspond to the transitions $NX \to NX$ and $NZ_c \to NZ_c$, respectively, with $I(J^P) = 1/2\,(1/2^-)$, while Figs. 4 and 5 show the corresponding results in case of $I(J^P) = 1/2\,(3/2^+)$. In all cases NX and NZ_c are treated as coupled channels. In all four graphics we can see that there are two peaks close to 4400 MeV and 4550 MeV,

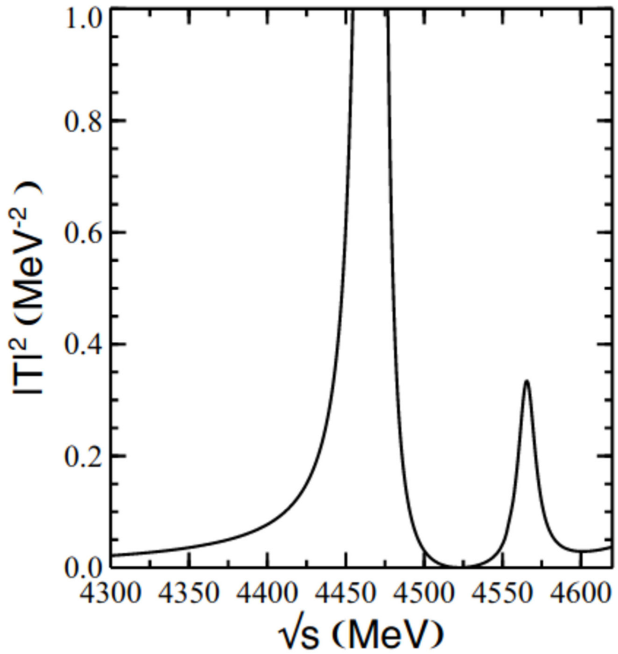

FIGURE 4. $|T|^2$ for the transition $NX \to NX$ with $I(J^P) = 1/2(3/2^+)$ as a functions of $\sqrt{s}$.

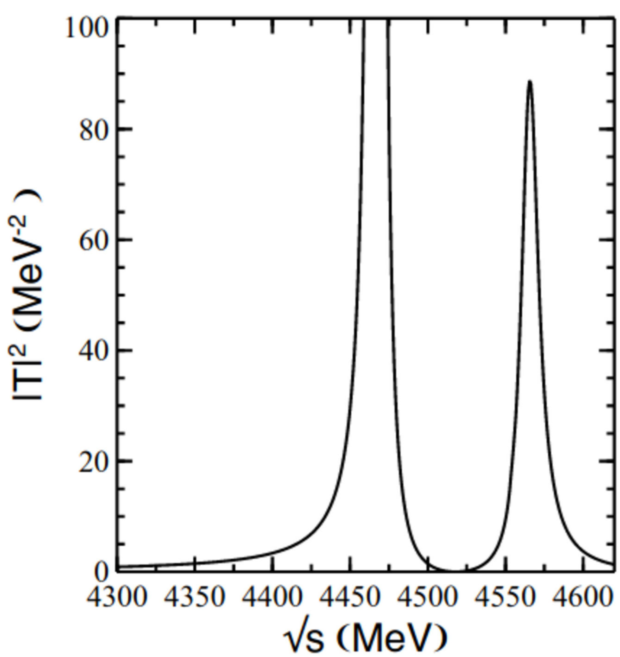

FIGURE 5. $|T|^2$ for the transition $NZ_c \to NZ_c$ with $I(J^P) = 1/2\,(3/2^+)$ as a functions of $\sqrt{s}$.

respectively. Varying the cut-off of Eq. (3) from 700 MeV to 770 MeV causes a small shift, about $3-5$ MeV, on the masses of the states obtained. Similar results are found when using the model of Ref. [15] to calculate the input two-body amplitudes. The results obtained with different models for the two-body interactions ($SU(4)$ heavy-quark spin symmetry model or $SU(8)$ model) as well as the different cut-off

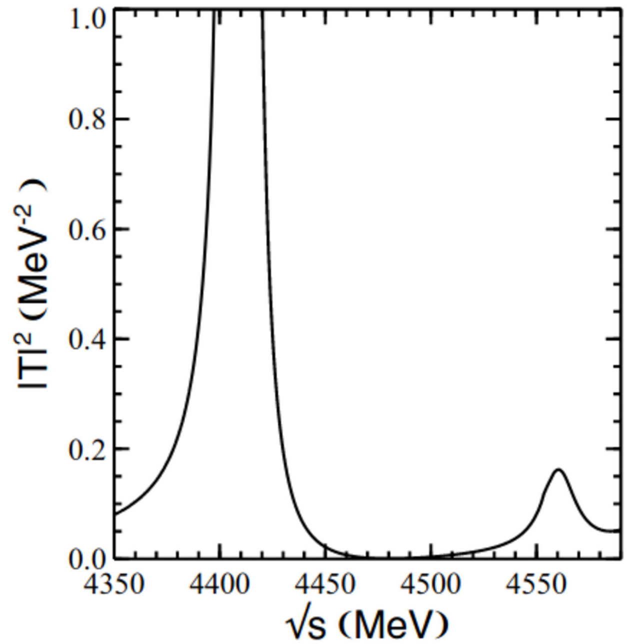

FIGURE 6. $|T|^2$ for the transition $NX \to NX$ with $I(J^P) = 1/2\,(1/2^+)$ as a functions of $\sqrt{s}$.

TABLE II. Masses and widths of the three-body states found in the study of the $ND\bar{D}^*$ system.

Spin-parity	Mass(MeV)	Width (MeV)
$1/2^+$	$4404-4410$	2
$1/2^+$	$4556-4560$	$\sim 4-20$
$3/2^+$	$4467-4513$	$\sim 3-6$
$3/2^+$	$4558-4565$	$\sim 5-14$

used in the form factor computation provide us uncertainties in the masses and widths of the states which we summarize in Table II.

4. Conclusions

From our study we can conclude that adding a Kaon or a Nucleon to a cluster formed by $D\bar{D}^*$ generates states with hidden charm and a three-body molecular nature, that is, states whose inner structure can be described by the hadronic interactions without considering the quarks and gluons interactions.

Our findings for the $KD\bar{D}^*/K\bar{D}D^*$ system imply that a K^* meson around 4307 MeV should be observed in experimental investigation, while for the $ND\bar{D}^*$ system N^* states with $J^P = 1/2^+, 3/2^+$ and masses around $4400-4600$ MeV are predicted.

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1. A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Exotic hadrons with heavy flavors: X, Y, Z, and related states, *PTEP* **2016** (2016) 6, 062C01, https://doi.org/10.1093/ptep/ptw045.
2. E. Kelmpt and A. Zeitsev, Glueballs, Hybrids, Multiquarks. Experimental facts versus QCD inspired concepts. *Phys. Rept.* **454** (2007) 1-202, https://doi.org/10.1016/j.physrep.2007.07.006.
3. E. Oset *et al.*, Weak decays of the heavy hadrons into dynamically generated resonances. *Int. J. Mod. Phys.* **E25** (2016) 1630001, https://doi.org/10.1142/S0218301316300010.
4. R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-Quark QCD Exotica. *Prog. Part. Nucl. Phys.* **93** (2017) 143-194, https://doi.org/10.1016/j.ppnp.2016.11.003.
5. S. Lars Olsen, T. Skwarnicki, and D. Zieminska. Non-standard heavy mesons and baryons: Experimental evidence. *Rev. Mod. Phys.* **90** (2018) 1, 015003, https://doi.org/10.1103/RevModPhys.90.015003.
6. P. A. Zyla *et al.*, (Particle Data Group), *PTEP* **2020** (2020) 083C01, https://doi.org/10.1093/ptep/ptaa104.
7. M. Wang, Recent results on exotic hadrons at lhcb, (2020), *presented on behalf of the lhcb collaboration at implications workshop 2020*.
8. B. B. Malabarba, K. P. Khemchandani, and A. Martínez Torres, N^* states with hidden charm and a three-body nature. eprint = “2103.09978” (2021).
9. X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martinez Torres, K^* mesons with hidden charm arising from $KX(3872)$ and $KZ_c(3900)$ dynamics. *Phys. Lett. B* **785** (2018) 112-117,https://doi.org/10.1016/j.physletb.2018.08.034.
10. D. Gamermann, J. Nieves, and E. Oset, Couplings in coupled channels versus wave functions: application to the X(3872) resonance. *Phys. Rev. D* **81** (2010) 014029, https://doi.org/10.1103/PhysRevD.81.014029.
11. A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, Few-body systems consisting of mesons. *Few Body Syst.* **61** (2020) 4, 35, https://doi.org/10.1007/s00601-020-01568-y.
12. A. Martinez Torres, E. J. Garzon, E. Oset, and L. R. Dai, Limits to the Fixed Center Approximation to Faddeev equations: the case of the $\phi(2170)$. *Phys. Rev. D* **83** (2011) 116002, https://doi.org/10.1103/PhysRevD.83.116002.
13. W. H. Liang, C. W. Xiao, and E. Oset, Baryon states with open beauty in the extended local hidden gauge approach. *Phys. Rev. D* **89** (2014) 5, 054023, https://doi.org/10.1103/PhysRevD.89.054023.
14. W. H. Liang, T. Uchino, C. W. Xiao, and E. Oset, Baryon states with open charm in the extended local hidden gauge approach. *Eur. Phys. J. A* **51** (2015) 2, 16, https://doi.org/10.1140/epja/i2015-15016-1.
15. C. Garcia-Recio *et al.*, The s-wave charmed baryon resonances from a coupled-channel approach with heavy quark symmetry. *Phys. Rev. D*, **79** (2009) 054004, https://doi.org/10.1103/PhysRevD.79.054004.
16. , G. Burdman and J. F. Donoghue, Union of chiral and heavy quark symmetries. *Phys. Lett. B* **280** (1992) 287-291, https://doi.org/10.1016/0370-2693(92)90068-F.
17. S. Weinberg, Effective chiral Lagrangians for nucleon-pion interactions and nuclear forces. *Nucl. Phys. B* **363** (1991) 3-18, https://doi.org/10.1016/0550-3213(91)90231-L.
18. M. B. Wise, Chiral perturbation theory for hadrons containing a heavy quark. *Phys. Rev. D* **45** (1992) 7, R2188, https://doi.org/10.1103/PhysRevD.45.R2188.